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OPTIMISATION OF AIRCRAFT POSITION IN THE FORMATION FLIGHT FOR THE DRAG REDUCTION

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Abstract

This article presents optimisation of necessary flight thrust in a V-shaped flight formation of small-unmanned plane "Sikorka". At the beginning is showed analyse of birds behaviour. Their formation flying was the cause of attention in order to minimalize fuel consumption. Afterwards there are overlooked scientific articles about the formation flying subject contain pure physic analyses, and articles about researches which was made in order to explain economic beneficial for airlines. Thus, the article presents mathematical model, which was optimised for three different starting position of a longitudinal axis. After optimisation there are presented results of the wingman position in regard of the leader. Influence of the calculation results on the formation flying was analysed, allowing for some conclusions about the future of the UAV's flights. The given process is aimed to achieve the best (optimal) solution from the point of view of the specific criterion. The following most important terms can be distinguished within the optimization process: decisive variables – parameters determining the basic project assumptions. The basic design variables and design constrains are described.

Keywords: *UAV, formation flying, optimisation, dynamic of flight, aerodynamic*

1. Introduction

In order to engage a client airlines around the world have always been forced not only to offer the highest quality of their services but first of all the most profit able price of a ticket. In order to reduce the ticket price, each provider tends to ensure cost-cutting measures in every aspect, not only at administrational but also at technical level. Utilization of Unmanned Aerial Vehicles will allow for reduction of employment among pilots that contributes to economical profit of the airline. In this operational model, the pilot siting in operational centre will operate multiple aircraft. Useful solutions shall also be taken from the animal world. Observation and analysis of formation flights of birds have resulted in launching the research in order to apply the given solution into aviation industry. Many scientific articles, showing the positive impact on fuel consumption reduction in formation flight, have been written during the past few years. The use of Unmanned Aerial Vehicles and formation flying will ensure multimillion-dollar savings from fuel consumption per year, globally, for each airline. The following solution for civil operations is pro-ecological.

2. Formation flying – Genesis

Since the beginning of humanity, the animal world has been seen as a role model in daily life. At some point in the past, while observing the behaviour of birds, performing formation flights, scientists have asked a question: "why do they fly in formation?". Research [1] has brought information that formation flight is 71% more effective than solo flight. This solution is achieved thanks to positive impact of circulation generated by leader on a follower. This has an impact on a change of induced drag and lift. It was observed that birds instinctively know their positions in a formation. Longitudinal separation depends on a specie's wingspan; however, it is most often around ¼ of their wingspan. The scientists have confirmed that the number of specimens for each arm is not significant and arms do not need to be symmetrical. Accurate analysis of northern bald ibis's formation flight was performed in research [2] in which GPS systems and Inertial Measurement Units were used. As results of them, it was observed that the ibises perform flights in a perfect V formation that is at an angle of 45 º with regard to themselves. During researches, it was calculated that aerodynamic drag was reduced by 65 percent. It was also observed that whole formation flies with slower speed and there is a decrease in heart action of each bird by 10 percent. The above-mentioned fact is a smaller demand for "power" for objects in formation. The research has ended up with conclusion that the leader is changing its speed constantly within a time. This provides the possibility to avoid situation, when one specimen will not be airworthy to continue the flight.

Fig. 1. Northern bald ibis's formation flight [13]

Fig. 2. Formation flight of A350 aircrafts [14]

3. Formation flying – Research

The research is aimed not only at possibility of understanding the animal world, but primarily at creation of a process for employment of formation flights into civil aviation, resulting in fuel savings by reduction of thrust of engines. One of the research methods was to create a mathematical model of bird's flight based on the example of UAV "Avitron" with appearance and behaviour resembling the real bird [3]. The main effect of the analysis is a confirmation that programmed formation flight, contrary to solo flights provides the possibility to perform longer flights. The other method is to analyse the scientific laws and tools, which provide the description of formation flights in mathematical and physical way. An example of that is a research [4], in which various types of formation flight's optimization algorithms were analysed in order to minimize fuel consumption. It was showed that for V formation consisting of five units, the number of iterations needed is equal to one hundred. The given result indicates that there is a possibility to create easy algorithms that can be computed by small and simple computers. Another type of formation flight analysis is assessment of their safety. Research [5] presented the computational model for development of vortices and their impact on the follower. The created model and its theoretical explanation allow understanding the effect of the wingman. The use of the strip method resulted in obtaining the distribution of forces and moments acting on the wing, and subsequently changing the angles of the flight parameters. As a result of the work, the influence of vortices on the wingman has been demonstrated, which, if properly configured, does not have to affect it negatively.

4. Formation flying – Application

In the course of time, the research has become more advanced. Nowadays pure mechanics or profit calculation is not only analysed. Real application examples were taken into consideration for the research. Scientists [6] performed a transatlantic flight simulation for two aircraft, taking-off, and landing on different airports. They presented that only a segment of whole flight will be performed as a formation flight, during which the positive aspect could be utilized. As a result of their calculations, where MTOW and the average velocity of large aircraft over 0.8 Mach number is assumed, they showed that the most optimal connection of aircraft types is Airbus A380-800 as a formation leader and Boeing 747-400 as a follower. The gain of above 6% on fuel consumption is achieved in the given configuration.

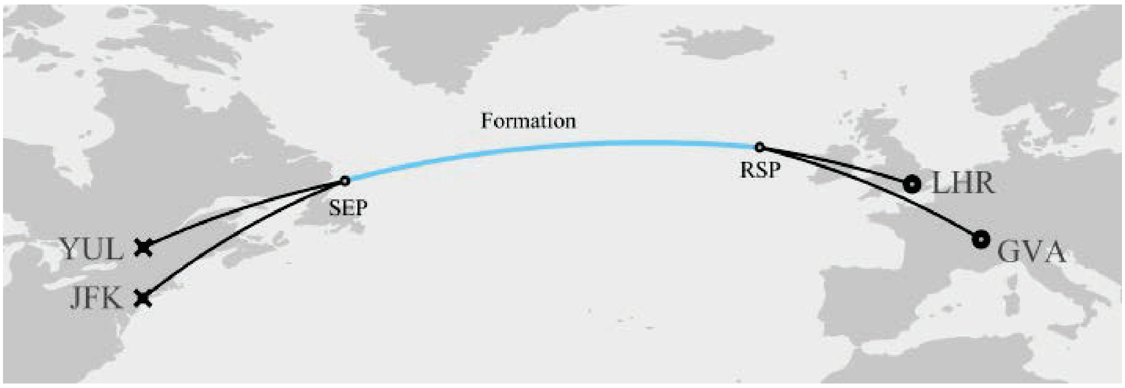

Fig. 2. Flight simulation for two aircraft [6]

While performing the analysis of formation flying and their positive impact on fuel savings it shall be kept in mind that reorganization of all civil aviation is performed when the opportunity occurs. Bearing in mind the above-mentioned non-compliance shown in [7], the flight optimization of FedEx cargo aircraft was presented. Evening flights from five airports located mostly on west coast of USA to Memphis airbase located on east coast of USA were taken into consideration. Scientists demonstrated that during the research two formations of three and two elements were created. The first one achieved 12.46% fuel consumption reduction and the next one 7.85%. For the company with five flight days per week on the given route, it would generate a gain of 2.8 million dollar and 700,000 gallons of fuel saving per year.

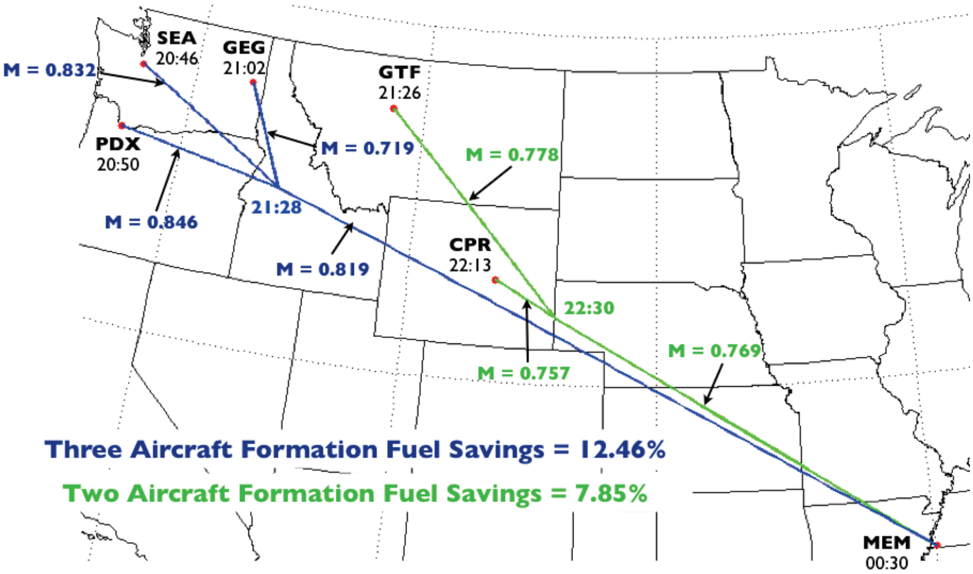

Fig. 3. The flight optimization of FedEx aircraft [7]

5. Optimization

The optimization process was conducted in the following analysis. The given process is aimed to achieve the best (optimal) solution from the point of view of the specific criterion. The most valuable result is called "optimum". The following most important terms can be distinguished within the optimization process: decisive variables – parameters determining the basic project assumptions, for instance: cross-section shapes, cross-section dimensions, material type, weight, physical, chemical and mechanical properties of the material, structural vibrations, power, energy consumption, functionality, efficiency, operational parameters, colour, ergonomic parameters and many other; the objective function – mathematical presentation of optimization idea selected within the first stage of design process. You may use one or two parameters simultaneously. Frequently the cost or weight of the structure is the decisive argument. The optimization process may be defined as the process of finding the minimum or maximum of the objective function; the design limitations (constrains) – requirements for the design, which has to be acceptable, for instance maximum weight of an aircraft.

The design process begins with a description of a problem that will be solved during calculations. For this purpose basic design variables and design constrains are described. Next step is a creation of an objective function, whose complexity depends on number of assumed parameters. The second phase in an optimization process is the appropriate selection of optimization procedure, in other word: a tool, which will be used to solve the given problem. Each problem requires the use of different methods in order to achieve the most effective solution.

Graphical interpretation of optimization problem can be presented on an example of steepest descent method and a conjugate gradient method. The steepest descent method is based on finding the function's minimum in an opposite way to function gradient at the point of departure. In order to determine the next iterative step, it is necessary to determine the gradient of the function at the starting point and the coefficient of the step [8].

Function gradient at the starting point:

$$s^k = -\nabla f\left(\mathbf{x}^k\right). \tag{1}$$

Factor of a given step:

$$\alpha^k = \min_{\alpha>0}\left(f\left(\mathbf{x}^k + \alpha\,\mathbf{s}^k\right)\right). \tag{2}$$

The departure point for calculating the next step is equal to:

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \alpha^{k}\mathbf{s}^{k}. \tag{3}$$

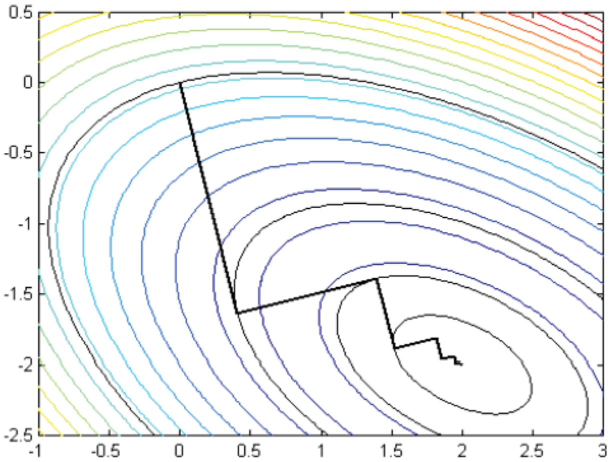

Fig. 4. Steepest descent method [8]

The process of the steepest descent method is based on an algorithm, which in every step in predefined direction is looking for the minimum value of the objective function. The search is finished when it is impossible to obtain smaller value than for step i-1. The disadvantage of the steepest descent method is slow convergence in case of badly chosen point of departure.

MATLAB software was used in order to solve an optimization process. Different types of optimization methods, such as: linear minimization, nonlinear minimization, multicriterion optimization or least squares optimization method can be used.

The basic optimization task of the function with limitations is saved in MATLAB program, in the form of:

$$\min_{x} f(x)\Big|_{g(x)\leq 0}, \tag{4}$$

where:
f(x) – optimized function,
g(x) – domain of constrains

6. Mathematical Model

In a plane during a flight, the pressure is lower than the surrounding atmospheric pressure on upper wing surface and higher on bottom wing surface. As a result of this action, the air flowing around the upper and bottom surface is tending to flow from inner part of the wing to the wing tip, symmetrically on both sides of the wing. Merging both streams will create big vortices, one per wing tip. Both of the same size. Measurable value of vorticity is operator called Γ circulation. It is a measure of the intensity of the vortex. The dependency between circulation and lift was described by Russian scientist Nikolay Zhukovsky in 1905 [9]. It defines that the lift depends only on a size of the vortex circulation, flow velocity and the density of the medium in the circular profile. Joukowsky potential can be used not only in the circular profile, but also in profiles of any shape.

$$\Gamma = \frac{m*g}{\rho*b_0*V}, \tag{5}$$

where:
m – a leader weight,
g – standard gravity,
ρ – air density,
b_0 – the span correction adopted due to the elliptical distribution of the circulation on the wing,
V – velocity.

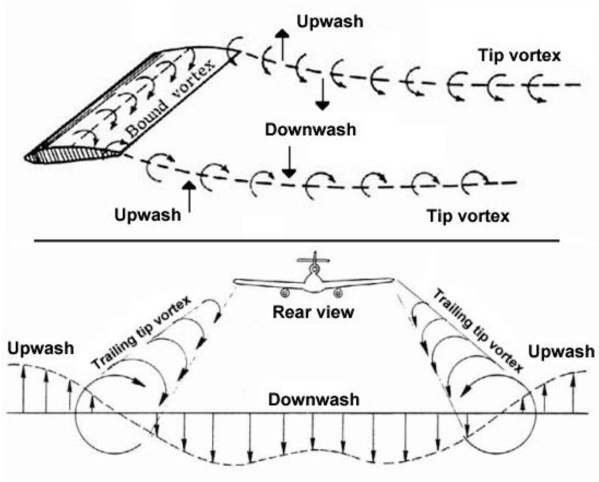

Fig. 5. Circulation generated by the flying plane [10]

Having the value of circulation generated by the flying plane, it can then use the Biot Savart's law to determine the induced velocity at any point P (x, y, z) away from the vortex.

$$\Delta q = \frac{\Gamma}{4\pi}\frac{\boldsymbol{dl} \times (\boldsymbol{r_0} - \boldsymbol{r_1})}{|\boldsymbol{r_0} - \boldsymbol{r_1}|^3}, \tag{6}$$

where:
Δq – value of the induced speed at the P point,
dl – vorticity component,
r0-r1 – the distance of the point from the vortex, calculated as the sum of the vectors from the centre of the coordinate system.

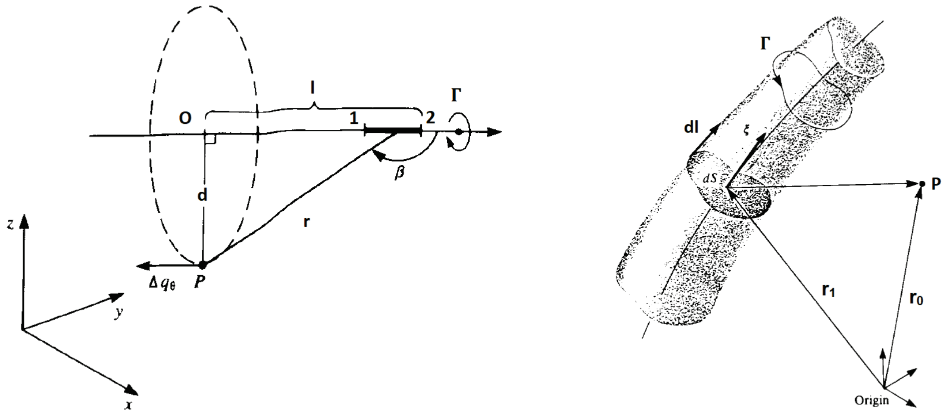

Fig. 6. a) Induced velocity generated by straight vortex [11], b) Induced velocity generated by vortex in P point [11]

Then we consider that the P point, adopted in the given coordinate system, and described in Fig. 6. It can be observed that the P point distance from the vortex is described directly by the r vector.

$$\Delta q_\theta = \frac{\Gamma}{4\pi}\frac{\sin\beta}{\boldsymbol{r}^2} dl. \tag{7}$$

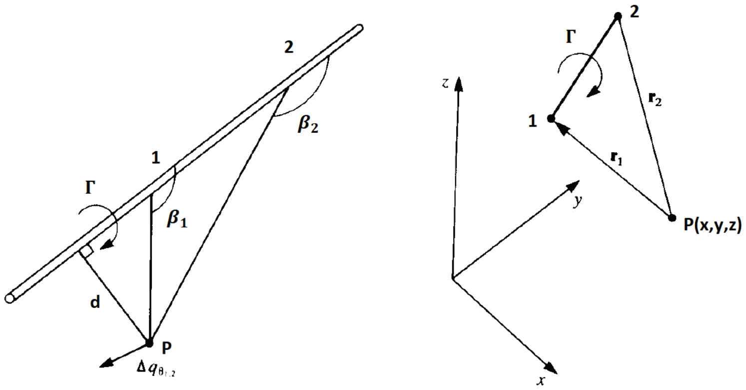

Fig. 7. a) Specification of points 1 and 2 in generated vortex,
b) Changing the description of the P point into distance vectors

Furthermore, by integrating between point 1 and 2, we get the following form of induced velocity:

$$(q_\theta)_{1,2} = \frac{\Gamma}{4\pi d}\int_{\beta_1}^{\beta_2} sin\beta d\beta = \frac{\Gamma}{4\pi d}(cos\beta_1 - cos\beta_2). \tag{8}$$

The next step is the transition from angles to vector values describing the distance of the P point from point 1 and 2, which was presented in Fig. 7b. For this purpose, the following dependences were determined, which in turn after the substitution allowed obtaining a full formula for the induced velocity at P point (x, y, z). Following dependencies were adopted in the model:

$$r_2 - r_1 = r_o, \tag{9}$$

After the transformations enabling the calculation of induced velocity in three axes: u, v, and w, the following equation was obtained:

$$\begin{aligned} u &= K(\boldsymbol{r}_1 \times \boldsymbol{r}_2)_{x,} \\ v &= K(\boldsymbol{r}_1 \times \boldsymbol{r}_2)_{y}, \\ w &= K(\boldsymbol{r}_1 \times \boldsymbol{r}_2)_{z,} \end{aligned} \tag{10}$$

where:

$$K = \frac{\Gamma}{4\pi}\frac{1}{|r_1 \times r_2|^2}\left(\frac{r_0 \cdot r_1}{r_1} - \frac{r_0 \cdot r_2}{r_2}\right), \tag{11}$$

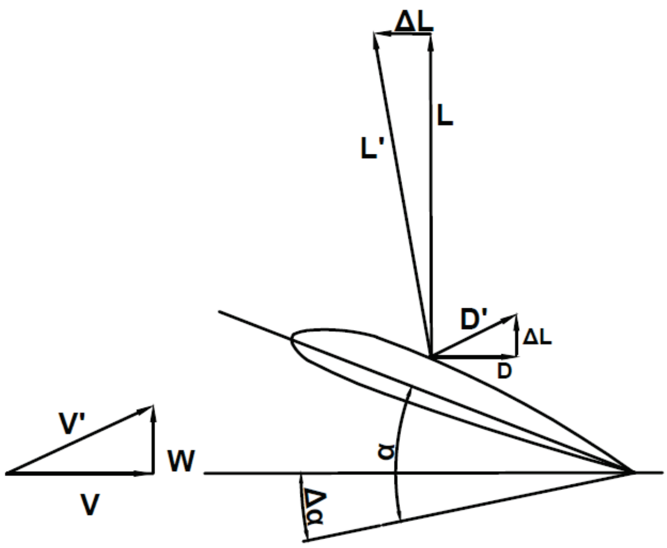

Fig. 9. Distribution of forces including induced velocity

The effect of the vertical component of the induced velocity is the generation of an additional pitch of the aircraft with respect to the original angle of attack. This additional angle is called the induced angle, calculated from the following operation:

$$\propto_i = \frac{w}{V}, \tag{12}$$

where:
w – vertical component of induced velocity[m/s],
V – flight velocity [m /s].

At the moment of the aircraft's pitch, an additional force is generated from the lifting force acting in the longitudinal direction of the aircraft. It is equal in value to the induced drag generated by the vortex operating on the wing.

$$\mathrm{L}' = \mathrm{L} * \mathrm{tg} \propto_\mathrm{i} . \tag{13}$$

In a fixed flight, a thrust of engines balances the aircraft's drag. The generated component of the lift force increases the value of the force pushing the aircraft. However, when viewed from the opposite side, the value of induced drag decreases the drag

$$T = D - L'. \tag{14}$$

While analysing above formulas, it can be observed that the new sequence necessary for the formation flight will be smaller than in the solo flight. Smaller thrust will affect the fuel consumption and cost savings

7. Adopted Assumptions

The following values for the previously designed small-unmanned aircraft were adapted for calculations:
– b = 4 m – wingspan,
– S = 1.6 m^2 – wing surface area,
– V = 13.7 m/s – optimal velocity,
– m = 15 kg – weight,
– ρ = 1.11 kg/m^3 – air density at 1000 m ASL,
– C_{X0} = 0.0652 – drag for angle of attack equal to 0^o.

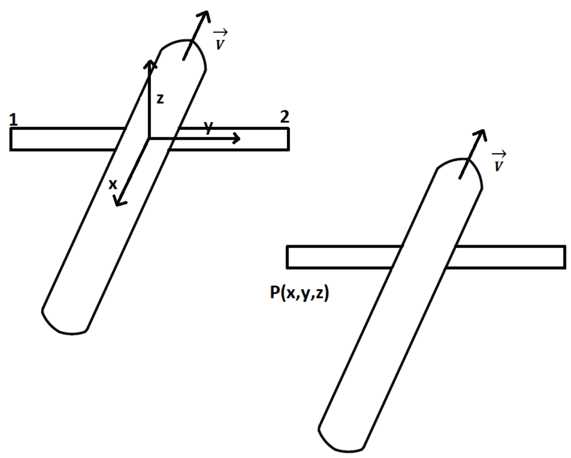

Fig. 10. Adopted coordinate system

The centre of the coordinate system was assumed in the centre of the airfoil. However, the P point (x, y, z) has been defined, from the leader side, as the point at the follower's wing tip. Point 1 (x_1, y_1, z_1) and 2 (x_2, y_2, z_2) indicate the extreme points of the leader's wing. In the analysis, a constant value of velocity induced over the entire length of the wing was assumed, calculated for the tip of the wingman wing on the same side as the leader.

8. Results

The results of position optimization on "y" and "z" axes will be presented for several different distances from the leader on the "x" axis. Graphs of the vertical component of the induced velocity, new component resulting from the "L" lift force and the necessary thrust in 3D and 2D will be presented. Three different starting points were adopted for optimization:

$$y = -b,\ z = -b,$$
$$y = 0,\ z = 0, \tag{15}$$
$$y = b,\ z = b.$$

The necessary thrust in undisturbed flight is equal to $T_0 = 10.8668$ N

8.1. The first research case: x = 2*b

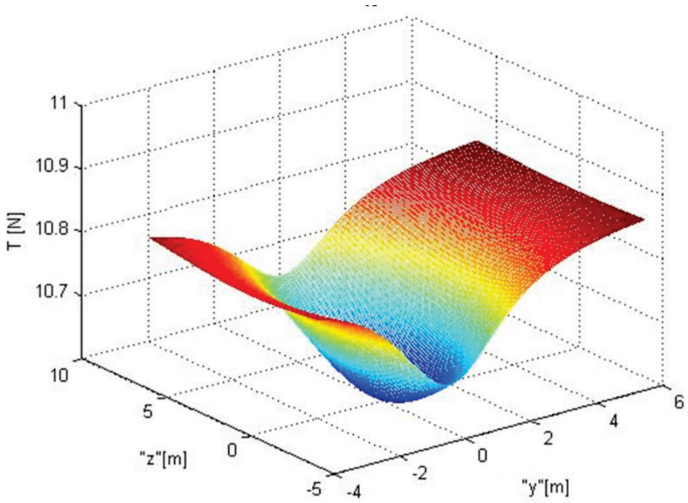

*Fig. 11. Thrust T at x = 2*b*

*Tab. 2. Optimization results for x =2*b*

	y = -b, z = -b	y = 0, z = 0	y = b, z = b
Number of iterations	8	1	8
y [m]	0	0	0
z [m]	0	0	0
w [m/s]	-0.0149	-0.0149	-0.0149
L [N]	0.2032	0.2032	0.2032
T [N]	10.6037	10.6037	10.6037

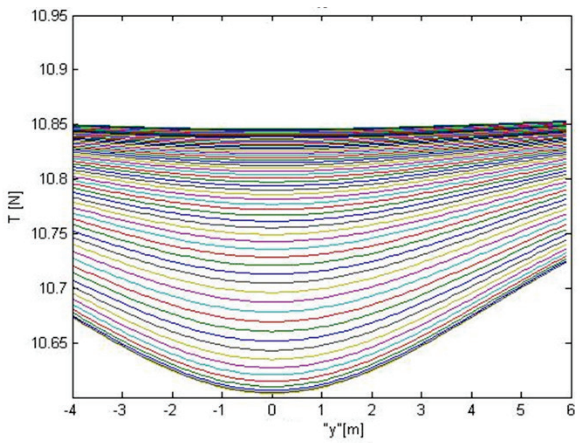

*Fig. 12. Thrust T 2D at x = 2*b*

8.2. The second research case: x = 1.32 m

x = 1.32 m is the distance for which the tip of the follower's wing is located just behind the leader's horizontal empennage

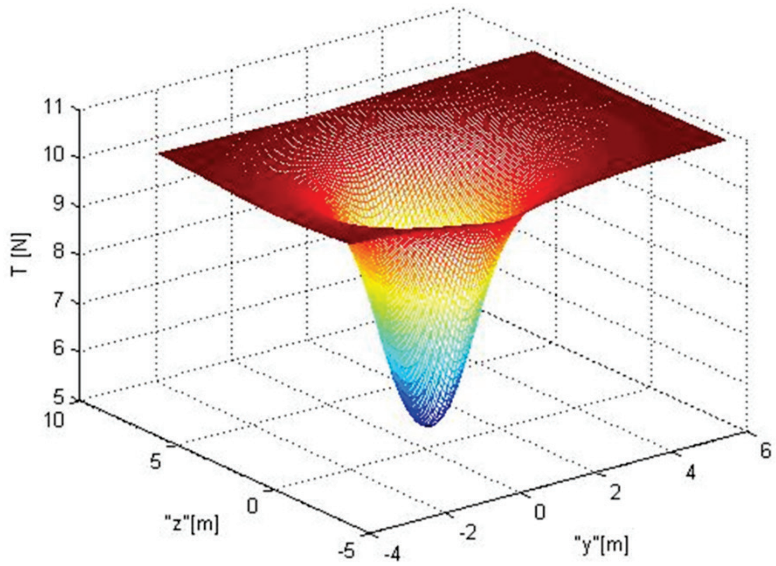

Fig. 13. Thrust T at x = 1.32 m

Tab. 3. Optimization results for x = 1.32 m

	y = -b, z = -b	y = 0, z = 0	y = b, z = b
Number of iterations	8	1	8
y [m]	0	0	0
z [m]	0	0	0
w [m/s]	-0.0149	-0.0149	-0.0149
L [N]	0.2032	0.2032	0.2032
T [N]	10.6037	10.6037	10.6037

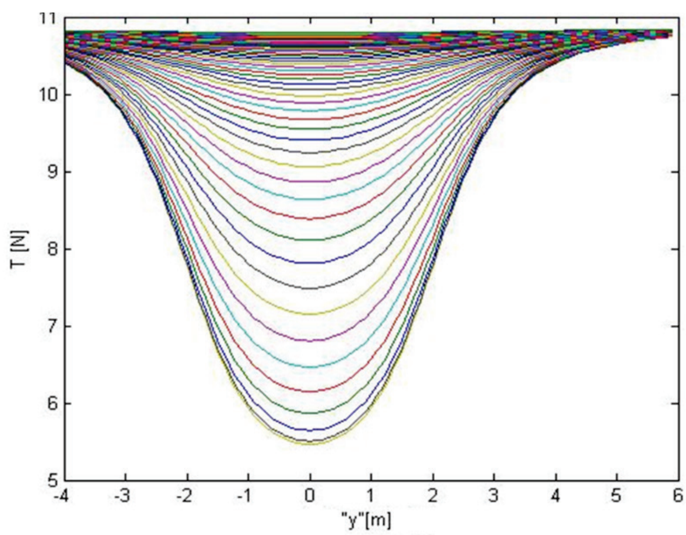

Fig.14. Thrust T 2D at x = 1.32 m

9. Summary

The article presents the problem regarding the optimization of the necessary thrust in order to fly in V formation. The observation of birds flying in V formations suggested the idea that this type of flight is the most optimal. Studies have shown that thanks to the application of a group flight with defined shape, birds are able to fly longer and further than a single individual bird. Employed mathematical model enabled the possibility to achieve expected results. The vortex radius generated by the aircraft with wingspan of 4 meters is equal to only about 0.0190 meter [9]. However, the further from the vortex core, the circulation impact decreases. In work [11], it was shown that the position of the wingman wing tip coinciding with the leader's axis of symmetry allows obtaining the maximum benefit from the flight in the formation. Performance of calculations for three different positions on the x-axis showed that the closer to the leader the more beneficial impact on the follower. However, when taking into consideration the safe longitudinal separation between aircraft and the behaviour of birds, the position between aircraft shall be assumed as: in the longitudinal axis as one span, in the lateral axis as the point for which the wing tips are in one line and they are having the same height. For the above-mentioned setting, the gain on the necessary thrust is around 7%. For an unmanned aircraft vehicle that takes about 5 kg of fuel on board, the obtained gain is barely noticeable. However, for aircraft with significantly greater wingspans, the fuel gain will already be calculated in tons, resulting with savings equal to millions of millions per year. According to the calculations made in [12] during in formation flight, there is not only economic advantage but also a pro-ecological one. For a transatlantic flight there is a possibility of reduction NOx emission by 33 kg in case of 7% fuel saving. The future of aviation in the perspective of many years will be shaped by unmanned aircraft. This will allow airlines to save on salaries for pilots. However, the use of formation flights, not only within one airline, but also on a global scale, will enable the possibility to save the fuel, which was demonstrated in the given article.

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