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A STOCHASTIC MODEL OF CHANGES IN THE OPERATIONAL STATES OF MEANS OF TRANSPORT

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Abstract

In the article, application of a Markov process for modelling the process of transport means operational state changes whose duration times can be estimated by Erlang distribution is presented. The research object is a system of city bus transport operation, used in a selected agglomeration. A homogenous Markov process was assumed a mathematical model of the process of transport means operation. Transformation of a stochastic process, for which distributions of random variables denoting duration times of the states are Erlang distributions, into a stochastic process characterized by exponential distribution of the state duration times, was performed at the costs of the state space extension. The use of a homogeneous Markov process for mathematical modelling of the operation process was dictated by the fact that the process reflects adequately, from the point of view of the research goal, the real operation process. In order to illustrate the process, a hypothetical, simplified calculation example was provided. Reliability of technical objects needs to be maintained at appropriate level, which is the responsibility of the repair and maintenance subsystem ensuring. This subsystem consists of two subsystems. The main tasks of the mobile emergency teams include providing vehicles, that is, buses that are outside the depot with the serviceability state as quickly as possible or hauling damaged buses to technical service stations.

Keywords: *homogeneous Markov process, operation process, operational state, transport means*

1. Introduction

The article shows application of Markov process for modelling operational state changes of vehicles characterized by non-exponential distribution of their duration times. The initial assumption of the research was that a homogenous Markov process is a mathematical process to be used for description of a technical object operation process. Transformation of a stochastic process with a finite state space, whose random variable distributions denoting the state duration times are Erlang distributions, was performed at the cost of extension of the state space of a random process, which is a mathematical model of the analysed technical objects operation process. A real city bus transport system of a selected urban agglomeration was used to illustrate the issues addressed in this study he analysed

2. Research object

The research object, which illustrates the issues presented in this study, is a system of public city bus transport operating in a given urban agglomeration. The main goal of the system operation is provision of effective and safe transport services by bus transport means over a given area and in a given quantitative scope.

Direct provision of the analysed services is the responsibility of an executive subsystem including elementary subsystems of the type <D – TO> (driver – bus), where a human is coupled, with a technical object by means of a series structures. Reliability of technical objects needs to be maintained at appropriate level, which is the responsibility of the repair and maintenance subsystem ensuring. This subsystem consists of two subsystems:

- repair and maintenance subsystem, including the so-called service station ST, where diagnostic, reviews and repair services are provided,
- subsystem of the so-called mobile emergency teams which provides repair services outside the service station.

The service station offers repairs and diagnostic services, such as:

- current services,
- periodic technical services,
- current repairs,
- technical state checks.

The main tasks of the mobile emergency teams include providing vehicles, that is, buses that are outside the depot with the serviceability state as quickly as possible or hauling damaged buses to technical service stations in case there is no possibility to repair them outside the depot.

In the next part of this study, it is assumed that the objects operated in the system are homogenous.

The analysed research object uses buses of different makes and types. Several traffic routes are served. One of the most important decision to be taken is selection of transport means to provide services along particular routes. In work [7], an example of a method for selection of a technical object to provide transport services, is presented. This method is based on analyses of a homogenous Markov process as a model of operation, whereas work [8] presents economic aspects of purchasing a transport means be used in a public transport system.

3. Major assumptions of the operation model

Random process X(t) with a finite number of S states and set of R+ parameters (subset of real numbers >=0) is a natural mathematical model of the operation process of different technical objects [4, 5]. Homogenous stochastic processes, including Markov and semi Markov processes as well as Markov and semi Markov decision processes, are commonly used for operational state change modelling [2, 5, 6, 9-12, 14, 15].

Three operational bus states, important from the point of view of the research goal, were accepted in result of the research object identification, including:

S_1 – operation state (transport task accomplishment),
S_2 – renovation state provided by mobile technical service teams,
S_3 – renovation state performed in the service station.

An assumption was accepted that stochastic process $\{X(t), t \geq 0\}$ is an initial mathematical model of the bus operation process. The analysed stochastic process $\{X(t), t \geq 0\}$ has a finite phase space S, $S = \{S_1, S_2, S_3\}$. The theory of homogenous Markov processes was assumed to be used for a description of the analysed technical objects. An analysis of the state space and events regarding buses made it possible to develop a directed graph for representation of the operation process carried out in the research object whose implementation is carried out by means of transition intensity matrixes Λ [3, 4, 13]:

$$\Lambda = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 \\ \mu_1 & -(\mu_1+\mu_3) & \mu_3 \\ \mu_2 & 0 & -\mu_2 \end{bmatrix}. \tag{1}$$

4. Selected results of initial investigations

An analysis of the technical object's being in the identified states was made as part of the initial investigations performed in the considered bus transport system. The investigations were carried out in real service conditions of bus operation by means of a passive experiment.

The experiment results presented in further part of the study apply to buses of the type conventionally marked by code A. The results describe 16 vehicles used in the analysed system and cover four-month period. Tab. 1 shows the results of initial investigations concerning the three analysed operational states [14]: correct operation of buses (S_1), renovation of buses provided by the mobile technical service teams (S_2), renovation performed in the service station (S_3).

Tab. 1. Values of selected statistics regarding duration time of the analysed operational states [14]

Statistics	Operational state		
	S_1	S_2	S_3
Number of observations	174	89	87
Mean value	295.06	1.11	1,71
Standard deviation	353.74	0.54	1,15
Minimum	7.22	0.05	0.23
Maximum	1824	2.75	6.17
Difference	1816.78	2.70	5.94
Variance	125131.0	0.29	1.32
Median	165.81	1.15	1.30
Mode	48.00	1.67	0.92

The analysis of duration times of technical services performed by the mobile technical teams (PT) included only services which caused the transport task delays whereas, the data concerning. Times of repairs performed in service stations (SS) included only those, which were caused by a bus damage detected during transport task performance and resulted in the so-called course.

5. Model of the operation process carried out in the research

Selection of mathematical apparatus to be used for description of the studied process operation was carried out on the basis of the following factors:
- the study goal,
- accuracy of the real process representation by the model,
- complexity degree of the used mathematical apparatus,
- possibility to obtain data concerning the operation process carried out in the research object.

On the basis of studies and analyses Markov process and the theory concerning this process analysis was found to be the most useful tool – from the point of view of the study goal – for modelling a real operation process carried out in the research object. Stochastic process X(t) being a homogenous Markov process with a finite set of S states can be fully described by:
- initial distribution X(t),
- matrix Λ of X process state change intensity.

Using the homogenous Markov process for mathematical modelling of the operation process, was assumed that, from the point of view of the research goal, the process represents quite well the modelled real operation process.

Using the theory of Markov process it is possible to determine, for the analysed operation model, probabilities $P_i(t)$, i = 1, 2, 3, of a technical object being in the identified operational states S_i. In order to develop a system of differential equations of A. N. Kolmogorov in the form:

$$P'(t) = P(t)\Lambda, \quad (2)$$

where:
P'(t) – column vector consisting of derivatives $P_i'(t)$,
P(t) – probability vector $P_i(t)$,

Λ – matrix of the process state change intensity.

Solution of equation system (2) can be written in the form matrix exponent:

$$P(t) = P(0)\, e^{t\Lambda}\,. \tag{3}$$

Lagrange–Silvester interpolation polynomial is used to solve this function. For this purpose, it is necessary to determine characteristic matrixes Λ, which are solutions to equation:

$$\Delta(\lambda) = 0, \tag{4}$$

where: $\Delta(\lambda)$ – characteristic polynomial of Λ,expressed by dependency:

$$\Delta(\lambda) = \lambda^n - \Delta_1\lambda^{n-1} - \ldots - \Delta_{n-1}\lambda - \Delta_n. \tag{5}$$

Matrix associated with matrix Λ is an important element, needed to determine matrix function (3). This matrix is determined as Adj $\|\lambda I - \Lambda\|$ and is determined from dependence:

$$\mathrm{Adj}\,\|\lambda I - \Lambda\| = \lambda^{n-1} B_0 + \lambda^{n-2} B_1 + \ldots + B_n, \tag{6}$$

where:

$B_k = \Lambda B_{k-1} - \Delta_k I$, for k = 1, 2, ... n–1,

$B_0 = I$,

I – unit matrix of n dimension.

In determining matrix Adj $\|\lambda I - \Lambda\|$ coefficients Δ_i, i = 1, 2, ..., n of equation (5), need to be defined.

For simplification, solutions of equation (4) were marked as λ_s. With the assumption that multiplicity of the root is equal to m_s, s = 1, 2, ..., n, indexes of real roots make up set I_r, whereas, associated roots make up set I_c. On the basis of works [1, 4], matrix P(t) can be written in the form:

$$P(t) = P(0)\left\{\sum_{s=1}^{u} \exp(\lambda_s t) \sum_{k=1}^{m_s} \frac{t^{k-1}}{(k-1)!} Z_{m_s - k}(\lambda_s)\right\}, \tag{7}$$

where:

$$Z_l(\lambda_s) = \frac{1}{l!}\left\{\frac{d^l}{d\lambda^l}\left[Adj\,\|\lambda I - \Lambda\| \prod_{\substack{r=1 \\ r\neq s}}^{u} (\lambda - \lambda_r)^{-m_s}\right]\right\}, \tag{8}$$

$\lambda = \lambda_s$,

$l = m_s$-k,

u – number of different roots of equation (4).

When all roots of equation (4) are real and single, the solution of (7) can be presented in the following form:

$$P(t) = P(0)\left\{\sum_{s=1}^{n} \exp(\lambda_s t) Adj\,\|\lambda_s I - \Lambda\| \prod_{\substack{r=1 \\ r\neq s}}^{n} (\lambda_s - \lambda_r)^{-1}\right\}. \tag{9}$$

Associated matrix Adj $\|\lambda I - \Lambda\|$ and coefficients of characteristic polynomial (5) of characteristic matrix Λmatrix were determined by Fadiejev algorithm.

Experimental tests were carried out in a real transport system in order to verify the assumptions that homogenous Markov process with a finite space of states can be used for the process of bus operation modelling.

One of the research goals was to define distributions of random variables T_i, i = 1, 2, 3, denoting duration times of the analysed operational states $S_i \in S$ of the considered buses. Zero hypothesis H_0 was verified for each state S_i (i = 1, 2, 3) in such a way that the empirical distribution of random variable T_i is consistent with the exponential distribution. Compliance test χ^2 (Pearson) was used for verification of the accepted hypothesis. Lack of grounds for rejection of the verified hypothesis were found only for random variable T_1 (for significance level α = 0.05). Whereas, no grounds were found for rejection of the verified hypothesis of compliance of empirical distribution of the remaining random variables with Erlang distribution. Then, random variable T_i can be presented in the form [3]:

$$T_i = T_{i1} + T_{i2} + \ldots + T_{ik}, \tag{10}$$

where: T_{ij} – independent random variables with exponential distribution with parameter λ.

Only transition to state $S_{i\,j+1}$, is possible form S_{ij} ($1 \leq j \leq k-1$).

Erlang distribution is characterized by distribution function in the form:

$$F(x) = 1 - e^{-\lambda x}\left(1 + \frac{\lambda x}{1!} + \frac{(\lambda x)^2}{2!} + \cdots + \frac{(\lambda x)^{k-1}}{(k-1)!}\right), x \geq 0, \lambda > 0, k \in N. \tag{11}$$

The minimum of square sum of Erlang distribution function $F(x)$ value deviation from the value of empirical distribution function $F_e(x)$ was accepted to be the criterion for selection of optimal values of parameters λ:

$$S(\lambda, k) = \sum_{i=1}^{m} [F(x_l) - F_e(x_l)]^2, l \in N. \tag{12}$$

In order to estimate the value of parameters k and λ, a calculation algorithm was developed.

In this way, transformation of process $\{X(t), t \geq 0\}$ with space of S states into process $\{Y(t), t \geq 0\}$ with state space for which distributions of operational state duration times are exponential was performed at the cost of the state space extension. Intensity of Y(t) process transitions is presented by dependence [1]:

$$\Lambda' = \begin{bmatrix} -\lambda_1 - \lambda_2 & \lambda_1 & 0 & \lambda_2 & 0 \\ 0 & -\mu_1 & \mu_1 & 0 & 0 \\ \mu_{21} & 0 & -\mu_{21} - \mu_{23} & \mu_{23} & 0 \\ 0 & 0 & 0 & -\mu_2 & \mu_2 \\ \mu_{31} & 0 & 0 & 0 & -\mu_{31} \end{bmatrix}. \tag{13}$$

Whereas, the graph of operational state changes is presented in Fig. 1.

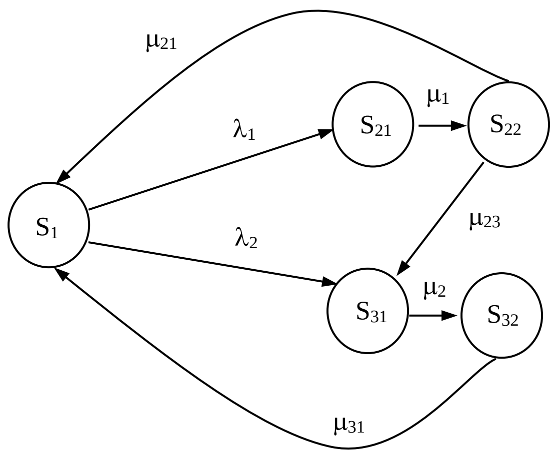

Fig. 1. Directed graph of operational states

6. Calculation example

Systems of public bus transport operation use buses of different types. Selection of the vehicle type to provide given transport services is an important issue to be coped with by the system decision makers. If there are different types of vehicles it is possible to obtain source data to estimate the model parameters values the issues addressed in the study can be used to facilitate selection of technical objects to be used for accomplishment of the assigned tasks. The condition described by the below dependence was accepted to be a criterion for selection of a given vehicle type to accomplish a given transport task.

$$Z(t) = \sum_{i=1}^{3} K_i(t) = \max , \tag{14}$$

where:

$K_i(t) = \int_0^{t_a} P_i(t) C_i(t) dt$ – profit (loss) connected with the bus being in the i-th operational state,

t_a – time of analysis,

$P_i(t)$ – momentary probability of a bus being in the i-th operational state,

$C_i(t)$ – cost connected with a technical object being in state S_i per time unit.

Further, it is accepted that:

$$C_i(t) = \text{const} = C_i,\ i = 1, 2, 3. \tag{15}$$

In order to illustrate the discussion, the values of summary profits obtained from provision of transport services Z(t) was determined. Details concerning the expected value $E(T_1)$ of the time of the analysed technical object correct operation are presented in Tab. 1. The values of costs incurred per time unit of the bus being in a given operational state are C_1 = 60.38 [PLN/h], C_2 = 107.62 [PLN/h], C_3 = 314.28 [PLN/h].

The remaining basic data describing the operation process model is presented in Tab. 2. Value $P_i(t)$, i = 1, 2, 3 was calculated on the basis of a developed computer program to be used for matrix method for differential equation solution for Markov process [14].

Figure 2 presents a dependence of the vehicle probability of being in state S_2 on time, whereas Fig. 3 shows dependence Z(t).

Tab. 2. Input data to be used for calculations

λ	λ_1	λ_2	μ_1	μ_{21}	μ_{23}	μ_2	μ_{31}
0.0079	0.0042	0.0036	1.7698	1.1150	0.6548	1.1976	1.1976

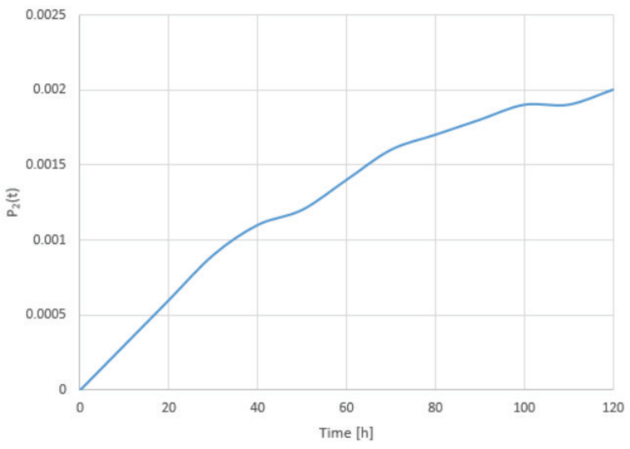

Fig. 2. Dependence of a technical object probability of being in S_2 on time

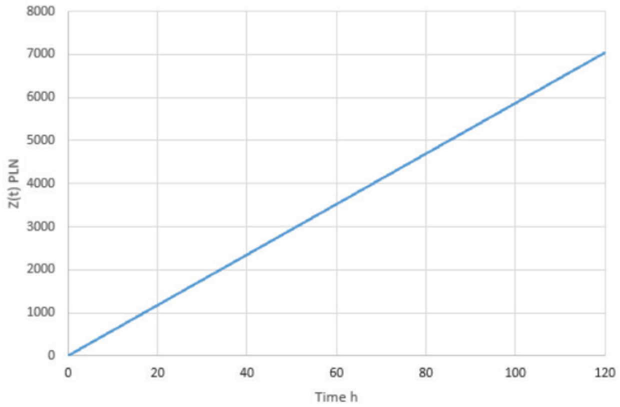

Fig. 3. Dependence of summary profits from bus operation

7. Conclusion

The aim of this study is to show the possibilities of using Markov process for modelling the operation process of technical objects characterized by not exponential distributions of operational state duration times.

The presented model of public bus transport operation is characterized by significant simplification. However, the description of its construction and analysis indicates that it can be used for an initial analysis of technical objects ability to perform transport tasks in a concrete system of operation.

Markov and semi Markov models depicting the process of technical objects operational state changes most commonly use boundary probabilities $p_i^* = \lim_{t \to \infty} p_i(t)$, i = 1, 2, ... It seems that determination of vales of the so-called momentary probabilities $P_i(t)$ significantly extends the application range of these models.

The issues discussed in the study relate to systems of public city bus transport systems. However, due to the assumed simplification degree of description apart from being used in transport systems, they can also be applied for analysis of the operation process of other systems.

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