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INVERSE PROBLEM OF SELECTION OF THE THEORETICAL CYCLE FOR THE REAL CYCLE OF INTERNAL COMBUSTION ENGINE

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Abstract

The effectiveness of work of an internal combustion engine can be assessed by means of the energy efficiency: theoretical, internal and effective... In the problem regarding the efficiency of obtaining a work from the tested SI engine, the theoretical Seiliger-Sabathe cycle was adopted as a reference model for the real engine cycle. For comparison, the OTTO cycle was also analysed. The engine indicating allows direct determination only of internal work. However, determining the work of the theoretical cycle first requires solving the problem of selecting the parameters of the theoretical cycle, according to the real cycle of the engine (inverse problem). In order uniquely to determine the course of the theoretical Seiliger-Sabathe cycle, it is necessary to determine the parameters of the starting point and the heat distribution number. The selection of the theoretical cycle for the real cycle, within the scope of determining the number of heat distribution, is to some extent of a contractual nature. Therefore, the problem of determining the number of heat distribution was solved by two own original methods. A comparison of the real cycle with the theoretical cycle determined for it is presented.

Keywords: *internal combustion engine, real cycle, theoretic cycle, heat distribution number, the reconciliation algorithm*

1. Effectiveness of obtaining a work from an internal combustion engine

The effectiveness of obtaining a work at the main stages of the energy conversion process in an internal combustion engine can be assessed by means of energy efficiencies [1-3]:

– efficiency of the theoretical cycle:

$$\eta_o = \frac{L_o}{m_p H_u}, \tag{1}$$

– internal efficiency:

$$\eta_i = \frac{L_i}{m_p H_u}, \tag{2}$$

– effective efficiency:

$$\eta_e = \frac{L_e}{m_p H_u}, \tag{3}$$

where:
L_o – work of the theoretical cycle,
m_p – mass of the fuel dose,
H_u – calorific value of fuel,
L_i – internal work,
L_e – effective work of an engine.

The perfection of thermodynamic processes in an internal combustion engine is assessed by means of the internal excellence ratio ξ_i [6]:

$$\xi_i = \frac{L_i}{L_o}. \tag{4}$$

The loss of work on mechanical friction and the work necessary to drive the auxiliary mechanisms of the engine is characterized by the mechanical excellence ratio ξ_m [4, 6]:

$$\xi_m = \frac{L_e}{L_i}. \tag{5}$$

The effective energy efficiency η_e of an internal combustion engine can therefore be expressed as the product of the efficiency of the theoretical cycle η_o and the excellent ratios:

$$\eta_e = \eta_o \, \xi_i \, \xi_m. \tag{6}$$

In order to determine the internal excellence ratio ξ_i of the engine, it is necessary to know the work of the theoretical cycle L_o and internal work L_i i.e. the work of the real cycle. Engine indicating makes it possible to determine directly only the internal work of L_i. However, to determine the degree of mechanical perfection, it is necessary to know the effective work of L_e. The effective work is determined on the basis of tests at the dynamometer station.

2. Selection of parameters of the theoretical cycle for the real cycle

Examination of the useful effects of the operation of an internal combustion engine requires the development of an algorithm for selecting the parameters of the adopted theoretical cycle according to the engine real cycle, determined experimentally based on indicator tests (reverse problem). The theoretical Seiliger-Sabathe cycle was adopted as a model of processes taking place in the tested SI engine. For comparison, the OTTO cycle was also analysed.

Determining the parameters of the starting point "1" and the heat distribution number Ψ it is necessary in order uniquely to determine the theoretical Seiliger-Sabathe cycle. The position of the point "1" of the cycle in the p-V coordinate system is determined by two coordinates: the volume V_1 and the pressure p_1. The volume V_1 is the maximum volume of the working space of the cylinder, resulting from the design parameters of the tested engine. The pressure p_1 is the pressure in the cylinder of the engine, averaged in the range of the extreme positions of the piston, during the intake stroke. This pressure is determined on the basis of the engine real cycle. A simple relation takes place between the pressure p_1 and the ambient pressure p_0 and the average pressure drop Δp_d of the fresh charge flow in the inlet system:

$$p_1 = p_0 - \Delta p_d. \tag{7}$$

The selection of the theoretical cycle for the real cycle, in the scope of determining the heat distribution number Ψ, has a contractual character to some extent. Therefore, this number was determined by two methods:

- the extremum method [7], which gives slightly, inflated values of the number Ψ,
- a developed method, conventionally named polytropic method [10], according to which the values of the number Ψ are underestimated.

Algorithms for determining the parameters necessary to calculate the heat distribution number Ψ are illustrated in Fig. 1.

2.1. Extremum method

The name of the method is related to the extreme of dependence of pressure on the volume $p = f(V)$. This dependence determines the real cycle of the engine (Fig. 1).

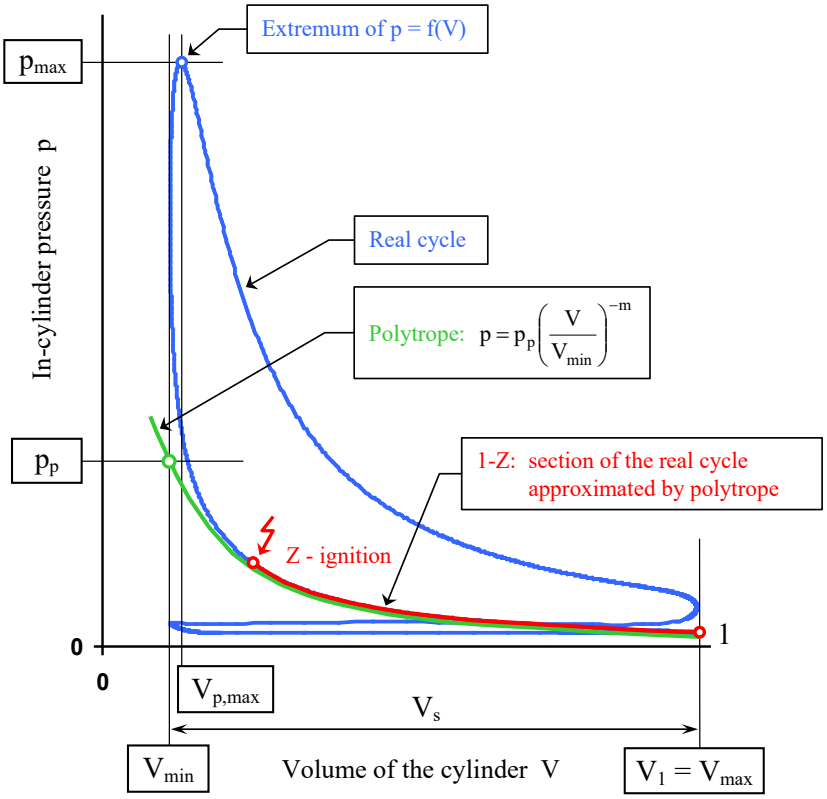

Fig. 1. Algorithms for determining the parameters necessary to calculate the heat distribution number Ψ according to the two methods used

The volume $V_{p,max}$, at which the pressure reaches the maximum value p_{max}, is used directly to determine the load parameter φ of the Seiliger-Sabathe theoretical cycle, assuming that:

$$\varphi = \frac{V_{p,max}}{V_{min}}. \tag{8}$$

Knowing the energy-stoichiometric parameter E, the load parameter γ is calculated according to the formula [6]:

$$\gamma = \frac{E(\kappa - 1) + \varepsilon^{(\kappa-1)}}{\varepsilon^{(\kappa-1)}\left[1 + \kappa(\varphi - 1)\right]}, \tag{9}$$

and then the heat distribution number Ψ is calculated:

$$\Psi = \frac{\gamma - 1}{\gamma - 1 + \kappa\gamma(\varphi - 1)}. \tag{10}$$

The above values allow determining the coordinates (p, V) of the characteristic points of the theoretical Seiliger-Sabathe cycle, selected for the experimentally determined real cycle.

2.2. Polytropic method

The essence of the proposed polytropic method is determination first the initial values of load parameters, assuming that:

– the initial value of the load parameter γ_0 is

$$\gamma_0 = \frac{p_{max}}{p_p}, \tag{11}$$

– and the initial value of the load parameter φ_0 is

$$\varphi_0 = \frac{V_{p,max}}{V_{min}}. \tag{12}$$

The parameters appearing in the above formulas are marked in Fig. 1. The algorithm for determining the pressure p_p is as follows:

– the section 1-Z, that is part of the compression stroke, is extracted from the real engine cycle (the mixture is ignited at point Z),
– measuring points, forming the section 1-Z, are approximated by the power function, which is a polytrope equation (hence the name of the method) in the form:

$$p(V) = p_p\left(\frac{V}{V_{min}}\right)^{-m}, \tag{13}$$

– assumption is made that if there was no ignition in the point "Z", the compression of the fresh charge would run further according to the designated polytrope, until the piston reaches top dead centre, i.e. the minimum volume V_{min}; the pressure p_p is therefore calculated from the polytropic equation (13) for the volume $V = V_{min}$:

$$p_p = p(V_{min}). \tag{14}$$

Knowing the stoichiometric-energy parameter E, the number Ψ of the heat distribution can be calculated from two independent formulas [10]:

$$\Psi = \frac{\varepsilon^{(\kappa-1)}(\gamma-1)}{E(\kappa-1)}, \tag{15}$$

or:

$$\Psi = \frac{E(\kappa-1) - \kappa\varepsilon^{(\kappa-1)}(\varphi-1)}{E(\kappa-1)[1+\kappa(\varphi-1)]}. \tag{16}$$

Two different values of the number Ψ will be obtained if the initial values of the parameters γ_0 and φ_0 are substituted for these formulas, because these are the values initially determined under assumptions (11) and (12). In order to avoid this ambiguity and obtain one value of the heat distribution number Ψ, it is proposed to use the reconciliation procedure of equations (15) and (16) [5].

In research on combustion engines, there are often situations when the number of calculated unknowns is smaller than the number of independent equations, binding these quantities with measurement data. Unknowns, determined directly from these equations, can be calculated in many ways, depending on the choice of a set of equations. The remaining equations will not be exactly met, which may be caused by simplifying assumptions as well as unavoidable measurement errors. Consequently, the same quantities will assume different values. In order to avoid these differences (to obtain one calculated value of the unknown) and to obtain compatibility of all equations, it is necessary to carry out the equation reconciliation procedure. The essence of this calculation is the correction of measurement data, after which the results of calculations of unknown quantities, determined from different sets of equations, will be the same.

Treating the problem in general, all independent equations, so-called equations of conditions, create a system of functions with the form [9]:

$$F_k = F_k(x_1,...,x_i,...,x_n \ , \ y_1,...,y_j,...,y_u), \qquad k = 1 \ldots r, \tag{17}$$

where:

x_i – measured or pre-determined quantity (i = 1 ... n),

n – number of measured quantities (basic components),
y_j – unknown (j = 1 ... u),
u – number of unknowns,
r – number of condition equations.

The equations of conditions that create the system (17) must satisfy the condition of mutual independence and the condition of determining the unknowns. Part of the equations of conditions does not meet after substituting the measurement results and approximate values of unknowns. The system is obtained:

$$F_k\left(x_{1,0} \ldots x_{i,0} \ldots x_{n,0} \;,\; y_{1,0} \ldots y_{j,0} \ldots y_{u,0}\right) = -w_k\,, \tag{18}$$

where:
$x_{i,0}$ – measurement result (or initial value) of i-th quantity (i = 1 ... n),
$y_{j,0}$ – approximate value of the j-th unknown (j = 1 ... u),
w_k – incompatibility of the k-th condition equation (k = 1 ... r).

Of course, the incompatibilities w_k of the equations of conditions, used for the approximate calculation of unknowns, are zero ($w_k = 0$). In order to obtain compatibility of all the equations of conditions, it is necessary to make corrections ν_i of the measurement results and corrections δ_j of approximate values of unknowns. The corrections are calculated from the system of equations [9]:

$$\begin{cases} \sum\limits_{i=1}^{n} a_{ki}\,\nu_i + \sum\limits_{j=1}^{u} b_{kj}\,\delta_j = w_k & \left(k = 1 \ldots r\right) \\ \dfrac{\nu_i}{m_i^2} = \sum\limits_{k=1}^{r} a_{ki}\,k_k & \left(i = 1 \ldots n\right) \\ \sum\limits_{k=1}^{r} b_{kj}\,k_k = 0 & \left(j = 1 \ldots u\right), \end{cases} \tag{19}$$

where:
k_k – coefficients at the determination of a conditional extreme (correlates),
m_i – mean absolute error of the measurement result of the i-th quantity.

The coefficients a_{ki} and b_{kj} of the above system of equations are partial derivatives of the function F_k, according to the measured quantities x_i and according to the unknown y_j respectively, calculated in the point with coordinates:

$$\left(x_{1,0} \ldots x_{i,0} \ldots x_{n,0} \;,\; y_{1,0} \ldots y_{j,0} \ldots y_{u,0}\right).$$

They are thus determined from dependence:

$$a_{ki} = \left(\frac{\partial F_k}{\partial x_i}\right)_0 \;, \qquad b_{kj} = \left(\frac{\partial F_k}{\partial y_j}\right)_0 . \tag{20}$$

For the considered problem, one unknown (heat distribution number Ψ) will occur in two independent equations (15) and (16), in the case of determination of the initial values of parameters γ_0 and φ_0 based on the real cycle. In this situation, it is possible to use the reconciliation procedure for these equations in order to obtain their compatibility [5]. These two dependencies constitute a system of condition equations (17).

The following designations were adopted:
ν_γ – correction of the initial value of the load parameter γ_0,
ν_φ – correction of the initial value of the load parameter φ_0,
δ – correction of the heat distribution number Ψ,
$m_\gamma = m_\varphi = 0.1$ – the average absolute error of the result of the initial calculation γ and φ.

Assuming the equation (15) for the approximate calculation of the value of the unknown heat distribution number (indicated as Ψ_0), the system of equations (19) allowing the calculation of corrections takes the form:

$$\begin{cases} -\varepsilon^{(\kappa-1)}\nu_\gamma + (\kappa-1)E\delta = 0 \\ \left[\kappa E(\kappa-1)\Psi_0 + \kappa\varepsilon^{(\kappa-1)}\right]\nu_\varphi + (\kappa-1)E\left[(\varphi_0-1)\kappa+1\right]\delta = w_2 \\ \dfrac{\nu_\gamma}{m_\gamma^2} = -\varepsilon^{(\kappa-1)}k_1 \\ \dfrac{\nu_\varphi}{m_\varphi^2} = \left[\kappa E(\kappa-1)\Psi_0 + \kappa\varepsilon^{(\kappa-1)}\right]k_2 \\ (\kappa-1)Ek_1 + \left[(\varphi_0-1)\kappa E(\kappa-1) + (\kappa-1)E\right]k_2 = 0 \end{cases} . \tag{21}$$

After solving the system of equations (21), taking into account the corrections received, the final corrected values of load parameters γ and φ are calculated:

$$\gamma = \gamma_0 + \nu_\gamma , \tag{22}$$

$$\varphi = \varphi_0 + \nu_\varphi , \tag{23}$$

as well as the heat distribution number Ψ:

$$\Psi = \Psi_0 + \delta . \tag{24}$$

The final value of the heat distribution number Ψ can now also be calculated from formulas (15) and (16), substituting the corrected values of parameters γ and φ for them.

Parameters of the theoretical Seiliger-Sabath cycle matched for the real cycle are used to analyse and assess the quantities characterizing the effectiveness of obtaining work from the internal combustion engine.

3. Summary – comparison between real and theoretical cycles

Comparison of the real cycle of the engine with the corresponding, theoretical cycles of Seiliger-Sabathe, determined by two methods for idling and full load (M_o = 59.6 Nm/rad), is illustrated in Fig. 2 and 3. For comparison, the figures also show a fragment of the OTTO cycle (maximum pressure range).

A characteristic feature of both theoretical cycles is a fairly large difference in their maximum pressures. For the extremum method, compared to the polytropic method, higher maximum pressure values are obtained, which is the effect of higher values of the heat distribution number Ψ. Increasing the number of heat distribution Ψ, at a fixed total amount of energy Q_d supplied to the cycle, is equivalent to increasing the amount of energy $Q_{d,V}$, supplied to the cycle with the isochoric process. At the same time, the amount of energy $Q_{d,p}$ supplied with the isobaric process decreases. At a constant volume of the medium, a small change in energy $Q_{d,V}$, causes a significant change in pressure at the end of the isochoric process. However, in energy aspect, the work values of theoretical cycles as well as the values of their efficiency are more important. In addition, these parameters, for both methods used, differ slightly.

The comparison of the theoretical cycles with the real cycle presented in Fig. 2 and 3 additionally leads to the conclusion that even for the SI engine, the more appropriate theoretical cycle is the Seiliger-Sabathe cycle, rather than the Otto cycle. This conclusion is justified by the

fact that the heat generation during combustion in the SI engine also takes place at the beginning of the expansion stroke. In addition, the highest maximum pressure values are obtained for the Otto cycle. For idling, the maximum pressure of the Otto cycle is almost five times higher than the real maximum pressure in the engine cylinder (Fig. 2), and for a full load (M_o = 59.6 Nm/rad) more than twice higher (Fig. 3). These differences when comparing the real and theoretical cycles of Seiliger-Sabathe are correspondingly smaller.

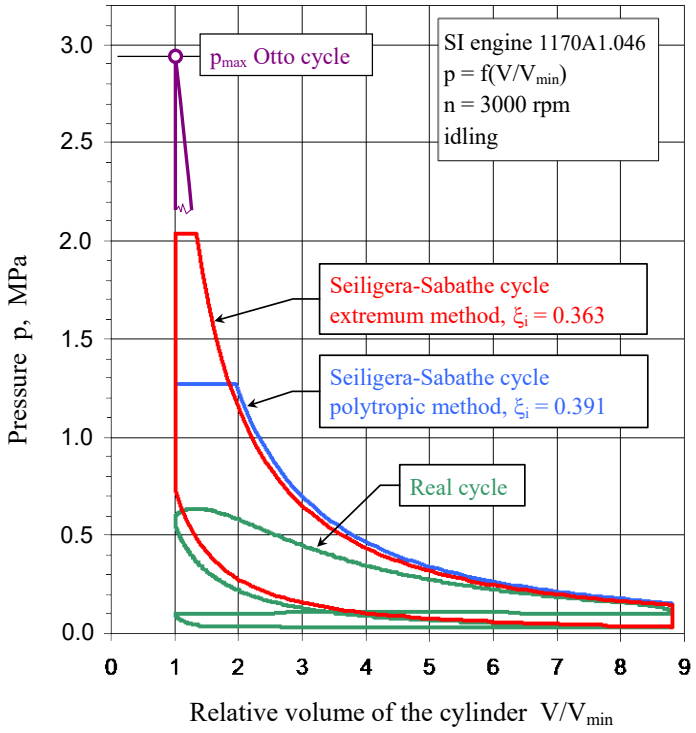

Fig. 2. Comparison of the real cycle with the corresponding Seiliger-Sabathe theoretical cycles for idling

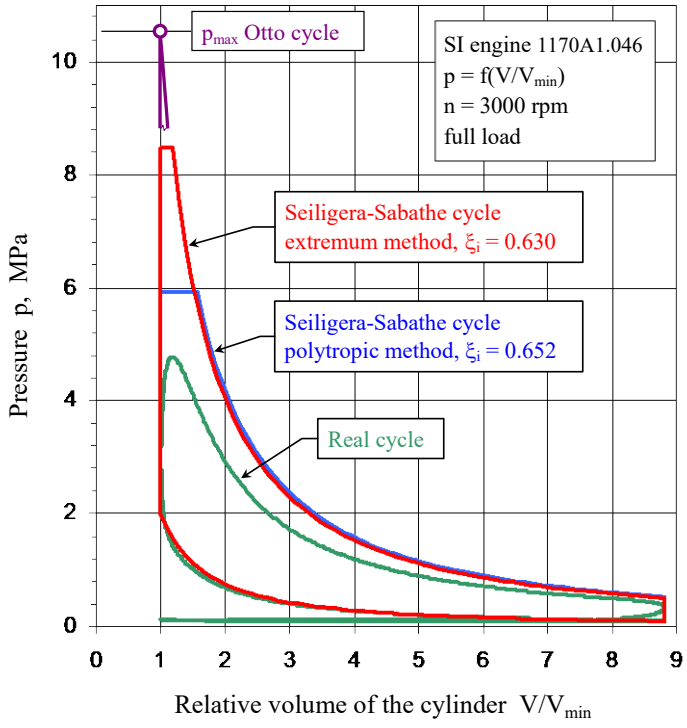

Fig. 3. Comparison of the real cycle with the corresponding Seiliger-Sabathe theoretical cycles for maximum torque

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