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ANALYSIS OF THE INFLUENCE OF THE CHANGES IN THE VALUE OF DYNAMIC VISCOSITY COEFFICIENT IN THE DIRECTION OF OIL FILM THICKNESS ON THE JOURNAL BEARING LOAD CARRYING CAPACITY

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Abstract

This article presents the results of numerical calculations of the hydrodynamic pressure distribution, load carrying capacity, friction force and friction coefficient of the slide journal bearing, if the assumed model of hydrodynamic lubrication takes into account the dependence of oil viscosity values on its temperature in all three directions of the adopted coordinate system, in particular, also across the thickness of the lubricant layer. This research considered the slide journal bearing lubricated with the Newtonian oil. The flow of oil was modelled as laminar and stationary. The bearing bushing had a full angle of wrap and its surfaces were smooth. In order to obtain hydrodynamic pressure distributions, the Reynolds type equation was numerically solved by application of the finite difference method (FDM). The numerical procedures for this research were prepared with the Mathcad 15 software. When adopting the classic models and simplifications for the hydrodynamic lubrication and a thin boundary layer, it is assumed, that the hydrodynamic pressure of lubricating oil does not depend on the position measured across the height of the lubrication gap. On the other hand, it is known, that the dynamic viscosity strongly depends on the temperature, which is a function of all three spatial variables. The aim of this work is to include, in the hydrodynamic lubrication model, the changes of viscosity in the direction of oil film thickness, and to investigate how it will affect the hydrodynamic pressure distribution and load carrying capacity of the journal bearing.

Keywords: *slide journal bearing, load carrying capacity, friction force, friction coefficient, numerical calculation*

1. Introduction

Modern constructions of the sliding friction nodes, and what is following this, modern, non-classical lubrication fluids, where apparent viscosity is a function of the shear rate, make it necessary to consider viscosity changes across thin layer of lubricant [4, 5, 12, 13]. Moreover, the increased values of adhesion forces in the layer directly adhering to the cooperating surfaces cause the effect of changing the oil viscosity across the thickness of the oil film [13]. Another example, which justifies the necessity to take the viscosity changes in the direction of layer thickness into account, is the fact of temperature changes in the direction of lubricant gap height, which results directly from the solution of the energy conservation equation [2, 4, 6, 12]. Such a change in temperature in a very thin layer of the lubricant always causes a significant change not so much in the viscosity of the lubricant but its gradients in the direction of the thickness of the gap.

Omission of taking oil viscosity changes across thickness of lubricant layer in previous papers of numerous authors [1, 3, 7, 8] in the field of hydrodynamic lubrication theory leads to two basic contradictions. The first one is the obvious variation of temperature across the thickness of the lubrication gap in the thin lubrication layer, which should, but does not imply similar viscosity changes across the thickness of the oil film in the previous solutions. The second contradiction arises when the constant viscosity across the thickness of the lubrication gap is assumed, which implies a direct influence of pressure on the temperature distribution in solutions, while the influence of temperature on the pressure distribution remains to be taken into account in further steps of approximate solutions.

In this article, author presents analytical solutions of the velocity vector components and temperature with taking dynamic viscosity changes across thickness of the lubricating layer into account. The article also presents the Reynolds type equation, which allows the numerical determination of the hydrodynamic pressure distribution with taking dynamic viscosity changes across thickness of the lubricating layer into account.

The description of the above problem requires consideration of the Reynolds type equation in conjunction with the non-linear energy conservation equation. From the practical engineering aspects, this model illustrates the interplay of temperature on hydrodynamic pressure and the effect of pressure on temperature.

The purpose of this work is to check what influence on carrying capacity, friction force and friction coefficient has taking viscosity changes across layer thickness into account. Therefore, for the considerations, a bushing with smooth surface and a full angle of wrap was adopted. Flow of the lubricating fluid is laminar, stationary, and non-isothermal. Mass and inertia forces have been omitted. The considerations were carried out for the cylindrical coordinate system. The classic Newtonian model of the lubricant was adopted, only the influence of temperature on the dynamic viscosity change was taken into account.

2. Analytical research

The solution to the problem of hydrodynamic lubrication of transverse slide bearings, omitting mass forces and taking viscosity changes with temperature into account, includes the solution of basic equations, i.e. equations of momentum conservation, stream continuity and energy conservation [1, 4, 9-13]:

$$\rho\frac{d\mathbf{v}}{dt} = \mathrm{Div}\,\mathbf{S}\,, \tag{1}$$

$$\mathrm{div}(\rho\mathbf{v}) = 0\,, \tag{2}$$

$$\mathrm{div}(\kappa\,\mathrm{grad}\,T) + \mathrm{div}(\mathbf{vS}) - \mathbf{v}\mathrm{Div}\mathbf{S} = \rho\frac{d(c_v T)}{dt}\,, \tag{3}$$

where:

c_v – specific heat at constant volume [J/(kg·K)],
t – time [s],
$\mathbf{v}$ – oil velocity vector [$m\cdot s^{-1}$],
T – oil temperature distribution in the lubrication gap [K],
ρ – oil density [$kg\cdot m^{-3}$],
κ – lubrication oil conductivity [W/(m·K)].

Relationship, which describes correlation between coordinates of stress tensor $\mathbf{S}$ and shear rate coordinates $\mathbf{A}_1$ of the lubrication oil of Newtonian properties, was assumed in the following form [4, 5, 8-13]:

$$\mathbf{S} = -p\mathbf{I} + \eta_T\mathbf{A}_1\,, \tag{4}$$

where:

$\mathbf{I}$ – unity tensor,
p – hydrodynamic pressure [Pa],
η_T – dynamic viscosity coefficient [Pa·s].

In the equation (4), $\mathbf{A}_1$ tensor is described by the following relation [4-6, 8-13]:

$$\mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^T,\ \mathbf{L} \equiv \mathrm{grad}(\mathbf{v})\,, \tag{5}$$

where:

L – tensor of the velocity vector gradient [s^{-1}],

In analytical considerations, the general function of viscosity variations depending on three variables of the coordinate system in the following form was adopted:

$$\eta_T = \eta_T(\phi, r, z), \tag{6}$$

where:

ϕ – perimeter coordinate,

z – longitudinal coordinate,

r – radial coordinate.

Simplified equation of momentum, stream continuity, and energy conservation for laminar and stationary lubrication after omission of the units in the magnitude of the radial relative clearance (ψ = 0.001) has the following form [9-13]:

$$0 = -\frac{1}{R}\frac{\partial p}{\partial \phi} + \frac{\partial}{\partial r}\left(\eta_T(\phi, r, z)\frac{\partial v_1}{\partial r}\right), \tag{7}$$

$$0 = \frac{\partial p}{\partial r}, \tag{8}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial r}\left(\eta_T(\phi, r, z)\frac{\partial v_3}{\partial r}\right), \tag{9}$$

$$0 = \frac{1}{R}\frac{\partial v_1}{\partial \phi} + \frac{\partial v_2}{\partial r} + \frac{\partial v_3}{\partial z}, \tag{10}$$

$$\frac{\partial}{\partial r}\left(\kappa\frac{\partial T}{\partial r}\right) + \eta_T(\phi, r, z)\left[\left(\frac{\partial v_1}{\partial r}\right)^2 + \left(\frac{\partial v_3}{\partial r}\right)^2\right] = 0, \tag{11}$$

$$0 \le \phi \le 2\pi,\ -b_m \le z \le b_s,\ 0 \le r \le h_p,$$

where:

v_1 – perimeter component of the oil velocity vector [$m \cdot s^{-1}$],

v_2 – radial component of the oil velocity vector [$m \cdot s^{-1}$],

v_3 – longitudinal component of the oil velocity vector [$m \cdot s^{-1}$],

T – oil temperature in the lubrication gap [K],

p – hydrodynamic pressure [Pa],

R – journal radius [m],

κ – heat transfer coefficient [W/(m·K)].

In order to solve the system of equations (7-11) we integrate equation (7) and (9) twice and determining the constant of integration by applying appropriate boundary conditions. To determine the radial component of the velocity vector, we transform the continuity equation of the stream (10) and then integrate it one time. The integration constant is determined from the appropriate boundary condition. Boundary conditions for lubricant velocity components have the form [9-13]:

$$v_1 = \omega R \text{ for } r=0, \text{ and } v_1 = 0 \text{ for } r = h_p, \tag{12}$$

$$v_2 = 0 \text{ for } r=0, \text{ and } v_2 = 0 \text{ for } r = h_p, \tag{13}$$

$$v_3 = 0 \text{ for } r=0, \text{ and } v_3 = 0 \text{ for } r = h_p. \tag{14}$$

wherein:

$$h_p(\phi, z) = \varepsilon \cdot \left[1 + \lambda\cos(\phi) + z \cdot \tan(\gamma)\cos(\phi)\right], \tag{15}$$

where:
h_p – dimensional lubrication gap,
γ – angle between journal axis and bushing axis,
ε – radial clearance [m].

Perimeter and longitudinal component of the velocity vector have the following [12, 13]:

$$v_1(\phi, r, z) = \left(\frac{1}{R}\frac{\partial p}{\partial \phi}\right) A_\eta + (1 - A_s)\omega R\,, \tag{16}$$

$$v_2(\phi, r, z) = -\int_0^r \frac{1}{R}\frac{\partial v_1}{\partial \phi} dr - \int_0^r \frac{\partial v_3}{\partial z} dr\,, \tag{17}$$

$$v_3(\phi, r, z) = \left(\frac{\partial p}{\partial z}\right) A_\eta\,. \tag{18}$$

In Eqs. (16, 18) are following notations:

$$A_s(\phi, r, z) \equiv \frac{\int_0^r \frac{1}{\eta_T} dr}{\int_0^{h_p} \frac{1}{\eta_T} dr}, \quad A_\eta(\phi, r, z) \equiv \int_0^r \frac{r}{\eta_T} dr - \frac{\left(\int_0^r \frac{1}{\eta_T} dr\right)\left(\int_0^{h_p} \frac{r}{\eta_T} dr\right)}{\int_0^{h_p} \frac{1}{\eta_T} dr}, \tag{19}$$

where: $0 \le \phi \le 2\pi$, $b_m \le z \le b_s$, $0 \le r \le h_p$, $h_p = h_p(\phi, z)$, $\eta_T(\phi, r, z)$.

Applying the second boundary condition (13_2) to the solution (17), we obtain the Reynolds type equation on the basis of which the hydrodynamic pressure distribution can be determined numerically. This equation has the following form:

$$\frac{1}{R^2}\frac{\partial}{\partial \phi}\left[\left(\frac{\partial p}{\partial \phi}\right)\left(\int_0^{h_p} A_\eta dr\right)\right] + \frac{\partial}{\partial z}\left[\left(\frac{\partial p}{\partial z}\right)\left(\int_0^{h_p} A_\eta dr\right)\right] = \omega \frac{\partial}{\partial \phi}\left[\int_0^{h_p} A_s dr - h_p\right], \tag{20}$$

where

$$\int_0^{h_p} A_s(\phi, r, z) dz \equiv \frac{\int_0^{h_p}\left(\int_0^r \frac{1}{\eta_T} dr\right) dr}{\int_0^{h_p} \frac{1}{\eta_T} dr}, \quad \int_0^{h_p} A_\eta(\phi, r, z) dr \equiv \int_0^{h_p}\left(\int_0^r \frac{r}{\eta_T} dr\right) dr - \left[\frac{\int_0^{h_p}\left(\int_0^r \frac{1}{\eta_T} dr\right) dr}{\int_0^{h_p} \frac{1}{\eta_T} dr}\right]\left(\int_0^{h_p} \frac{r}{\eta_T} dr\right), \tag{21}$$

for: $0 \le \phi \le 2\pi$, $b_m \le z \le b_s$, $\eta_T(\phi, r, z)$.

The solution of the energy equation (11) assuming a constant value of the heat transfer coefficient and the adoption of classical boundary conditions ($T(\phi, r, z) = T_0 + f_c$ for $r=0$; $T(\phi, r, z) = T_0 + f_p(\phi, z)$ for $r = h_p$; $\kappa \frac{\partial T}{\partial r} = -q_c$ for $r = 0$) allows to obtain the following form of the temperature distribution in the lubricating oil:

$$T(\varphi, r, z) = -\frac{q_c r}{\kappa} + f_c - \frac{1}{\kappa}\int_0^r\left(\int_0^r W_{T1}(\varphi, r, z) dr\right) dr\,, \tag{22}$$

$$W_{T1}(\phi,r,z) \equiv \frac{1}{\eta_T}\left\{\left[\left(\frac{1}{R}\frac{\partial p}{\partial \phi}\right)\left(r-\frac{\int\limits_0^{h_p}\frac{r}{\eta_T}dr}{\int\limits_0^{h_p}\frac{1}{\eta_T}dr}\right)-\frac{\omega R}{\int\limits_0^{h_p}\frac{1}{\eta_T}dr}\right]^2+\left[\left(\frac{\partial p}{\partial z}\right)\left(r-\frac{\int\limits_0^{h_p}\frac{r}{\eta_T}dr}{\int\limits_0^{h_p}\frac{1}{\eta_T}dr}\right)\right]^2\right\}. \tag{23}$$

Load carrying capacity, friction force and friction coefficients are determined from the following dependencies [9-13]:

$$C = R\sqrt{\left(\int\limits_{-b}^{+b}\left(\int\limits_0^{\phi_k} p\cos\gamma\sin\phi\, d\phi\right)dz\right)^2+\left(\int\limits_{-b}^{+b}\left(\int\limits_0^{\phi_k} p\cos\gamma\cos\phi\, d\phi\right)dz\right)^2}, \tag{24}$$

$$Fr = R\int\limits_{-b}^{+b}\left[\int\limits_0^{\phi}\left(\eta_T\frac{\partial v_1}{\partial r}\right)_{r=h_p} d\phi\right]dz, \tag{25}$$

$$\mu = \frac{Fr}{C}. \tag{26}$$

3. Numerical calculations

Numerical calculations of hydrodynamic pressure were made using the finite difference method, solving the equation (20) for two variants. The first variant assumes that the dynamic viscosity depends only on the peripheral and longitudinal variable $\eta(\phi,z)$. In the second calculation variant, it was assumed that the viscosity depends on peripheral, radial and longitudinal variable $\eta(\phi,r,z)$. In both variants, the dependence of viscosity on temperature is described by the classical exponential function. Calculations have been performer in Mathcad 15 software, using own calculation procedures. Having hydrodynamic pressure values, load carrying capacity, friction forces and friction coefficients were determined for the relative eccentricity $\lambda = 0.1$ do $\lambda = 0.9$ and dimensionless bearing length $L_1 = 1$. For the calculations, journal radius $R = 0.02$ was assumed, angle between journal axis and bushing axis $\gamma = 0$, radial clearance $\psi = 0.002$. Journal radial speed was also assumed $\omega = 400\ s^{-1}$ and coefficient of viscosity changes in temperature $\delta_T = 0.04267\ K^{-1}$. Characteristic dimensional value of the dynamic viscosity for the characteristic temperature $T_o = 363$ K was $\eta_o = 0.01546$ Pas. For the calculations, also heat transfer coefficient $\kappa = 0.15$ W/(m·K) was assumed.

Calculated on the basis of equation (24) the values of the load carrying capacity in the function of relative eccentricity are presented in Fig. 1, while the friction force values determined on the basis of equation (25) are shown in Fig. 2. Friction coefficient (26) in a function of relative eccentricity is shown in Fig. 3.

Percentage differences in load carrying capacity, friction force and friction coefficient for both calculation variants (variant I: $\eta(\phi,z)$ – viscosity independent from variable r; variant II: $\eta(\phi,r,z)$ – viscosity dependent from variable r) are shown in Tab. 1. The change values have been calculated based on the following dependencies:

$$\Delta C = \frac{C[\eta(\phi,z)]-C[\eta(\phi,r,z)]}{C[\eta(\phi,z)]}\cdot 100\%,$$

$$\Delta Fr = \frac{Fr[\eta(\phi,z)]-Fr[\eta(\phi,r,z)]}{Fr[\eta(\phi,z)]}\cdot 100\%, \tag{27}$$

$$\Delta\mu = \frac{\mu[\eta(\phi, z)] - \mu[\eta(\phi, r, z)]}{\mu[\eta(\phi, z)]} \cdot 100\% .$$

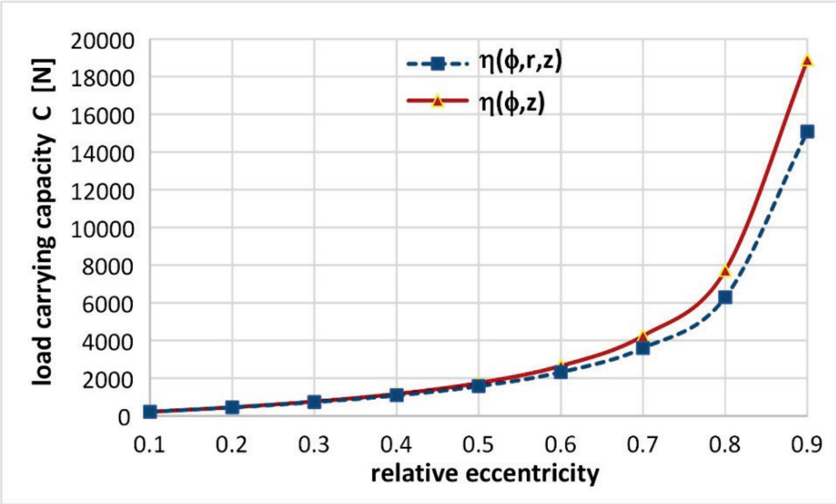

Fig. 1. Values of the load carrying capacity for two calculation variants in a function of relative eccentricity

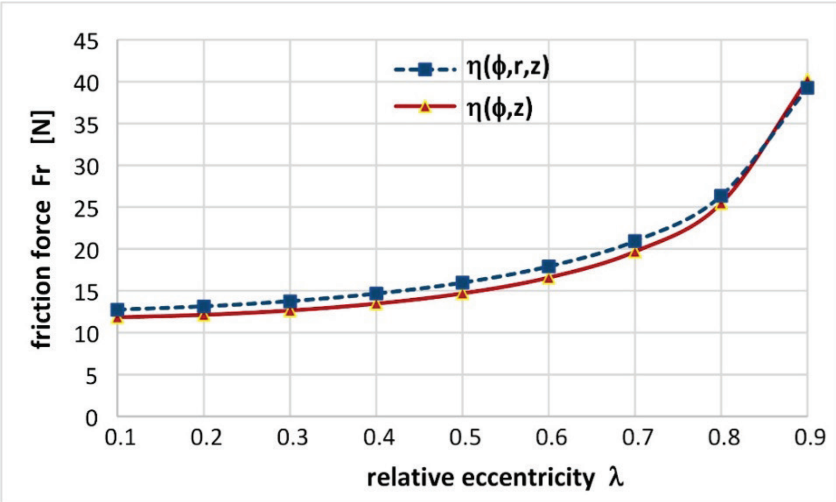

Fig. 2. Values of the friction force for two calculation variants in a function of relative eccentricity

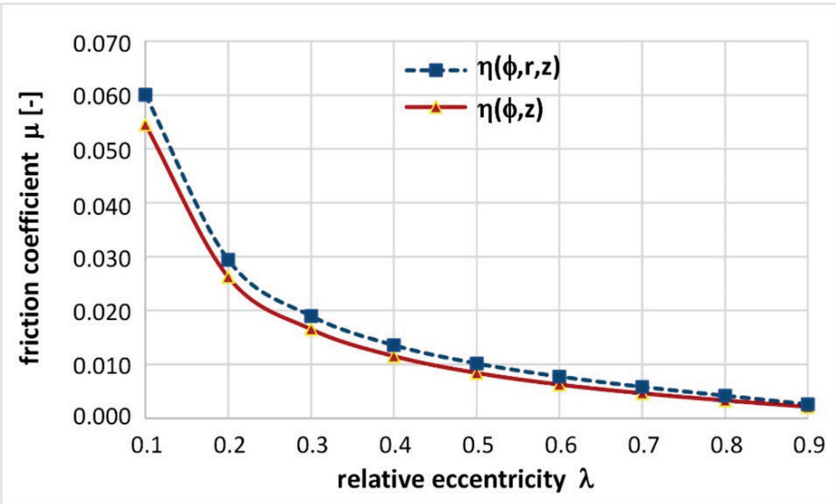

Fig. 3. Values of the friction coefficient for two calculation variants in a function of relative eccentricity

Tab. 1. Percentage changes of the load carrying capacity, friction force and friction coefficient in a function of relative eccentricity

Relative eccentricity	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ΔC [%]	2.3	3.6	5.4	7.5	10.1	12.8	15.4	18.4	20.1
ΔFr [%]	-7.6	-8.3	-8.7	-8.8	-8.7	-8.0	-6.3	-3.5	2.4
Δμ [%]	-10.1	-12.3	-14.9	-17.6	-20.8	-23.7	-25.7	-26.8	-22.2

4. Conclusions and observations

The following conclusions can be drawn from the analysis of the obtained results:

– load carrying capacity, with taking viscosity changes from the perimeter, radial and longitudinal variable into account, is smaller than the load carrying capacity when taking viscosity changes only from perimeter and longitudinal variable into account. The difference is the smallest for small relative eccentricities of the order of 2-3%, and the biggest for large relative eccentricities (18-20%),
– friction force, with taking viscosity changes from the perimeter, radial and longitudinal variable into account, is bigger than the load carrying capacity when taking viscosity changes only from perimeter and longitudinal variable into account. The difference is fairly constant for almost full range of relative eccentricities (6-9%). For the higher relative eccentricities, the difference in values of friction forces decreases,
– friction coefficient, with taking viscosity changes from the perimeter, radial and longitudinal variable into account, is bigger than the load carrying capacity when taking viscosity changes only from perimeter and longitudinal variable into account. The difference is smallest for small relative eccentricities of the order of 10%, and the biggest for large relative eccentricities 27%.

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