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THE APPLICATION OF ANT ALGORITHM IN THE ASSIGNMENT PROBLEM OF AIRCRAFTS TO STOPS POINTS ON THE APRON

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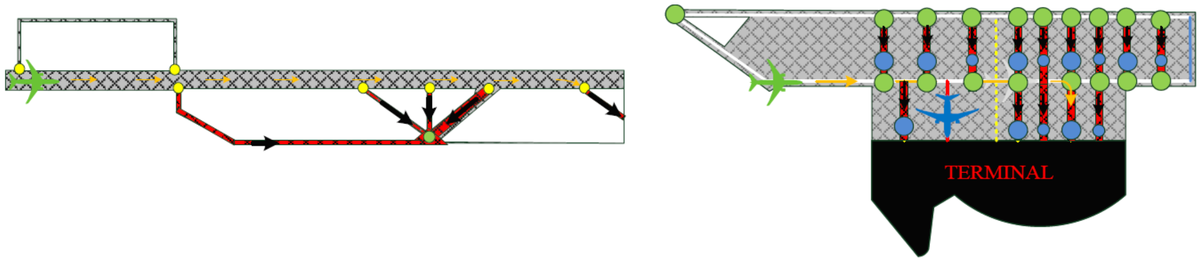
Abstract

The article refers to the problem of assigning the aircrafts to stops points on the apron. The aim of this paper is to develop the algorithm, which will be used to this assignment. The assignment problem of aircrafts to stops points is the complex decision problems, which refer to the problems of designating the minimal path in the graph. The assignment model was described, i.e. decision variables, constraints and the criterion function. Decision variables take the binary form and determine the connections between the elements of infrastructure of the apron, i.e. touchdown points, intermediate points and stop points. Constraints take into account the number of aircrafts in the given period, the number of unoccupied stop points. The criterion function determines the minimum driving time of the aircraft on the apron. In order to designate the routes in international transport the heuristic algorithm, i.e. ant algorithm was developed. The steps of building this algorithm were presented. This algorithm was verified in the C# programming language. The results generating by the presented algorithm were compared with the results generating by the random algorithm.

Keywords: *air transportation, air traffic control, aircraft operations, airport*

1. Introduction

One of the main problems in modelling airport operations is to assign the aircrafts to stops points on the apron after their landing. The assignment of aircrafts to stops points should be made in fast time; it means that in operations realized in the airport the main aim is to minimize the time from the moment of landing of the aircraft to the moment of stopping it. The assignment of aircrafts to stops points in this paper is interpreted as finding the best connection (the route) between the point in which the aircraft exits the runway (touchdown points) and the ending point in which the aircraft has a stop on the apron. It is problematic to indicate the first touchdown point for the aircraft after landing, intermediate points, which lead to the stop point. There are many stop point on the apron. One can assign the aircraft to the stop point in such a way that the route resulting from this assignment was as small as possible. The graphical presentation of touchdown points on the runway (the yellow point), and the intermediate points (green points) was shown on Fig. 1a, stop points (blue points) on Fig. 1b.



a) b)

Fig. 1. The elements of the infrastructure of the apron [15].

The aim of this paper is to develop the algorithm, which will be used to assign the aircrafts to stops points on the apron. The presented assignment problem is the decision problem, which refer to the problems of designating the minimal path in the graph. Taking into account the fact that the infrastructure of the apron can be presented in the form of a graph, determining the route of the aircraft running from the touchdown points to the stop points requires the determination of the minimum path in this graph [12], [13],[16], [22], [23], [14]. The classic problem of determining the minimum path in the graph is solved by the Dijkstry's, Bellman-Ford's, A*, Floyd-Warshall's, Johnson's algorithms. Taking into account the complexity of the presented issue, there is a need to develop a new algorithm adequate to the presented problem of determining the routes of aircrafts on the apron. In this paper, the assignment problem was designated by the use of an ant algorithm. The authors did not find the application of this approach in this type of the assignment. Fast time of generating the result by this algorithm is its main feature, what is desired in the process of assigning the aircrafts to stops points on the apron.

This process depends on many factors, e.g., number of aircrafts [27]. The algorithm for designating this type of problem must be adapted to frequent changes of these factors and generate the solutions in a quick way. On the airport, the time of generating the solution by the algorithm plays the most important role. The ant algorithm generates the results in a quick way and therefore this algorithm was selected in this problem. Although the ant algorithms, which belong to a group of heuristic algorithms, do not guarantee the optimal solution, they are enabled to close on the optimal solutions so-called suboptimal ones. Despite this inconvenience, the genetic algorithms are a practical tool for optimization and they are used in a variety of complex decision making problems, e.g., vehicles routing problems [17], [10],[9], [8], [7].

The presented assignment problem is a new approach to the allocation of aircrafts to stops points located on the apron and is different from classical assignment problem. The classical assignment problem is a well-known issue, widely described in the literature [3], [4], [5]. The main aim of this problem is to assign the vehicles and the people to the tasks according to a given criterion function, e.g., minimal transportation cost or completion time. The number of the vehicles, the people, and the tasks are known. This problem can be modified and take different forms, e.g., the number of the vehicles is equal to the number of the tasks, or the number of the vehicles is smaller or bigger than the number of the tasks. The assignment is considered in a variety of situations like the assignment of the gates at vehicle transhipment terminals [6] and setting up class schedules [1], [23], [25].

In air transport, the application of the assignment issue for determining flight plans, crew allocation to a given flight or the type of an aircraft for a given route is provided in [11], [20], [21]. The problem is to set flight plan schedules so that all the routes are completed in the minimum time.

The assignment problem in literature is solved by various algorithms simplex algorithm [2], Hungarian method [19], heuristics algorithms, e.g., genetic algorithms [24], [18].

The nature of the presented assignment referring to designating the minimal path in the graph underlines the fact that the available allocation algorithms cannot be applied in the analysed problem. The problems of the assignment in literature take a different form depending on the subject analysed.

2. The main assumption of the assignment model

The assignment model may be presented in the following form:

$$\boldsymbol{AM} = \langle \boldsymbol{G}, \boldsymbol{Q}, \boldsymbol{F}, \boldsymbol{O} \rangle,$$

where:

$\boldsymbol{G}$ – the graph of the infrastructure of the apron,

$\boldsymbol{Q}$ – the number of aircraft on the apron in a given time period,

$\boldsymbol{F}$ – the set of characteristics of structure elements,

$\boldsymbol{O}$ – the organization is understood as the assignment the aircrafts to stop points.

The infrastructure of the apron presents connections between the highlighted taxiway points, such as touchdown points, intersection points of taxiways (so-called intermediate points) and service points for airplanes (stop points). The number of aircraft on the apron in a given time period is interpreted as the number of aircraft landing and taking off in the given period. The set of characteristics of structure elements determines, e.g. distance between point elements of the infrastructure. The organization is understood as the assignment the aircrafts to stop points.

In order to determine the distance between the individual elements of the graph, the following connections were introduced:

- the connections between touchdown points (**TP**) and intermediate points (IP);
- the connections between intermediate points (IP);
- the connections between intermediate points (IP) and stop points (SP).

The graphical presentation of all elements of the graph and the routes of the aircraft was presented in Fig. 2.

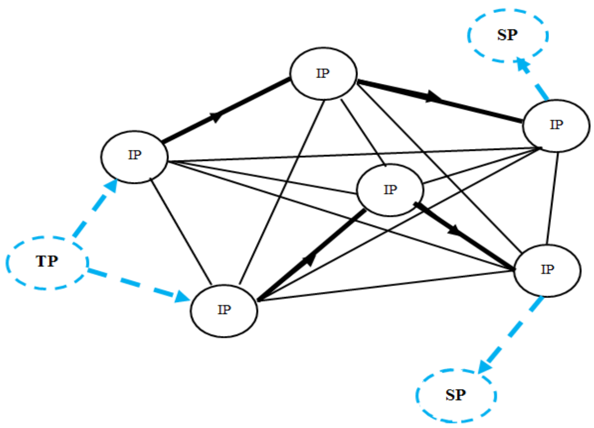


Fig. 2 The graph of the infrastructure of the apron

In order to model the assignment problem of aircrafts to stops points on the apron one can distinguish the additional data input:

- Travel time of the aircraft between the elements of the infrastructure of the apron;
- Time of landing of aircraft;
- Delays in intermediate points resulting from the priority of passage of aircraft;
- The number of unoccupied stop points.

The decision variables take the following forms:

- The variable which determines the connections between touchdown points (**TP**) and intermediate points (IP) in a given time period;
- The variable which determines the connections between intermediate points (IP) and intermediate points (IP) in a given time period;
- The variable, which determines the connections between intermediate points (IP) and stop points (SP).

All of the variables shown are binary variables; the value 1 determines the connection between points, otherwise 0.

The criterion function considering defined decision variables minimizes the total time of landing of the aircraft on the apron. This time depends on the minimum distance from the touchdown point to the stop point.

Limitations result from the number of aircraft subject to the process of allocating them to stop points, the number of stop points, the number of touchdown points, the number of intermediate points.

3. The ant algorithm for the assignment problem

Theory of ant algorithms introduces the concept of artificial ants. Each ant builds its own route i.e. an ant visits touchdown points, goes to the intermediate points. The endpoint of the ant route is the selected stop point.

At first the data input needs to be determined: the set of ants was defined as ***MR***={1,…,*mr*,…MR}, the number of iteration of the algorithm ***I***={1,…,*i*,…I}, the set of aircrafts ***AR***={1,…,*ar*,…MR}. The starting point of each route is in touchdown points. The ending point is in the stop points.

The transition probability on the basis, which the ant goes to another point in the route, takes the following form:

$$PR^{mr}{}_{yz}(t)=\begin{cases}\dfrac{[\tau_{yz}(t)]^{\alpha}\cdot[\eta_{yz}(t)]^{\beta}}{\sum_{l\in\Omega^{mr}}[\tau_{yl}(t)]^{\alpha}\cdot[\eta_{yl}(t)]^{\beta}}, & z\in\Omega^{mr},\\ 0, & z\notin\Omega^{mr},\end{cases} \tag{1}$$

where:

$\tau_{yz}(t)$ – the intensity of pheromone trail between the *y* -the a *z* -the point in *t* -the iteration;

$\eta_{yz}(t)$ – the heuristic information, e.g. $\eta_{yz}(t) = 1/w(y,z)$, where $w(y,z)$ − the travel time between *y* − the point of route and *z* -the point of the route taking into account the delays in intermediate points;

α, β – parameters determining the effect of pheromones and the heuristic information on the behavior of ants;

Ω^{mr} – the set of unvisited points *l* in the route of the ant.

Random choice of the route between the point *y* and point *l* begins from calculating the probability of the transition to the points according to the pattern (1). The next step is to calculate the distribution for each transition path and draw the number *r* from the range [0,1]. The route *tr* about the value of the distribution q_{tr} which fulfils the condition $q_{tr-1} < r \leq q_{tr}$ is selected, where tr is the number of the route between y – the point and l – the point.

After the realization of routes by all ants, the pheromone update must be made. Three types of the pheromone update can be distinguished: ant – density, ant – quantity, ant – cycle. In order to update the pheromone the ant – cycle was used as the most efficient version of the ant algorithm [8], [7]. At the beginning, it is assumed that the trail on the links between the points is equally strong. In subsequent iterations, the pheromone trail is calculated according to the formula:

$$\tau_{\mathbf{yz}}(t+1) = (1-\rho)\cdot\tau_{\mathbf{yz}}(t) + \sum_{mr=1}^{\mathrm{MR}} \Delta\tau_{\mathbf{yz}}{}^{mr}(t), \tag{2}$$

where

mr – another ant $mr \in \boldsymbol{MR}$ $mr \in \boldsymbol{MR}$, ρ – a factor pheromone ($0 < \rho \leq 1$),

$\tau_{yz}(t+1)$ – the strengthening of the pheromone, for the first iteration this strengthening takes the value τ_0 for each connections,

$$\Delta\tau_{yz}{}^{mr}(t) = \begin{cases} \dfrac{1}{L^{mr}(t)}, & \text{when the route } (y,z) \text{ was used by } mr \text{ – this ant,} \\ 0\ , & \text{otherwise,} \end{cases} \tag{3}$$

where:

$L^{mr}(t)$ – the time of the route in t iteration realized by mr – this ant, if the segment of routes (y,z) was realized by mr − this ant then $\Delta\ \tau_{yz}{}^{mr}(t)$ equals $1/\ L^{mr}(t)$, otherwise 0.

The graphical presentation of the algorithm was presented on the Fig. 3.

The steps of the assignment algorithm may be described as:

- Step 1. Introduction of input data regarding the algorithm and infrastructure.
- Step 2. Select the first ant from the population and the first iteration.
- Step 3. Determining the number of free stop points in a given time period. This step is updated for each aircraft.
- Step. 4 Select at random way the touchdown point.
- Step. 5 Calculate the transition probability $PR^{mr}{}_{yz}(t)$.
- Step. 6 The ant goes to the point z.
- Step. 7 Repeat the Step 5 if the point z is not the stop point.
- Step. 8 Select another ant if the condition referring to the number of ants in population is met. The algorithm goes to the step 4.
- Step. 9 Update of pheromone if the condition referring to the number of ants in population is not met.
- Step. 10 The algorithm goes to the next iteration if the condition referring to the number of iterations is met. The algorithm goes to the step 4.
- Step. 11 Select the minimum route from the population if the condition referring to the number of iterations is met. The minimum route is the solution of this problem. On the basis of this route the stop point is designated.
- Step. 12 The assignment for another aircraft if the condition referring to the number of aircrafts is met, if the condition referring to the number of aircrafts is not met the algorithm finishes work.
- Step. 13 The algorithm goes to the Step 2.

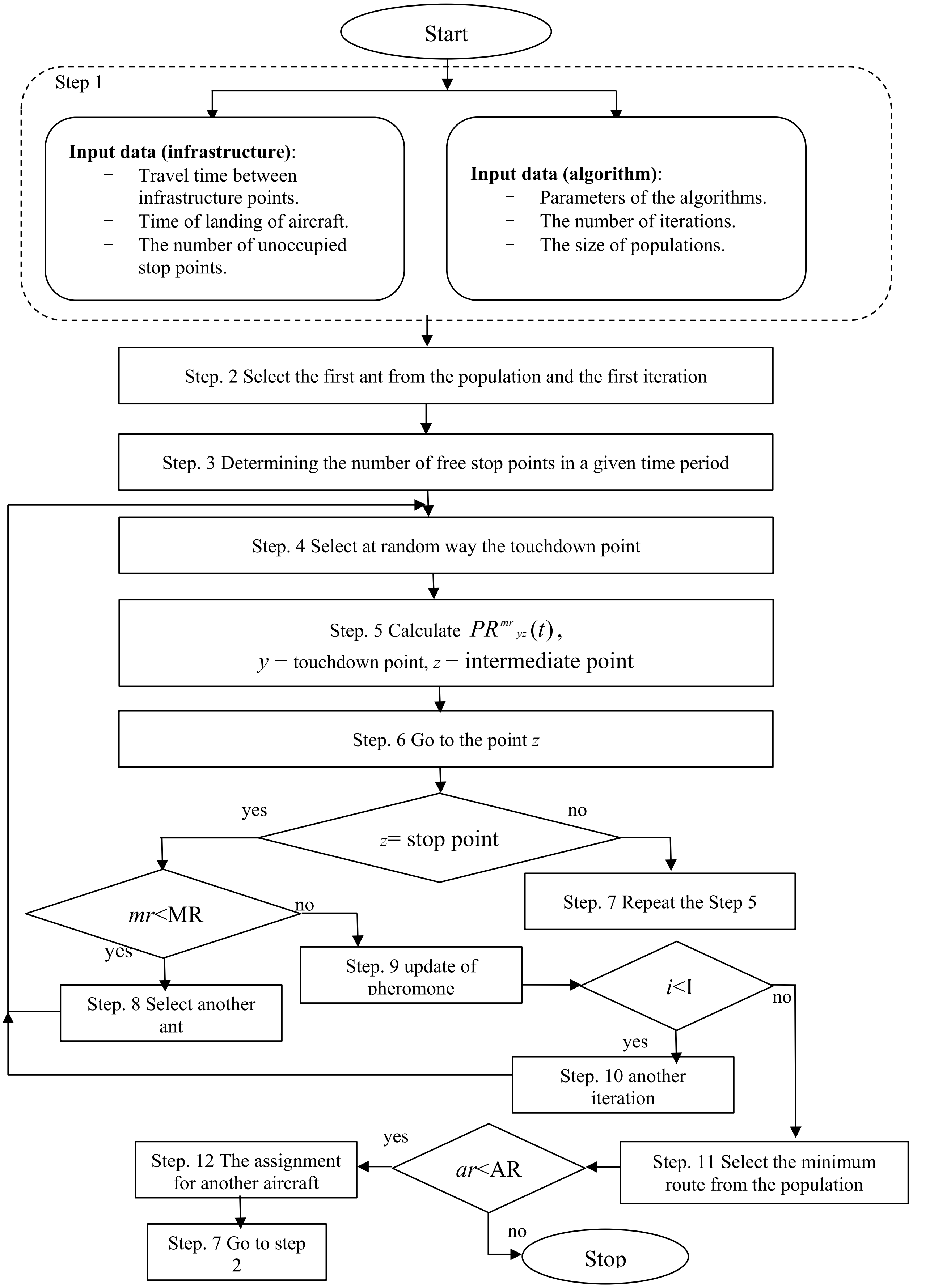


Fig. 3 The ant algorithm for the assignment problem

4. Results

The algorithm was implemented by the use of the real input data in the C# programming language. In the example, the Okęcie Airport in Warsaw was analysed. The first step of implementation of the ant algorithm was to find the set of the best parameters, which characterizes this algorithm. The number of iterations was set to 200. The size of population (ants) was set to 300. The combinations of parameters were presented in Tab. 2.

Tab. 2. The combinations of the parameters for the ant algorithm

Test	α	β	γ	Test	α	β	γ	Test	α	β	γ
1	1	0.5	0.2	17	1	1	0.2	33	1	5	0.2
2	1	0.5	0.4	18	1	1	0.4	34	1	5	0.4
3	1	0.5	0.6	19	1	1	0.6	35	1	5	0.6
4	1	0.5	0.8	20	1	1	0.8	36	1	5	0.8
5	3	0.5	0.2	21	3	1	0.2	37	3	5	0.2
6	3	0.5	0.4	22	3	1	0.4	38	3	5	0.4
7	3	0.5	0.6	23	3	1	0.6	39	3	5	0.6
8	3	0.5	0.8	24	3	1	0.8	40	3	5	0.8
9	5	0.5	0.2	25	5	1	0.2	41	5	5	0.2
10	5	0.5	0.4	26	5	1	0.4	42	5	5	0.4
11	5	0.5	0.6	27	5	1	0.6	43	5	5	0.6
12	5	0.5	0.8	28	5	1	0.8	44	5	5	0.8
13	10	0.5	0.2	29	10	1	0.2	45	10	5	0.2
14	10	0.5	0.4	30	10	1	0.4	46	10	5	0.4
15	10	0.5	0.6	31	10	1	0.6	47	10	5	0.6
16	10	0.5	0.8	32	10	1	0.8	48	10	5	0.8

The results of all tests for parameters were presented in Tab. 3.

Tab. 3. The results of the ant algorithm for each test

Test	Results [minutes]	Test	Results [minutes]	Test	Results [minutes]
1	11:25	17	11:35	33	11:24
2	8:45	18	10:05	34	10:35
3	8:20	19	9:15	35	11:45
4	9:55	20	9:00	36	11:23
5	11:03	21	9:20	37	8:45
6	11:30	22	9:35	38	8:55
7	9:255	23	8:56	39	8:34
8	8:50	24	8:45	40	8:34
9	10:25	25	8:00	41	9:36
10	10:20	26	8:15	42	9:05
11	8:24	27	8:35	43	8:09
12	9:53	28	8:20	44	9:25
13	8:48	29	8:25	45	11:23
14	9:46	30	11:20	46	11:12
15	11:04	31	11:25	47	10:27
16	11:37	32	12:11	48	10:19

In order to verify the correctness of the ant algorithm (AA), its results (for the best parameters – test 25) were compared with random values (AL). The ant algorithm in each case generated a better solution than the random algorithm. The results are shown in Tab. 4.

Tab. 4. The comparison of the algorithms.

Test	AA	AL	Test	AA	AL	Test	AA	AL
1	7:50	11:23	17	8:05	10:45	33	8:25	14:05
2	7:35	10:55	18	8:10	10:30	34	8:10	14:26
3	8:20	12:55	19	8:55	11:15	35	8:35	13:35
4	7:40	13:03	20	8:10	12:24	36	8:35	12:35
5	8:35	14:55	21	8:25	12:34	37	8:20	14:34
6	8:00	15:25	22	9:15	17:45	38	8:25	17:25
7	8:20	14:05	23	7:55	18:45	39	8:25	15:55
8	8:35	16:25	24	8:35	17:25	40	8:45	16:05
9	8:20	16:25	25	8:06	14:23	41	9:00	18:00
10	8:35	15:24	26	8:55	15:23	42	9:15	17:15

The solution generated by the ant algorithm for complex decision problems is a sub-optimal solution, which confirms Tab. 4. However, considering the complexity of the assignment problem, the solution is accepted from a practical point of view.

5. Conclusion

The early convergence to the sub-optimal solution in presented algorithm is blocked by the use of the update pheromone ant – cycle in the ant algorithm. The sub-optimal results generated by the algorithm depend on many factors, e.g.: the combinations of parameters of the algorithm, the number of iterations or ants in population. It should be underlined that results of the ant algorithm depend on the type of the data input. The main advantage of this algorithm is that the results are generated in a quick way, what is very important for airports. The processes occurring in the airports are the dynamic processes. For this reason, this algorithm must be started a few times depending on the number of aircraft on the apron. In this case, the calculation speed plays a huge role what underlines utility of this algorithm in the airports.

It should be noted that the presented ant algorithm has been used to solve the specific problem of the assignment. This problem is original problem, not analysed in the literature, so it is not possible to compare the results generated by this algorithm with the results obtained by the other algorithms.

It should be emphasized that the presented algorithm is the starting point for testing other algorithms within the defined research problem. The comparison of random results with the results generated by the proposed ant algorithm emphasized the effectiveness of its operation in the discussed problem. The generated results by the ant algorithm are the basis for further work on the development of new algorithms in the context of the examined problem.

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