

THE METHOD OF LUBRICATION FOR VEGETABLE GREASE

Krzysztof Wierzcholski

Technical University of Koszalin
Faculty of Technology and Education
Śniadeckich Street 2, 75-453 Koszalin, Poland
tel.: +48 94 3478344, fax: +48 94 3426753
e-mail: krzysztof.wierzcholski@wp.pl

Abstract

This article covers new aspects and a method of hydrodynamic lubrication of various friction pair using vegetable non-Newtonian greases. Biological greases and oils are applied in slide journal bearing lubrication occurring in farm and consumption machines working in pollution sensitive areas. Further applications can be found in humanoid robots joint lubrication as well as lubricants in various friction pairs, for example in gear and other friction nodes. The following belong to new vegetable greases: Accu-Lube LB10000 as medium to heavy duty machining oil, Accu-Lube LB 4000 – light to medium duty machining oil. Bio-vegetable greases have very important characteristic features and they offer the following benefits: prevention of corrosion protection-preventing premature failure, maximum penetration-ensuring protection at the cable core, extreme wash-off resistance that guarantees protection in wet conditions. Furthermore, the presented vegetable greases have biodegradability properties; hence, this fact has important meaning in temporary environmental problems. The aim of this article is to present a new general analytical and numerical solution of the hydrodynamic lubrication problem, to determine the influence of the vegetable grease and its biological additions on the temperature distribution in the internal surfaces of the slide bearing sleeve and pressure distribution in the bearing gap or on the cooperating surfaces of the friction pair. The research methods and materials used in this article include the following: the Mathcad 15 Professional Program and new semi-analytical methods applied in connection with the theory of hydrodynamic lubrication extended to the curvilinear orthogonal surface and coordinates. Particular solutions are introduced to the cylindrical coordinates. The obtained results of the lubrication of the movable sleeve surface include a new model of the hydrodynamic lubrication theory. As a consequence of the new method of calculation indicated the intervals are provided of the increases and decreases of the dynamic viscosity of greases and the intervals of the hydrodynamic pressure increments and decrements versus of the pH (power hydrogen ion concentration) of greases and wettability of vegetable greases into cooperating surfaces.

Keywords: *biological greases, hydrodynamic lubrication, biodegradability aspects, corrosion protection, new model of calculations*

1. Introduction

The mechanism of bio-lubrication for the cooperating friction pair surfaces of various geometrical shapes coated with vegetable superficial layer in rollers, jacking systems, pins or coated with phospholipids membrane in humanoid robots, requires to accept the form of elaboration in the curvilinear coordinates in the presented considerations.

Bio-greases or lubricant fluids have non-Newtonian properties due to the presence of the phospholipids and vegetable particles, esters or silicones. The very thin lipid bilayer is resting on the superficial layer surface. It creates a thin polar membrane of lipid molecules. These membranes are flat sheets that form a continuous barrier around the cells. The lipid bilayer is a barrier that keeps ions and proteins where they are needed and prevents them from diffusing into areas where they should not be found. On the superficial bilayer, negatively charged ions are lying with sodium counter captions strongly hydrated in bio-vegetable fluid. The bio-bilayer lining the hydrophilic negatively charged superficial surfaces in various friction pairs is presented in Fig. 1.

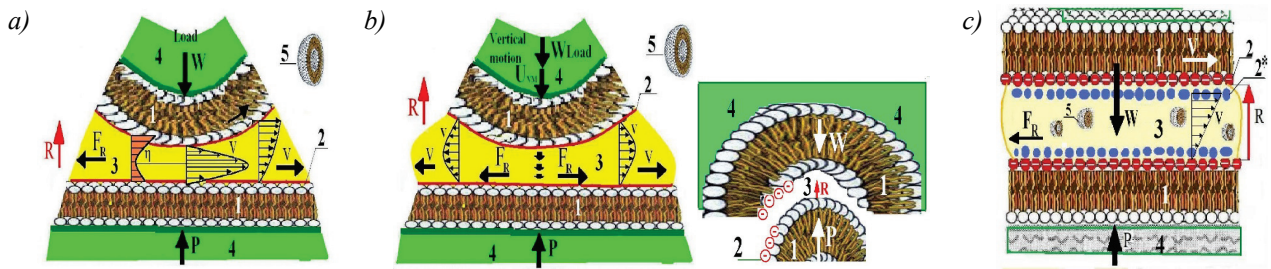


Fig. 1. The friction pair gap limited by the various shapes of bio-vegetable superficial or phospholipid (PL) bilayer a) the elliptic or spherical shapes of mentioned bilayer in lubrication by rotation, b) the parabolic shapes of mentioned bilayer in lubrication by squeezing, c) right-linear shapes of bilayer in lubrication by rotation where: 1 – bio-bilayer, 2 – hydrated $-PO_4^-$ group, 2* – hydrated sodium ions, 3 – vegetable grease, 4 – porous or vegetable fibril region, 5 – Liposom occurring in bio-greases, whereas: W – Load, P – hydrodynamic pressure force, R – Repulsion force, V – velocity of the upper surface, F_R – Friction force, η – viscosity distribution of vegetable fluid

Figure 1 shows the scheme of forces acting in the numerous gaps between two cooperating surfaces in friction pairs, bearing, joint with various shapes. These gaps are limited by the upper and lower bio-vegetable membrane and are filled with vegetable greases. Here, the repulsive force R is visible caused by the negatively charged mentioned membrane, especially of the $(-PO_4^-)$ groups with sodium positive ion strongly hydrated in the presence of vegetable fluid. Such charged surfaces are observed on both external vegetable surfaces contacting with the greases. Furthermore, we have the load carrying capacity force denoted by letter P and caused by the hydrodynamic pressure obtained from the rotation motion of the cooperating moving surface or by the squeezing during the lubrication process. In general, the senses and lines of forces R and P are the same, especially in the case of squeezing, boosted squeezing and weeping lubrication. The load force W of the presented friction pair has in general the reverse sense and the same line as forces P and R . Hence, as well, the carrying capacity force P as the repulsive force R counteracts and opposes the load force W .

The transverse sections of curvilinear joint gaps surfaces presented in Fig. 1b, c have in general among other spherical, cylindrical, parabolic, hyperbolic shapes dependent of the kind of the friction pair and dependent of the surface irregularities and roughness. The irregularities of surfaces have values from 1 to 5 micrometres.

2. The remarks about a new bio-vegetable lubrication

Here, general remarks are presented about a semi analytical method of the solution of the asymmetrical, stationary, laminar, non-Newtonian bio-lubricant flow lubrication between two rotational squeezing and deformable, curvilinear orthogonal movable friction pair surfaces coated with a superficial bio-vegetable layer or a PL bi-layer. The inertia forces of the vegetable greases are negligibly small because the speed of liquid particles in between the cooperating surfaces or in the humanoid joint gap during the limb motion attains values that are usually not greater than 3 m/s. To include or to exclude the direct influences of electric intensity field generated on the superficial surface on the tribology properties and to indicate the indirect influence of electric intensity on the lubricant viscosity changes, we neglect body forces except for Lorentz forces. Based on our own studies and scientific literature data, it can be stated that temperature variations that usually occur in living cooperating bodies in an interval from 34°C to 43°C have an influence on the tribology parameters during the bio-grease lubrication if and only if we consider the temperature influences on the vegetable fibres consistency and pH ions concentration in bio-greases and indirectly on the bio-liquid viscosity effects. In the case of jacking systems, open gears, pins humanoid robot the environment temperature can be higher and attains the value of even 120°C. Hence, in this case the temperature has always-direct influences on the tribology and bio-tribology parameters. Thus, the mathematical 3D model of the mentioned systems and

hydrodynamic humanoid joint lubrication always includes the conservation of energy equation and simultaneously a Young-Kelvin-Laplace equation that describes the interfacial energy and indirectly the lubricant dynamic viscosity in the friction pair superficial layer. Here, the convection energy terms and pressure dissipation energy terms are neglected. Joule heat terms are considered. The 3D vegetable grease flow between two above-mentioned solid cooperating surfaces in the electromagnetic field will be described in a vector form by the three equations of momentum equilibrium with a fluid continuity equation (1), and in a scalar form by an energy conservation equation (2), and a Young-Kelvin-Laplace equation (3). The aforementioned equations are as follows [7, 11]:

$$\text{Div } \mathbf{S} + \rho_e \mathbf{E} = 0, \quad \text{div}(\rho \mathbf{v}) = 0, \quad (1)$$

$$\text{div}(\kappa \text{ grad } T) + \phi_F = \mathbf{J}^2 / \sigma, \quad (2)$$

$$\gamma = \gamma_{\max} + 2s_c R_g T \ln \left(\sqrt{\frac{K_a}{K_b}} + 1 \right) - s_c R_g T \ln \left[\left(\frac{K_a}{a_H^+} + 1 \right) \left(\frac{a_H^+}{K_b} + 1 \right) \right], \quad (3)$$

where: \mathbf{S} [Pa] – stress tensor in the vegetable lubricant, \mathbf{E} [V/m] – electric intensity vector, \mathbf{J} – electric current density in A/m², T [K] – lubricant temperature, ρ_e [C/m³ = As/m³] – electric space charge in lubricant, κ [W/mK] – thermal conductivity coefficient for bio-lubricant, ϕ_F [W/m³] – dissipation of energy, \mathbf{S} [Pa] – stress tensor in the lubricant, R_g – gas constant (8.3144598 J/Kmol), $s_c = (N_A \cdot A)^{-1}$ [mol/m²] – concentration of vegetable or PL particles, γ [J/m² = N/m] – interfacial energy, γ_{\max} – is the maximum interfacial energy connected with PL-layer, K_a [J] – acid equilibrium constant (denotes how much energy is needed to stretch the bi-layer), K_b [J] – base equilibrium constant (denotes how much energy is needed to bend or flex the bi-layer), a_H [J] – protons energy activity, A [m²] – the soft surface of the friction pair coated by the vegetable or PL molecules, $N_A = 6.024 \cdot 10^{23}$ – Avogadro number, ρ [kg/m³] – bio lubricant density in, σ [S/m] – electrical conductivity of the vegetable superficial layer or the PL bi-layer.

Due to the presence of the bio-vegetable or PL-layer on the cooperating surfaces, we conclude that the lubricant has non-Newtonian and the especially pseudo-plastic properties. It will be shown that the constitutive equations of Reiner-Rivlin type for the abovementioned lubricant describes well the non-linear dependences between stresses and shear rates, whereas the apparent viscosity and consistency coefficient depends mainly on the shear rate of lubricant flow, electrostatic field occurring on the vegetable superficial layer or PL-bi-layers, the wettability of soft porous friction surfaces porous material, power hydrogen ion concentration in bio-lubricant, and less from the temperature in biological and more in a robot joint. For the lubricant, in general the relationship between the stress tensor \mathbf{S} and the displacement velocity tensor $2\mathbf{T}_d = \mathbf{A}_1$ i.e. constitutive equations are assumed in the following form [5]:

$$\mathbf{S} = -p\delta + \eta_T \mathbf{A}_1, \quad (4)$$

whereas the unit tensor δ , strain tensor \mathbf{A}_1 have the following components: δ_{ij} , Θ_{ij} . We denote δ_{ij} – Kronecker Delta, p [Pa] – pressure. For non-Newtonian lubricant of the Rivlin-Reiner type, the constitutive dependencies between the apparent viscosity η_T [Pas] and shear rate have the following form [5, 11]:

$$\eta_T = 2^{n-1} m(n) \left| \frac{1}{2} \mathbf{I}_1^2(\Theta) - \mathbf{I}_2(\Theta) \right|^{\frac{n-1}{2}}, \quad \mathbf{I}_1(\Theta) = \Theta_{kk}, \quad \mathbf{I}_2(\Theta) = \frac{1}{2} e_{ijk} e_{imn} \Theta_{jm} \Theta_{kn}, \quad (5)$$

where: \mathbf{I}_1 , \mathbf{I}_2 [s⁻¹], [s⁻²] are the invariants of shear rate Θ_{ij} [s⁻¹], n – dimensionless flow index depended on bio-fibres concentration in vegetable greases ($0.8 < n < 1.2$), $m = m(n, pH, T, We)$ – fluid consistency coefficient in Pasⁿ, e_{ijk} – tensor Levi-Civity, We – superficial layer wettability for

co-operating friction pair surfaces, pH – power hydrogen ion concentration in vegetable greases. The relations between shear rate Θ_{ij} [s^{-1}] and vegetable greases velocity components v_i [m/s] for $i = 1, 2, 3$ are assumed in the classical form.

3. Basic equations after boundary simplifications

By expanding the basic equations in (1-5) in α_i ($i = 1, 2, 3$) directions in orthogonal coordinate system $(\alpha_1, \alpha_2, \alpha_3)$, and taking into account layer boundary simplifications, i.e. neglecting the negligibly small terms of order 0.001 presenting the quotient of the characteristic gap height ε_T to the radius of curvature of cooperating friction pair surface or with PL membrane, denoting radial clearance ψ , we obtain the system of non-linear basic partial differential hydrodynamic equations. The aforementioned system describes the solutions of the hydrodynamic friction pair surface for bio-vegetable lubrication with PL bilayer in local curvilinear and coordinates for various shapes of co-operating surfaces, where non-Newtonian, incompressible synovial fluid with apparent dynamic viscosity $\eta_T(\alpha_1, \alpha_2, \alpha_3, t)$ varies in length α_1 , width α_3 and gap-height directions α_2 , and in t-time. Such a system has the following form [11, 12]:

$$0 = -\frac{1}{h_1} \frac{\partial p}{\partial \alpha_1} + \frac{\partial}{\partial \alpha_2} \left[\eta_T(\alpha_1, \alpha_2, \alpha_3) \frac{\partial v_1}{\partial \alpha_2} \right] + \rho_e E_1, \quad (6)$$

$$0 = \frac{\partial p}{\partial \alpha_2}, \quad (7)$$

$$0 = -\frac{1}{h_3} \frac{\partial p}{\partial \alpha_3} + \frac{\partial}{\partial \alpha_2} \left[\eta_T(\alpha_1, \alpha_2, \alpha_3) \frac{\partial v_3}{\partial \alpha_2} \right] + \rho_e E_3, \quad (8)$$

$$0 = \frac{1}{h_1} \frac{\partial v_1}{\partial \alpha_1} + \frac{\partial v_2}{\partial \alpha_2} + \frac{1}{h_1 h_3} \frac{\partial}{\partial \alpha_3} (h_1 v_3) = 0, \quad (9)$$

$$\frac{\partial}{\partial \alpha_2} \left(\kappa \frac{\partial T}{\partial \alpha_2} \right) + \eta_T(\alpha_1, \alpha_2, \alpha_3) \left[\left(\frac{\partial v_1}{\partial \alpha_2} \right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2} \right)^2 \right] = \mathbf{J}^2 / \sigma, \quad (10)$$

$$s_c(w_e, p_H, T) = \frac{\gamma(w_e, p_H, T, t) \left(\frac{\gamma_{max} - 1}{\gamma} \right)}{R_g T} \ln \left[1 + \left(\frac{\sqrt{L_a} - \sqrt{L_b}}{\sqrt{L_k} + 1} \right)^2 \right]^{-1}, \quad (11)$$

where:

$$L_a \equiv \frac{K_a}{a_H^+}, L_b \equiv \frac{a_H^+}{K_b}, L_k \equiv L_a L_b, (L_a + 1)(L_b + 1) > (\sqrt{L_k} + 1)^2, 0 < L \equiv \frac{(\sqrt{L_k} + 1)^2}{(L_a + 1)(L_b + 1)} < 1. \quad (12)$$

Apparent viscosity has the following form:

$$\begin{aligned} \eta_T(\alpha_1, \alpha_2, \alpha_3, t) &= \eta_T(n, p_H, We, T, E) \equiv m(n, p_H, We, T, E) \left[\left(\frac{\partial v_1}{\partial \alpha_2} \right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2} \right)^2 \right]^{\frac{n-1}{2}} = \\ &= \frac{\gamma_{max} + kA^{-1} T \ln L}{\delta_v \cdot v_0} \left(\sqrt{\left(\frac{\partial v_{11}}{\partial \alpha_{21}} \right)^2 + \left(\frac{\partial v_{31}}{\partial \alpha_{21}} \right)^2} \right)^{n-1}, \end{aligned} \quad (13)$$

In the calculations performed by virtue of the aforementioned equations, we denote: Boltzaman

constant $k = 1.38054 \cdot (10^{-23})$ [J/K], A [m^2] bearing surface coated with the bio-molecule, T [310 K] -temperature, γ [mJ/m^2] interfacial energy, We – wettability of bio-sleeve in the interval from 30° for hydrophiling, to 100° for hydrophobic superficial layer, v_0 – characteristic value of linear velocity of bio-lubricant in the biobearing gap from 0.25 to 4.0 [m/s], δ_v – dimensionless coefficient introduced by the author in the range ($2 < \delta_v < 6$) determining in bio-greases the variations of concentration of nano-meter long vegetable fibres from $\delta_v = 2$ for $c_c = 1,000,000$ mol/ mm^3 to $c_c = 1.00$ mol/ mm^3 and less for $\delta_v = 6$, E – electric intensity charges. We denote $v_{11} = v_1/v_0$ – the dimensionless velocity component in α_1 curvilinear circumferential coordinate direction, $v_{31} = v_3/v_0$ – the dimensionless velocity component in α_3 -curvilinear longitudinal direction, $\alpha_{21} = \alpha_2/\epsilon_0$, ϵ_0 – the characteristic dimensional value of the joint gap height. Assuming the influence of the bio superficial layer on the friction pair surfaces in the presence of the vegetable greases, we take into account the following typical experimental data: power acid equilibrium constant $pK_a = 2.58$, power base equilibrium constant $pK_b = 5.98$, the typical value of protons energy activity in the gap a_H [J] giving inequality $0 < L < 1$, the maximum interfacial energy of the lipid membrane $\gamma_{max} = 4.0$ mJ/ m^2 , average cooperating surfaces velocity $v_0 = 1.0$ [m/s], vegetable concentration fibres in bio-fluid $\delta_v = 6$ so that $\delta_v v_0 = 6$ m/s, bio-lubricated friction pair surface $A = 0.00025$ m^2 , vegetable grease density $\rho = 1010$ kg/ m^3 and h_1, h_3 are the Lamé coefficients.

A new hydrodynamic model for vegetable greases presented in this article enables one to obtain numerous remarks referring to the lubricant as superficial layer solid material properties of cooperating surfaces. Some of them are illustrated in Tab. 1.

Tab. 1. The value variations of multivariable dynamic viscosity function for various properties of vegetable greases connected with numerous kinds and features of co-operating surfaces of friction pair bodies coated with vegetable superficial layer or PL-bilayer during the bio lubrication

No.	Non-Newtonian vegetable greases or bio-lubricant		Features of bio-superficial layer assigned to the successive parameters or its tendencies of changes	Kind of lubrication assigned to the successive parameters
	The phenomenon of such parameters as: vegetable greases viscosity, temperature, wettability, hydrogen ions concentration and its changes	The values of intervals of successive parameters or their tendencies for assumed data		
1	Viscosity η decrements with temperature increases $35^\circ C < T < 80^\circ C$	$0.30 \text{ Pas} > \eta > 0.01 \text{ Pas}$, for $We = 40^\circ$, $pH = 6$, $v\delta_v = 6 \text{ m/s}$, $c_c = 5 \cdot 10^5 \text{ mol/mm}^3$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing
2	Wettability We of solid body of superficial layer	$70^\circ > We > 50^\circ$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing
3	Viscosity η decrements along wettability grade interval of superficial layer in range from 80° for hydrophobic – to 30° for hydrophilic $80^\circ > We > 30^\circ$	$0.40 \text{ Pas} > \eta > 0.03 \text{ Pas}$, for: $pH = 6$, $T = 37^\circ C$, $v\delta_v = 6 \text{ m/s}$, $c_c = 5 \cdot 10^5 \text{ mol/mm}^3$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing
4	Viscosity: a) increments, b) decrements, with pH index increases and decreases	a) $0.30 \text{ Pas} < \eta < 0.55 \text{ Pas}$ if: $2.0 < pH < 3.7$, b) $0.55 \text{ Pas} > \eta > 0.20 \text{ Pas}$ if: $3.7 < pH < 12$, for a,b: $We = 50^\circ$, $T = 40^\circ C$, $v\delta_v = 6 \text{ m/s}$, $c_c = 5 \cdot 10^5 \text{ mol/mm}^3$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing
5	Viscosity η decrements for shear rate values Θ increments for vegetable grease and other pseudo-plastic oil	$0.30 \text{ Pas} > \eta > 0.03 \text{ Pas}$, if: $5 \text{ s}^{-1} < \Theta < 10^4 \text{ s}^{-1}$, for: $We = 40^\circ$, $T = 37^\circ C$, $v\delta_v = 6 \text{ m/s}$, $c_c = 5 \cdot 10^5 \text{ mol/mm}^3$, $2 < pH < 12.0$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing
6	Viscosity η increments with increments of vegetable fibres molecules concentration for $v\delta_v = 6 \text{ m/s}$	$0.03 \text{ Pas} < \eta < 0.40 \text{ Pas}$, if: $10 < c_c < 10^6 \text{ mol/mm}^3$, ($8 > \delta_v > 2$), for: $3.7 < pH < 12$, $70^\circ > We > 50^\circ$	Isotropic, homogeneous, Elastic, constant Young modulus E from 12 to 50 MPa	Rotation, squeezing

4. The sketch of analytical solutions

Boundary conditions for bio-lubricant velocity components have the following form [12]:

$$v_1 = \omega h_1 \text{ for } \alpha_2 = 0, \quad v_1 = 0 \text{ for } \alpha_2 = \varepsilon_T, \quad (14)$$

$$v_2 = 0 \text{ for } \alpha_2 = 0, \text{ and } v_2 = 0 \text{ for } \alpha_2 = \varepsilon_T, \quad (15)$$

$$v_3 = 0 \text{ for } \alpha_2 = 0, \quad v_3 = 0 \text{ for } \alpha_2 = \varepsilon_T. \quad (16)$$

Imposing the condition (14)-(16) on the general solution of Equation (6)-(9) we obtain the following particular solutions [11, 12]:

$$v_1(\alpha_1, \alpha_2, \alpha_3) = \left(\frac{1}{h_1} \frac{\partial p}{\partial \alpha_1} - M_1 \right) A_\eta + (1 - A_s) \omega h_1, \quad (17)$$

$$v_2(\alpha_1, \alpha_2, \alpha_3) = - \int_0^{\alpha_2} \frac{1}{h_1} \frac{\partial v_1}{\partial \alpha_1} d\alpha_2 - \int_0^{\alpha_2} \frac{1}{h_1 h_3} \frac{\partial (h_1 v_3)}{\partial \alpha_3} d\alpha_2, \quad (18)$$

$$v_3(\alpha_1, \alpha_2, \alpha_3) = \left(\frac{1}{h_3} \frac{\partial p}{\partial \alpha_3} - M_3 \right) A_\eta, \quad (19)$$

where: $0 \leq \alpha_1 \leq 2\pi\theta_1$, $0 \leq \theta_1 \leq 1$, $b_m \leq \alpha_3 \leq b_s$, $0 \leq \alpha_2 \leq \varepsilon_T$ and $M_i = \rho_e E_i$.

If we impose the second boundary condition (15) for the component of lubricant velocity in the gap height direction i.e. $v_2=0$ for $\alpha_2=\varepsilon_T$ then it is easy to find out that the pressure function p in the curvilinear coordinates $(\alpha_1, \alpha_2, \alpha_3)$ satisfies the following modified Reynolds equation [12]:

$$\begin{aligned} \frac{1}{h_1} \frac{\partial}{\partial \alpha_1} \left[\left(\frac{\partial p}{\partial \alpha_1} - h_1 M_1 \right) \left(\int_0^{\varepsilon_T} A_\eta d\alpha_2 \right) \right] + \frac{1}{h_3} \frac{\partial}{\partial \alpha_3} \left[\frac{h_1}{h_3} \left(\frac{\partial p}{\partial \alpha_3} - h_3 M_3 \right) \left(\int_0^{\varepsilon_T} A_\eta d\alpha_2 \right) \right] = \\ = \omega h_1 \frac{\partial}{\partial \alpha_1} \left[\int_0^{\varepsilon_T} A_s d\alpha_2 - \varepsilon_T \right], \end{aligned} \quad (20)$$

for: $0 \leq \alpha_1 \leq 2\pi$, $b_m \leq \alpha_3 \leq b_s$ where oil dynamic viscosity varies in the gap height direction, i.e. $\eta_T(\alpha_1, \alpha_2, \alpha_3)$. Substituting expressions (17), (19) into the energy equation (10), and taking into account the constant thermal conductivity of the lubricant, after terms ordering and calculations the energy equation tends to the following form [12]:

$$\frac{\partial^2 T}{\partial \alpha_2^2} + \frac{\eta_T}{\kappa} \left\{ \left[\left(\frac{1}{h_1} \frac{\partial p}{\partial \alpha_1} - M_1 \right) \frac{\partial A_\eta}{\partial \alpha_2} - \omega h_1 \frac{\partial A_s}{\partial \alpha_2} \right]^2 + \left[\left(\frac{1}{h_3} \frac{\partial p}{\partial \alpha_3} - M_3 \right) \frac{\partial A_\eta}{\partial \alpha_2} \right]^2 \right\} = \frac{M_T}{\kappa}, \quad (21)$$

for: $0 \leq \alpha_1 \leq 2\pi$, $b_m \leq \alpha_3 \leq b_s$, $0 \leq \alpha_2 \leq \varepsilon_T$, $M_T = \mathbf{J}^2/\sigma$.

In Eqs. (17)-(21) the following notations are introduced:

$$A_s(\alpha_1, \alpha_2, \alpha_3) \equiv \frac{\int_0^{\alpha_2} \frac{1}{\eta_T} d\alpha_2}{\int_0^{\varepsilon_T} \frac{1}{\eta_T} d\alpha_2}, \quad A_\eta(\alpha_1, \alpha_2, \alpha_3) \equiv \frac{\int_0^{\alpha_2} \alpha_2 d\alpha_2 - \frac{\left(\int_0^{\alpha_2} \frac{1}{\eta_T} d\alpha_2 \right) \left(\int_0^{\varepsilon_T} \alpha_2 d\alpha_2 \right)}{\int_0^{\varepsilon_T} \frac{1}{\eta_T} d\alpha_2}}{\int_0^{\varepsilon_T} \frac{1}{\eta_T} d\alpha_2}. \quad (22)$$

5. Conclusions

1. For the constant temperature of 313 K and the constant wettability of about 50° of a porous soft superficial layer on the cooperating surfaces lubricated with vegetable bio-lubricant, for power

hydrogen ion concentration pH from about 2 to 4 lubricant dynamic apparent viscosity increases and hydrodynamic pressure increases, and for pH from 4 to 12 lubricant dynamic apparent viscosity and hydrodynamic pressure decreases (see row 4 in Tab. 1).

Proof. The mentioned corollary follows from the calculations of the lubricant dynamic viscosity η_T on the grounds of the interfacial energy illustrated in Fig. 2 and obtained in an experimental and numerical way [11, 12]. The calculated lubricant dynamic viscosity η_T introduced into Equation (20) results in hydrodynamic pressure increments and decrements indicated in the presented conclusion 1 and it completes the conclusion 1.

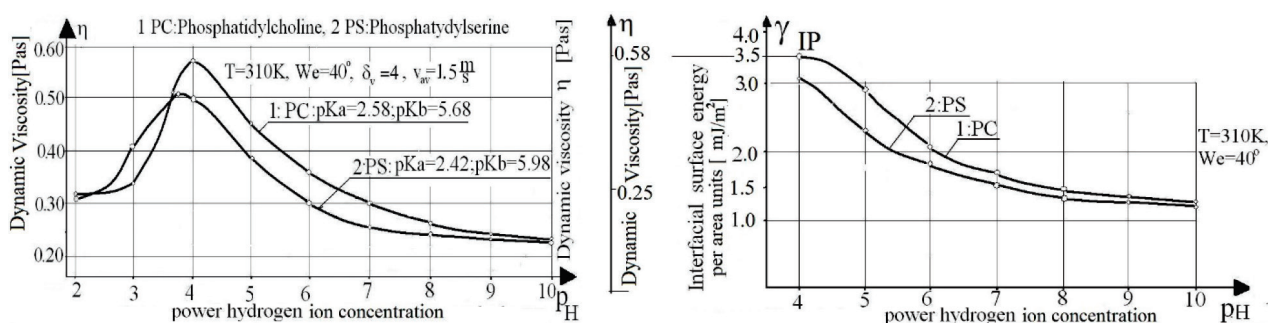


Fig. 2. Dynamic viscosity increments for $0 < pH < 4$ and decrements for $pH > 4$ of vegetable versus concentration of power hydrogen ions pH with lipids of type PC and PS, for constant temperature 37°C - 40°C , superficial layer wet-ability $We = 40^\circ$ - 50° average surfaces, velocity of 1.5 m/s, and for vegetable nano-fibres concentration $c_c = 500000 \text{ mol/mm}^3$

2. The well-known law for pseudo plastic liquids about viscosity decrements with shear rate increments valid for considered vegetable greases. In addition, viscosity of vegetable greases decreases versus temperature increments.

Proof: From formula (12), it follows that L function attains values in the interval (0,1), hence the term $(T/A)\ln L$ has always negative values. Thus, temperature increments decrease the numerator of the fraction defining the apparent viscosity (13), hence viscosity decreases. Analogously, surface A increments denote increments of bio-fibres or PL concentration, hence the negative value $(T/A)\ln L$ decreases. Thus, the numerator increases in Formula (13) of the fraction describing viscosity, i.e. viscosity increases. Because the velocity increments of vegetable greases denote shear rate increments, the denominator in Formula (13) increases, and the fraction defining the viscosity decreases, i.e. viscosity decreases. This fact completes the proof of conclusion in point 2.

Referencese

- [1] Anders, N., Olaf, S., Roger, E., et. al., *Bilayer thickness and membrane protein function: An energetic perspective*, Annular Review of Biophysics and Biomolecular Structure, 36 (1), pp. 107-130, 2007.
- [2] Arkin, R. C., *Behaviour-based robotics*, MTP Press, ISBN 0-262-01165-4, 1998.
- [3] Asada, H., Slotine, J.-J. E., *Robot analysis and control*, ISBN 0-471-83029-1, Wiley, 1986.
- [4] Chagnon, G., Rebouah, M., Favier, D., *Hyperelastic energy densities for soft biological tissues*, Rev. J. of Elasticity, Vol. 120, Iss. 2, pp. 129-160, 2015.
- [5] Deneveth, J. M., McLean, S. G., Arruda, E. M., *Evaluation of hyperelastic models for the non-linear and non-uniform high strain-rate mechanics of tibial cartilage*, J. Biomech., Vol. 46 (10), pp. 1604-1610, 2013.
- [6] Mashaghi, A., et al., *Hydraton strongly affects the molecular and electronic structure of membranephospholipids*, Vol. 136, 114709, 2012.
- [7] Pawlak, Z., Urbaniak, W., Hagner-Derengowska, M., Hagner, W., *The probable explanation for the low friction of nature joints*, Cell Biochemistry and Biophysics, Vol. 71, No. 3, pp. 1615-1621, 2015.

- [8] Pawlak, Z., Urbaniak, W., Gadomski, A., Yusuf, K. Q., Afara, I. O., Oloyede, A., *The role of lamellate phospholipid bilayers in lubrication of joints*, Acta of Bioengineering and Biomechanics, Vol. 14, No. 4, pp. 101-106, 2012.
- [9] Poprawski, W., *Biodegradable greases applied for bearing of mobile machinery*, Inżynieria Maszyn, R. 19, z. 2, 2014.
- [10] Wierzcholski, K., Miszczak, A., *The robot joint lubrication with ultra thin hyperelastic superficial layers*, Journal of KONES Powertrain and Transport, Vol. 23, No. 3, pp. 555-562, Warsaw 2016.
- [11] Wierzcholski, K., *Joint cartilage lubrication with phospholipids bilayer*, Tribologia, 2 (265), pp. 145-157, 2016.
- [12] Wierzcholski, K., *Mathematical principles and methods of biological surfaces lubrication with PL bilayer*, Bio-Medical Work-Shop, Kazimierz Wielki, Univ. Bydgoszcz 2018.

Manuscript received 04 June 2018; approved for printing 05 September 2018