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MODELLING OF VANE AND ROTOR BLADE ROWS IN SIMULATIONS OF GAS TURBINE PERFORMANCE

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Abstract

A method of modelling of nozzle and rotor blade rows of gas turbine dedicated to simulations of gas turbine performance is proposed. The method is applicable especially in early design stage when many of geometric parameters are yet subject to change. The method is based on analytical formulas derived from considerations of flow theory and from cascade experiments. It involves determination of parameters of gas flow on the mean radius of blade rows. The blade row gas exit angle, determined in turbine design point is a basis for determination of details of blade contour behind the throat position. Throat area is then fixed based on required maximum mass flow in critical conditions. Blade leading edge radius is determined based on flow inlet angle to the blade row in the design point. The accuracy of analytical formulas applied for definition of blade contour details for assumed gas exit angle was verified by comparing the results of analytical formulas with CFD simulations for an airfoil cascade. Losses of enthalpy due to non-isentropic gas flow are evaluated using the analytical model of Craig and Cox, based on cascade experiments. Effects of blade cooling flows on losses of total pressure of the gas are determined based on analytical formulas applicable to film cooling with cooling streams blowing from discrete point along blade surface, including leading and trailing edges. The losses of total pressure due to film cooling of blades are incorporated into the Craig and Cox model as additional factor modifying gas flow velocities.

Keywords: *aircraft engines, mechanical engineering, engine parts, simulation and modelling*

1. Introduction

One-dimensional analysis is an effective approach in design of a turbine engine on early stage of its development. It allows for determination of the dimensions of gas flow path in a turbine, determination of distribution of gas flow velocities, temperatures and flow angles along the mean line of flow path. In addition, the first approximation of geometry of blade rows, involving number of blades, stagger angles, blade axial chord may be obtained from this analysis. At this stage of the design, coordinates of blade profiles may be left undefined. Changes of gas flow variables behind vane and rotor blade rows, including gas path angles, temperature, and pressure can be described as functions of a single variable, degree of reaction, which may be defined as ratio of static enthalpy drop on the rotor to static enthalpy drop in the turbine stage [1]. A computational method, involving one-dimensional analysis of gas flow along the medium radius of the flow path may be applied not only to preliminary design of a turbine, but also to simulation of performance of a turbine in conditions other than its design point. An important problem to solve in analysis of off-design conditions is determination of gas exit angle from a blade row (vane or rotor row), which is a function of geometry of the blade row and flow variables. For this purpose, blade and blade row geometry must be defined in sufficient detail. A review of methods of determination of gas exit angle from a blade row is presented in [2]. It can be seen in this review, that all mentioned methods require definition of blade pitch and throat opening in the design conditions. Some of these methods require definition more geometric details, including radius of blade curvature on the convex side and trailing edge thickness. Losses of enthalpy due to non-isentropic flow depend on the geometry of the blade row as well. For the presented work, the Craig and Cox loss model [3] was selected and extended in order to account for blade film cooling. This model requires knowledge of gas flow angles and basic geometric details of blades and blade rows, (pitch, throat opening, backbone length, trailing edge thickness). It uses also aggregated geometric data (contraction ratio) which may be approximated based on blade statistics. This approach is particularly useful at early stages of the design, as it allows for some flexibility in definition of blade contour. It is convenient also in estimating performance of modified variants of some baseline engine.

Another important issue present in analysis of turbine design and off-design conditions is blade cooling. Blade cooling, involving outflow of cool gas from surface of blades including leading edge cooling, film cooling and transpiration cooling has a strong effect on enthalpy losses, due to mixing of the cooling streams with the hot gas, producing loss of total pressure. Solutions for modelling of blade rows addressing problems mentioned above are described in the following points of this article.

2. Determination of gas exit angle from a blade row

For the determination of blade row exit angle, the formulas proposed by Aronov [4] were selected because of the continuity of the analytical solution while passing critical flow conditions within the blade row. The gas exit angle, shown as β_2 in Fig. 1 is dependent on geometric angles β_k, β_s, β_c, ε and throat opening b.

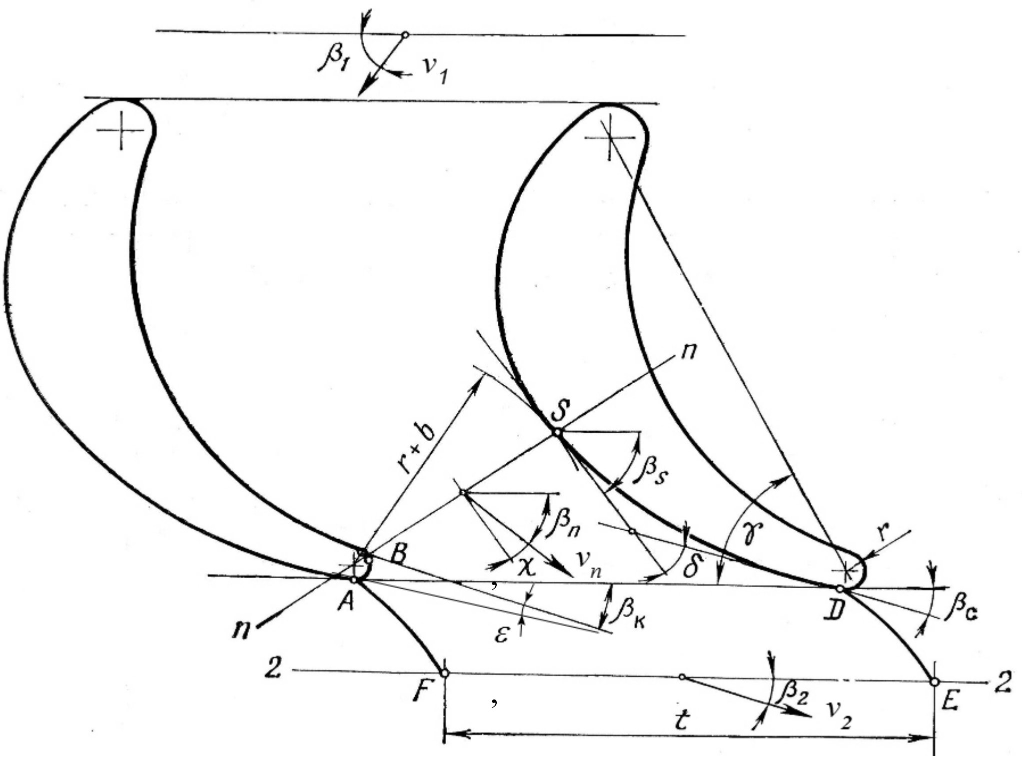

Fig. 1. Design angles of a blade row in its exit region, defined for a turbine design point [4]

These geometric data must be fixed for a turbine design point. The relations between these angles are as follows:

$$\beta_n = 0.5(\beta_s + \beta_k), \tag{1}$$

$$\chi = \beta_s - \beta_n = \beta_s - 0.5\beta_s - 0.5\beta_k = 0.5(\beta_s - \beta_k). \tag{2}$$

In the present method it has been assumed, that three angles: δ, β_k and ε (angle between tangents to blade trailing edge) were fixed in the design point. All the other angles have been determined based on the values of the three angles mentioned. The angle δ must not exceed some critical value above which flow separation occurs in the convex side of the blade. In the present method it has been arbitrarily assumed that $\delta<=18°$, $3°<=\varepsilon<=5°$ and β_k has been determined in a procedure safeguarding the assumed gas exit angle β_2 in design point. The procedure applied formulas presented in [4]:

– for the conditions of subcritical flow in the throat area:

$$\beta_2 = arc\ sin\left[\frac{b}{t}cos\chi\frac{y\left(\lambda_{02}\psi_{pr}\frac{cos\beta_2}{cos\beta_n}\right)\pi\left(\lambda_{02}\frac{\psi_{pr}cos\beta_2}{\psi_{1-n}cos\beta_n}\right)}{y(\lambda_{02}\psi_{pr})\pi(\lambda_{02})}\right], \tag{3}$$

– in critical and supercritical flow conditions at the exit of blade row:

$$\beta_2 = arc\ sin\left[\frac{b}{t}cos\chi\frac{y(1)\pi\left(\frac{1}{\psi_{1-n}}\right)}{y(\lambda_{02}\psi_{pr})\pi(\lambda_{02})}\right]. \tag{4}$$

In formulas (3) and (4) λ_{02} is a value of non-dimensional, isentropic velocity λ in the cross-section "2-2" of Fig. 1. The non-dimensional velocity λ is defined as ratio of actual velocity v to sound velocity in critical-flow conditions, at Mach number equal to unity:

$$\lambda = \frac{v}{\sqrt{\frac{2k}{k+1}RT_t}}, \tag{5}$$

where k is the ratio of specific heats: $k=c_p/c_v$, R is gas constant, T_t is gas total temperature.

The reason for two equations for β_2 angle is that when exit gas velocity passes from subcritical values to supercritical ones, the non-dimensional velocity λ in the throat cross-section remains equal to unity.

The functions $\pi(\lambda)$ and $y(\lambda)$ are gas-dynamic functions of non-dimensional velocity [5]. $\pi(\lambda)$ is the ratio of static pressure to total pressure, and $y(\lambda)$ is a ratio of non-dimensional mass flow to $\pi(\lambda)$ and is evaluated as:

$$y(\lambda) = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\frac{\lambda}{1-\frac{k-1}{k+1}\lambda^2}. \tag{6}$$

The symbols ψ_{pr} and ψ_{1-n} in formulas (3) and (4) stand for enthalpy loss coefficients across the blade row and for the fragment of a blade row from inlet to the throat opening. ψ_{1-n} is a function of thickness of blade trailing edge. For more details of the evaluation of the ψ_{pr} and ψ_{1-n} coefficients, the reader is referred to [4].

Formulas (3) and (4) were derived from the mass and momentum conservation equations, using assumptions of zero tangential stress along the S-D fragment of blade contour, as shown in Fig. 1. Accuracy of these formulas has been investigated computationally for a cascade of C3X profiles, designed for work as first-stage vanes [6]. For this purpose, a CFD model of the cascade has been prepared, as shown in Fig. 3. The boundary values of total pressure and total temperature at inlet to cascade have been fixed, while static pressure in the outlet plane has been sequentially reduced, in order to obtain different exit flow velocities and exit angles. The flow solution has been obtained with Fluent solver, using four-equation Transition SST turbulence model. The exit flow angle has been computed from components of exit velocity in the pressure-outlet surface, as area-weighted average over the exit plane and compared with the analytical solution of equations (3) and (4). The results of these computations are shown in Fig. 3. It can be seen that differences between the numerical and analytical solutions for β_2 angle are within 1 degree.

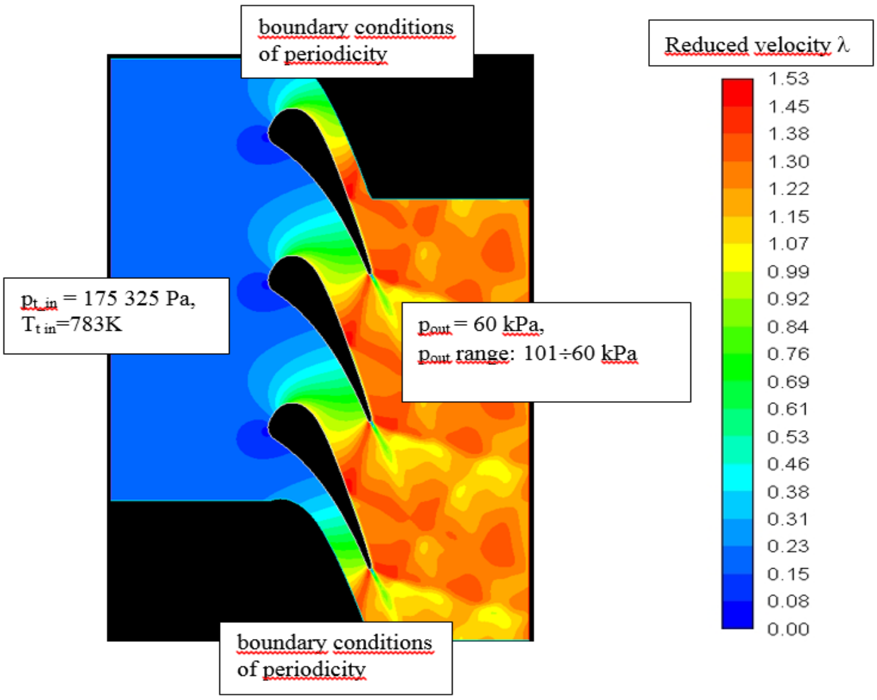

Fig. 2. Boundary conditions applied in CFD analysis and contour of reduced velocity λ in flow solution with supercritical exit conditions

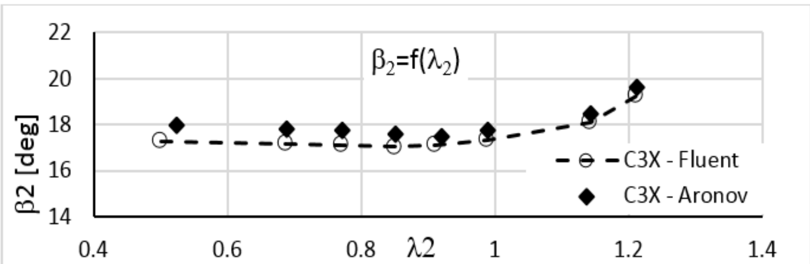

Fig. 3. Comparison of gas exit angle β_2 obtained in the numerical solution of viscous flow with results of analytical formulas for different values of reduced velocity λ_2

3. Accounting for the effects of blade cooling streams on the gas in the flow path

Blade cooling streams blowing from orifices on blade surface have two effects on the main flow: the first one is decrease of the total and static temperature of the main flow and the second one is increase of the enthalpy loss due to mixing with the main flow, through losses in gas total pressure. The first effect has been evaluated in accordance to Mattingly et al. [7]. The total temperature of the mixed flow is a weighted average of the hot-gas and cool-gas total temperatures, with the weights being the mass flows of both flows. The second effect is equally significant. According to [8] the loss in airfoil efficiency as an effect of total pressure loss due to mixing of mainstream and coolant flows may exceed losses due to factors present in non-isentropic flow around uncooled blades. This is illustrated in Fig. 4 for increasing ratio of coolant flow to mass flow. The loss in total pressure due to mixing of the cooling stream blowing from a surface orifice with the mainstream flow is evaluated by formula proposed by Hartsel [8]:

$$\frac{\Delta P_{tg}}{P_{tg}} = \frac{k}{2} M^2 \frac{W_c}{W_g}\left[1 + \frac{T_{ce}}{T_{tg}} - 2\frac{U_c}{U_g} cos\beta\right], \tag{7}$$

where:

P_{tg} – mainstream total pressure,

ΔP_{tg} – mainstream total pressure loss due to particular cooling stream,
k – ratio of heat constants,
M – Mach number in the mixing zone,
W_c – cooling mass flow from particular orifice (or row of orifices at similar chord location and outflow conditions),
W_g – part of main stream mass flow subject to mixing with coolant flow (estimated, acc. to [8]),
T_{ce} – coolant total temperature at exit of orifice,
T_{tg} – mainstream gas total temperature,
U_c – velocity of coolant flow,
U_g – velocity of mainstream flow,
β – angle of coolant stream with respect to the mainstream.

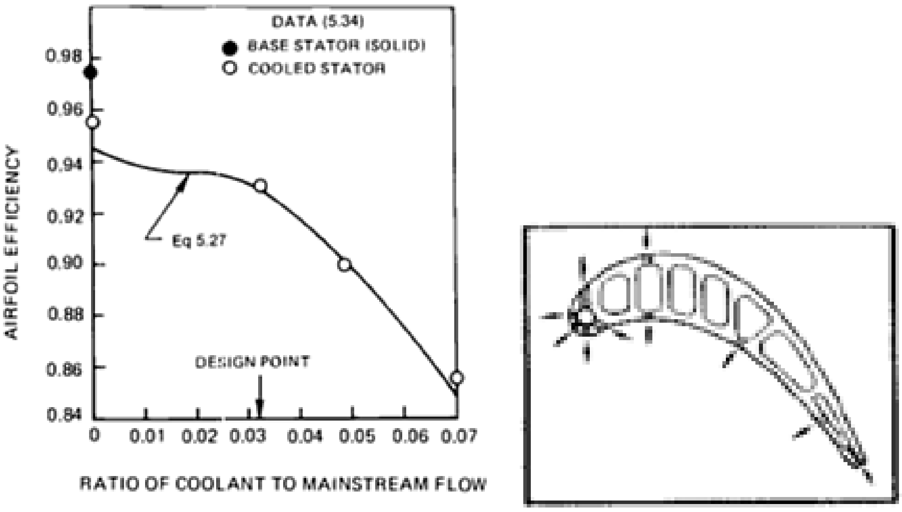

Fig. 4. Decrease of airfoil efficiency due to streams emanating from blade surface according to [8]. The solid line shows results of eq. (7)

In the next step, the total pressure losses have been converted into mainstream enthalpy losses. For each coolant outflow with parameters (W_c, T_{ce}, U_c, β,) producing total pressure loss Δp_0 the enthalpy loss has been evaluated using the formula:

$$\Delta H = \frac{1}{2}V_{2\,is}^2 - \frac{1}{2}V_2^2 = c_p T_{t1}\left[\left(\frac{p_2}{p_{t2}}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_{t1}}\right)^{\frac{k-1}{k}}\right], \tag{8}$$

where V_2 is isentropic velocity of the main stream in the vicinity of the coolant outflow, pt1 and Tt1 are total pressure, total temperature at inlet to the blade row, pt2 is the total pressure including losses due to cooling at the orifice exit, and p_2 is the static pressure at the orifice exit. In evaluation of local flow conditions within the blade row an assumption of linear change of pressure along axis of symmetry has been used. This simplifying assumption allows for application of equations (7) and (8) without information on blade coordinates. It has also been assumed, that cooling streams influence the gas exit angle only through the change of the exit velocity from blade row. This has yet to be verified by CFD modelling.

The losses of enthalpy evaluated with eq. (8) have been summed up with losses of enthalpy due to other factors accounted for in the Craig and Cox model and are converted to mainstream velocity loss coefficients:

$$\psi = \frac{V}{V_{is}} = \sqrt{1 - 2\frac{\sum \Delta H}{V_{is}^2}}. \tag{9}$$

4. Algorithm for simulation of turbine performance

The formulas for gas exit angle from a blade row and for enthalpy losses accounting for blade cooling have been implemented in an algorithm for determination of gas parameters and extracted work in simulations of turbine performance, which is presented in Fig. 5. The algorithm has been implemented in the Excel environment in functions written in the VBA language. Input to the computational procedure consists of the assumed gas parameters at the turbine stage inlet as well as of set of parameters representing flow path and blade rows: axial chords, blade pitch in the rows, throat opening, contraction ratios, stagger angles, contour angles at the trailing edge. The algorithm is a part of a larger system, which when operating in the turbine design mode allows for determination of geometry of gas flow path and blade angles shown in Fig. 1.

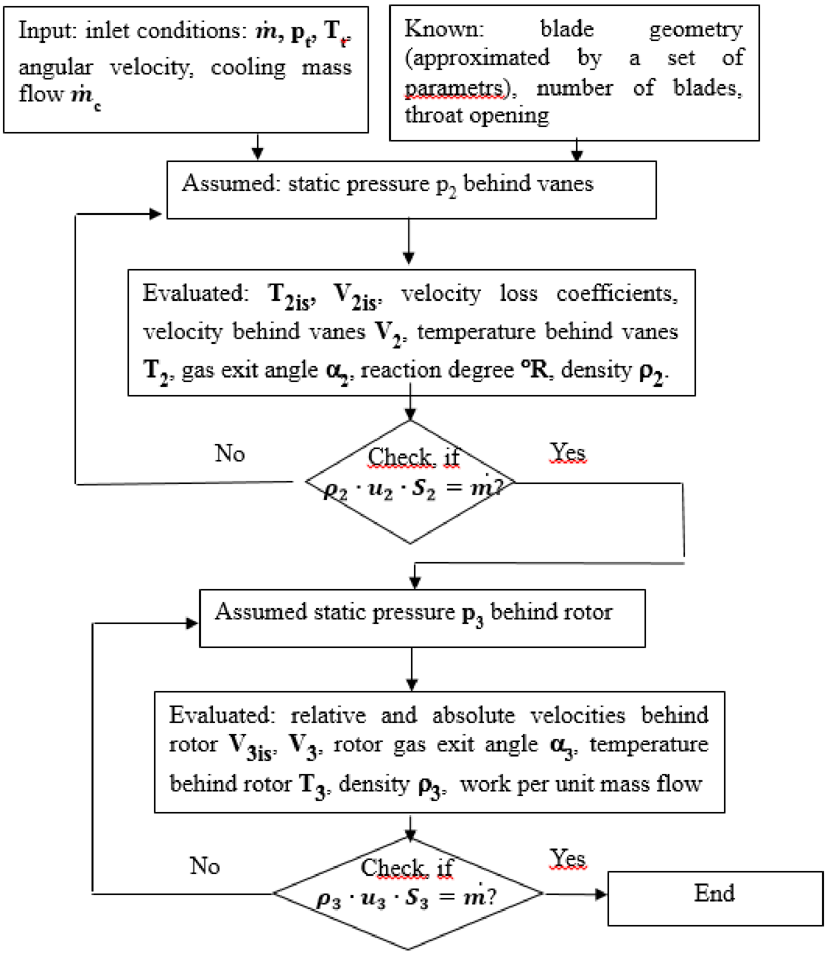

Fig. 5. General algorithm for determination of gas flow parameters in computations of turbine performance

For each of the blade rows a two-step procedure is used, where in the first step isentropic-change parameters are evaluated behind each row ($T_{2\,is}$, $V_{2\,is}$ behind vanes and $T_{3\,is}$, $V_{3\,is}$ behind the rotor).

$$T_{2is} = T_t \left(\frac{p_2}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}, \tag{10}$$

$$V_{2i} = \sqrt{2c_p(T_{02} - T_{2i})}. \tag{11}$$

Next, the viscous-flow temperature and velocity is obtained, using velocity loss coefficient ψ, evaluated with the classical Craig and Cox model [3], extended by accounting for effects of coolant outflow from blade surface.

$$V_2 = V_{2i} \cdot \psi, \tag{12}$$

$$T_2 = T_{01} - \frac{V_2^2}{2c_p}. \tag{13}$$

The exit velocities and blade-row exit flow angle β_2 is evaluated in a reference system connected with the blade row: stationary system for vane row, and relative system for rotor row. In case of rotor row of different inlet and exit diameter the inertia forces cause additional change of exit velocity, evaluated in the relative, rotational system:

$$\Delta V_{rot} = \omega^2 (r_3^2 - r_2^2), \tag{14}$$

where ω is angular velocity, r_2 is rotor inlet diameter, r_3 is rotor exit diameter.

Work per unit mass of gas is evaluated using Euler equation [7]:

$$L = \omega(r_2 v_2 + r_3 v_3). \tag{15}$$

The presented method has been applied for determination of gas parameters and work extracted from a test case of turbine stage with flow path geometry shown in Fig. 6, for which another solution, obtained with NPSS software [9] was available. Cooling mass flow was equal to 16% of the total stage mass flow. The gas conditions at the inlet to the stage represent an off-design point of the engine, at high-altitude end-of-climb conditions. The geometric parameters of the rotor blades were fixed earlier, using a design-point version of the algorithm presented in Fig. 5. The design version of the simulation algorithm uses degree of reaction as variable in the iteration procedure in order to obtain gas angles at the inlet and exit of the blade row, which are the basis for determination blade angles shown in Fig. 1. The results of the analysis for the off-design work point with the algorithm presented in Fig. 5. are presented in Tab. 1.

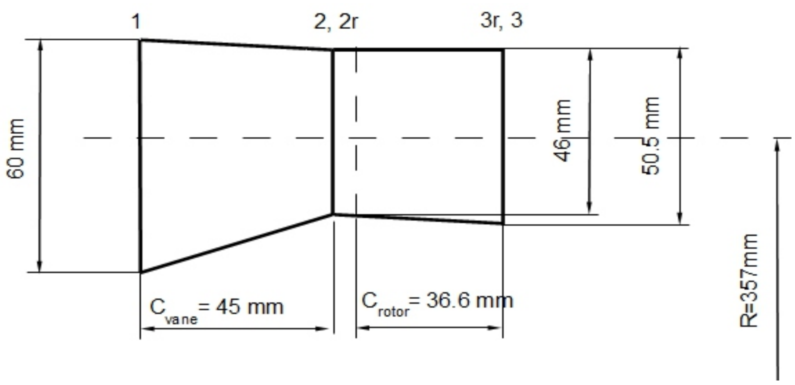

Fig. 6. Flow-path dimensions of the test turbine stage

Tab. 1. Comparison results obtained with presented method with results of NPSS for a test turbine stage

Computational stations:	1	2	2r	3r	3	difference in exit values w.r.t. NPSS results
pt [Pa]	1424539	1384867	965595	828705	397287	1.21%
pt [Pa] (NPSS)	1424539				396731	
Tt [K]	1674.57	1592.76	1460.81	1460.81	1224.68	0.14%
Tt [K] (NPSS)	1674.57				1210.01	
exit angle w.r.p. to symmetry axis [°]	0	78.80	-4.36	71.59	38.93	
V [m/s]		573.38	113.47	822.68	333.94	
Useful stage work [J/kg]					442386.04	-4.49%
NPSS stage work [J/kg]					463188.59	

The numbers 1, 2, 3 in Fig. 6 and Tab. 1 designate numbers of computational stations: inlet to vane row, exit from vane row, exit from stage, where gas parameters are evaluated in the global coordinate system. Numbers 2r and 3r designate inlet to and exit from rotor row, where gas parameters are evaluated in rotational reference system. The useful stage work accounts for efficiency losses evaluated according to Craig and Cox model. As far as accuracy of the results is concerned, the exit total pressure and total temperature were given explicit in the results of the NPSS software, while stage work in the NPSS results was evaluated from shaft power and overall efficiency. Higher error in stage work than in changes of gas parameters obtained with the present method indicates that stage efficiency in the present method has yet to be calibrated.

5. Conclusions

A method of modelling of turbine blade rows for evaluation of gas-turbine performance, accounting for effects of blade cooling, based on one-dimensional analysis of gas flow on the mean radius has been presented. Blade rows geometry is represented by set of parameters, which include the radii, axial chord, stagger, contraction ratio, trailing-edge radius, and contour angles in the throat and trailing edge region. These geometric parameters, along with inflow data are input to analytical formulas for evaluation of blade-row gas exit angles and gas flow velocities accounting for losses of enthalpy due to non-isentropic flow. The model of a blade row is extended by parameters describing outflow of cooling gas from orifices in blade surface: W_c, T_{ce}, U_c, β. For determination of losses of enthalpy of gas, a classical loss mode of Craig and Cox [3] was extended to account for losses caused by film cooling of vane and rotor blades. Comparison of the results of formulas for blade-row gas exit angle with numerical solution of viscous flow and comparison of results for a turbine test stage with results of validated simulation software prove viability of the presented approach.

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