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STRESS DISTRIBUTION IN AN ANISOTROPIC BEAM SUBJECTED TO LOAD

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Abstract

The article presents the physical equations describing the isotropic and anisotropic materials. Orthotropic material and monotropic are special varieties of anisotropy. Constructional steel, pine wood and polyester-glass composite were tested. The beams were made from these materials. The beams were subjected to external loads. The external load caused internal forces in beams. Calculations of stress distribution were carried out by finite element method (Patran – Nastran software). The calculation results allowed for precise illustrate the distribution of stress especially in layered materials. The load is basically a transmitted through the strong layers of composite. This is illustrated in the figures. Wood is materials of a layered structure and is classified as inhomogeneous materials. Whereas steel is considered as a homogeneous material. Passing from the level of microscopic inhomogeneity to the macroscopic homogeneous level is called homogenization. This method formulates the macroscopic description by homogenizing microscopic properties. For the purpose of mathematical description of a material, the real centre can be substituted by a homogeneous centre. The homogenization method is commonly used to describe the properties of rocks, wood, composites, reinforced concrete, as well as human osseous tissues. The description of the mechanical properties of isotropic materials is based on the theory of elasticity, while the anisotropic materials are based on the anisotropic theory of elasticity. Calculations of anisotropic materials are quite complicated (large number of physical quantities) and sometimes-approximate results are obtained.

Keywords: *isotropic material, anisotropic material, beam deflection, stress distribution in the beam*

1. Introduction

In natural conditions, there are materials, whose properties are independent of the direction of load exerted on them, but there are also materials whose properties depend on load direction. The first type mentioned is an isotropic material while the second type is called an anisotropic material. An example of an isotropic material in the macro scale is steel whereas an anisotropic material is exemplified by a monocrystal, which has various properties in proper crystallographic directions. Taking into account the micro scale, steel can also be regarded as an anisotropic material consisting of various phases, grains etc.

The article deals with construction materials applied in technical objects that are in the macro scale. The most common property used for assessing the material is the concept of specific rigidity and specific strength. Research has shown that when considering strength and rigidity, steel is not as good as composite materials, which are anisotropic materials. Composites are either natural or artificial materials, containing at least two different components, separated from each other or filling its volume evenly [13].

Wood is a natural composite consisting of alternating layers of an early and late wood. A specific case of anisotropy is known as orthotropy, in which it is possible to distinguish 3 perpendicular symmetry layers as well as monotropy, also called transverse isotropy characterized by a plane whose material has isotropic properties [13].

The description of mechanical properties of isotropic materials is based on the theory of elasticity, while for the anisotropic materials the anisotropic theory of elasticity is applied. The calculations for anisotropic materials are quite complicated as they involve a large number of physical quantities and sometimes lead to approximate results. There are numerous papers related to the strength of isotropic materials, whereas very few works deal with anisotropic materials [13].

This work presents the calculation results of stress distribution in beams made of 3 types of material: steel as an isotropic material, natural wood as an orthotropic material, and plastic as a monotropic material. Beams made of these materials were subjected to bending. The distribution of stresses in the form of contour lines was shown in the beams cross sections. Computer visualization allowed to illustrate in detail the stresses of beams made of different materials.

2. Physical equations for orthotropic materials

The general form of a physical equation for the theory of liner elasticity is as follows [3, 4, 7]:

$$\sigma_{ij} = Q_{ijkl}\ \varepsilon_{kl} \tag{1}$$

or after inversion:

$$\varepsilon_{ij} = S_{ijkl}\ \sigma_{kl}\,, \tag{2}$$

where σ_{ij} is the stress tensor component, ε_{kl} is deformation tensor component, Q_{ijkl} is rigidity matrix element, and S_{ijkl} is susceptibility matrix element. Q_{ijkl} or S_{ijkl} in the three-dimensional space have 81 fixed values.

An example of an orthotropic material is wood. This material belongs to the layered composites of orthotropic symmetry.

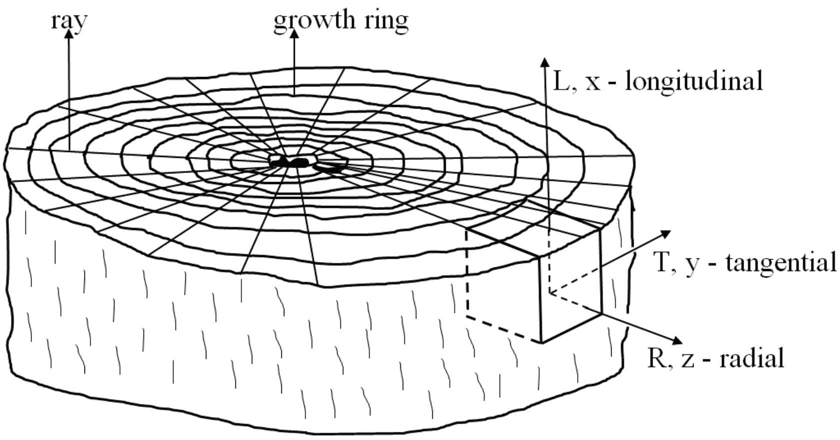


Fig. 1. The tree trunk and its anatomical directions

Due to the diversified rings arrangement, it is possible to distinguish three basic directions that are three planes of symmetry in the wood: longitudinal – $L(x)$, tangential – $T(y)$, radial – $R(z)$, Fig. 1. If the wood specimen is cut at a good distance from the tree centre so that the curve of the rings is negligibly small, the wood properties are of an orthotropic character [3, 4, 7, 8].

For the composites of orthotropic symmetry, the physical equations are as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}, \tag{3}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}. \tag{4}$$

Orthotropy is a case of anisotropy for which the number of independent components of rigidity matrix is 9. For the materials of transverse isotropy, further simplifications can occur because only one plane can be defined for them that are material isotropic plane as its material properties are identical in all directions. For example, if this plane is (x_1, x_2) plane, the physical relations will assume the following equation form:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & Q_{13} & 0 & 0 & 0 \\ Q_{13} & Q_{13} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (Q_{11}-Q_{12})/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{5}$$

or after inversion:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}. \tag{6}$$

The number of independent constants in rigidity matrix and susceptibility matrix is to be reduced to 5. In case of isotropic materials whose properties are identical in all directions, the physical equations are as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & (Q_{11}-Q_{12})/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (Q_{11}-Q_{12})/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (Q_{11}-Q_{12})/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \tag{7}$$

or after inversion:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}. \tag{8}$$

The number of independent components of the rigidity and susceptibility matrices is reduced to 2. The forms of rigidity and susceptibility matrices do not have physical interpretations and on the basis of their value, it is not possible to predict the activity of a material in case of simple strength tests like tensile test or shearing. However, it is possible to assign such properties to engineering constants also called technical constants, which comprise the generalized elasticity module, generalized Poisson coefficient and shear modules [3, 4]. Therefore, in the generalized Hooke law, defining the relation between the deformation state and the stress, there is a susceptibility matrix including two material constants for isotropic bodies (E, ν). On the other hand, for orthotropic bodies, there are nine material constants (E_1, E_2, E_3, ν_{12}, ν_{23}, ν_{31}, G_{12}, G_{23}, G_{31}) [5, 6, 8, 12].

Apart from the material division into the anisotropic and isotropic materials, it is also possible to classify materials as homogeneous materials and inhomogeneous ones. It is believed that a material is homogeneous if its physical properties are identical in a given place of its isolated section. Each material has a discontinuous structure, however in a mathematical description it is assumed that it constitutes a continuous centre. It is permissible due to the fact that the dimensions of even the smallest technical object considerably exceed the dimensions of the microstructure components.

Wood and other composite materials of a layered structure, showing various mechanical and physical properties belong to inhomogeneous materials. Whereas, steel is considered as a homogeneous material.

Passing from the level of microscopic inhomogeneity to the macroscopic homogeneous level is called homogenization. This method formulates the macroscopic description by homogenizing microscopic properties. For the purpose of mathematical description of a material, the real centre can be substituted by a homogeneous centre. The microstructure is usually not so obvious in the macroscopic description; it is sort of “hidden” under the values of substitute parameters. Therefore, in order to analyse particular values of substitute parameters and their variability, it is necessary to analyse the influence of microstructure on their values [2, 11].

The homogenization method is commonly used to describe the properties of rocks, wood, composites, reinforced concrete, as well as human osseous tissues [11]. The substitute material constants of pine wood and polyester-glass composite were presented in the paper [7, 9, 10].

3. Material characteristics and course of research

Construction steel, natural pinewood and polyester- glass composite were utilized in order to define the stresses distribution in the beams subjected to outer loads. Previous results of research conducted by the author and literature data [7, 9, 10] were applied to calculate the beams of the above-mentioned materials.

Figure 2 shows the two-layer sample in the Cartesian co-ordinate system. The ratio of the layers of late wood and early wood was assumed in the calculations $h_1/h_2 = 0.5$. The model scheme of natural wood beam support and load is shown in Fig. 3. The value of beam deflection caused by the load of F force was measured. In the Figure: x – denotes longitudinal direction, z – transverse direction and z – denotes radial direction of the layers forming a composite [7, 8].

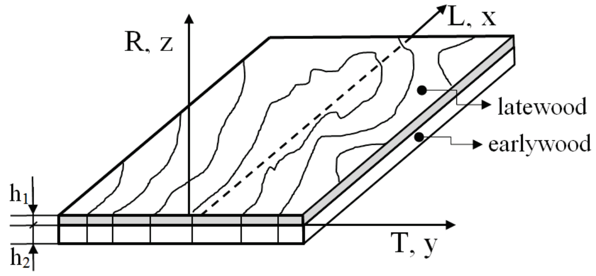


Fig. 2. The orientation directions of a wood sample in the Cartesian co-ordinate system
h_1 – thickness of latewood layer, h_2 – thickness of early wood layer

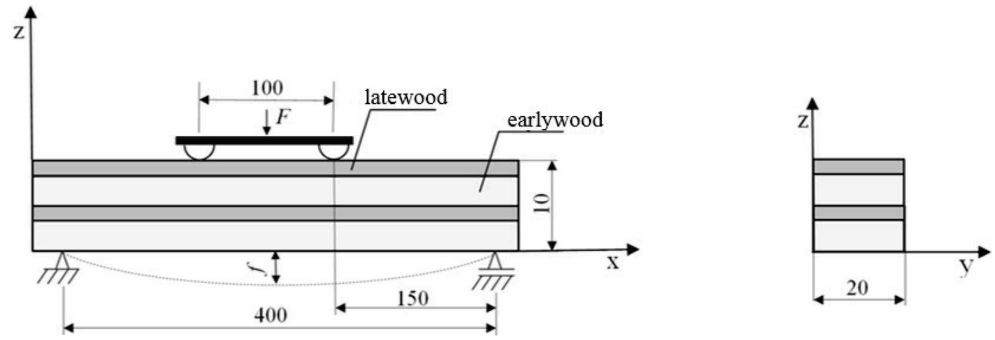
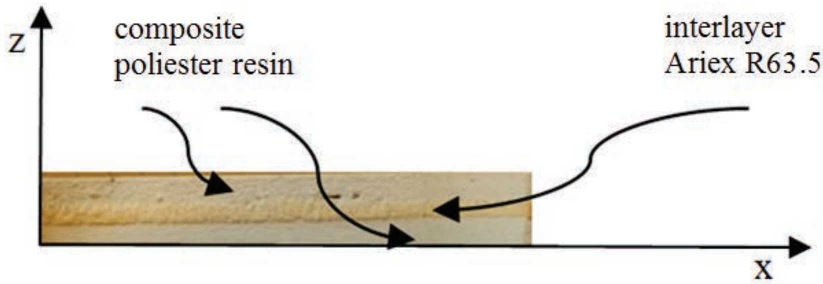


Fig.3. Support and load system of a beam

Wooden beam is made up of alternate layers of soft and hard wood. Particular layers are monotropic materials, while the beam being a construction material is an orthotropic material [7]. The polyester-glass composite, which underwent strength tests, was composed of three layers (Fig. 4) that is 2 outer layers of polyester resin 4.07 mm thick and the central interlayer 2 mm thick made of Ariex R63.50 [9].



Rys. 4. Composite made of polyester resin and Ariex R63.50

Ariex is a Swiss product used as interlayer material, which is a thermoplastic polymer foam, having closed cells. It is characterized by high deformation quality and perfect adhesion. This material is a typical core material, which absorbs vibrations in case of constructions that are dynamically loaded. It is applied in marine, aviation, transport, and railway industry [9]. The outer and inner layers have the quality of isotropic material, while the composite constitutes a monotropic material. The mechanical properties of this composite are shown in the work [9].

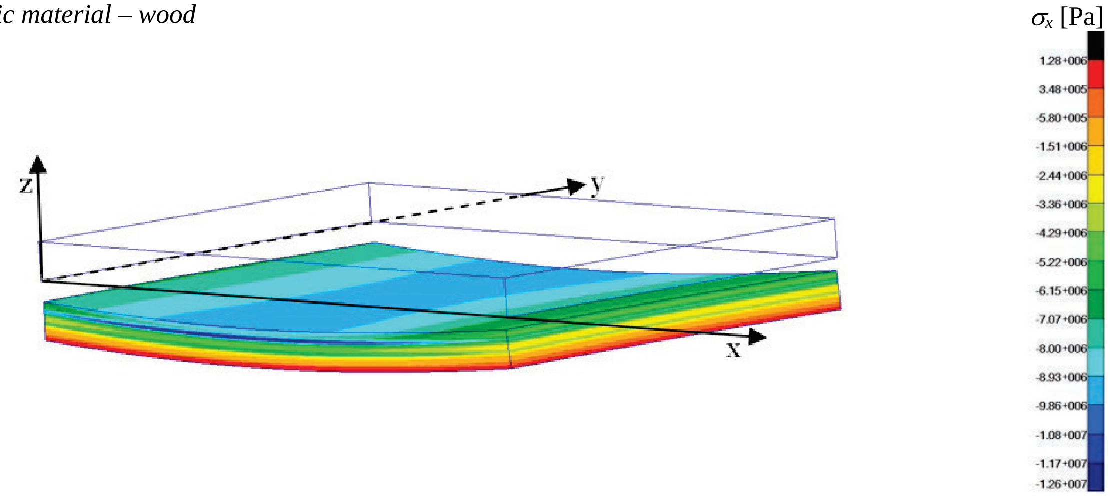
The geometry of composite samples and the steel samples was identical to the wood samples presented in Fig. 2. Under load, the samples underwent bending (Fig. 3.). As a result of bending the samples, the deflection and inner forces occurred.

3. Research results and their analysis

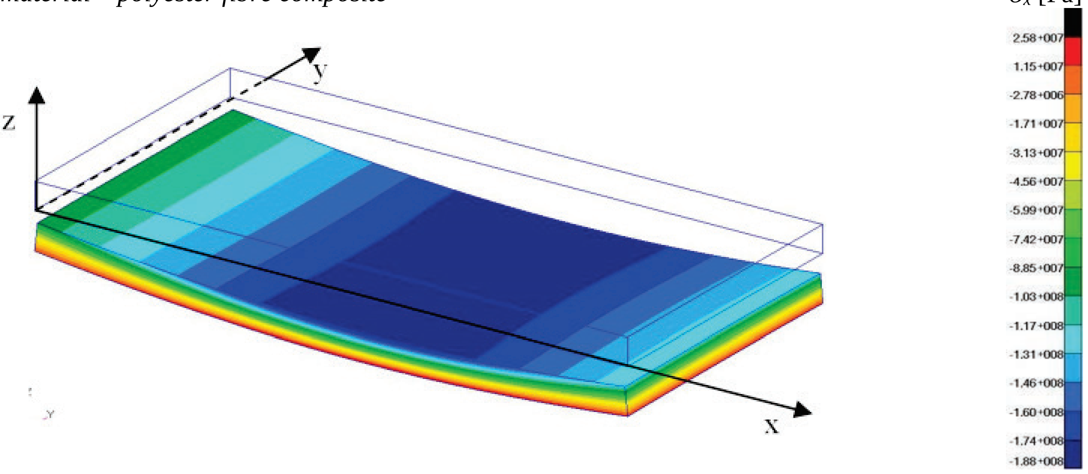
The calculations of stresses distribution were conducted by finite elements method (Patran – Nastran software) similarly as in case of disk of flat state of stresses. Each layer was described on its thickness with three nine nodes elements. The MES model contains approximately 75 thousand elements and over 250 thousand freedom levels. The calculations results allowed to display the precise distribution of stresses in particular beams made of wood and of layered composite. Example calculations of the results of stresses distribution were presented in Fig. 5 and 6.

The performed calculations allowed visualizing the distribution of stresses and the deflection of beams in any given parts – Fig. 5. The visualization of stresses distribution σ_3 allows observing different stress distribution in a beam under load due to the composition of their material. In layered materials, the main stress is transferred by stronger layers, in case of wood by hard wood layers, while in case of plastic by polyester resin layers. Fig. 5 shows the stresses distribution and the deflection of the beam central part, which makes it possible to observe considerable differences in stresses distribution, and deflection in a selected cross section of a beam.

a) orthotropic material – wood



b) monotropic material – polyester fibre composite



c) isotropic material – structural steel

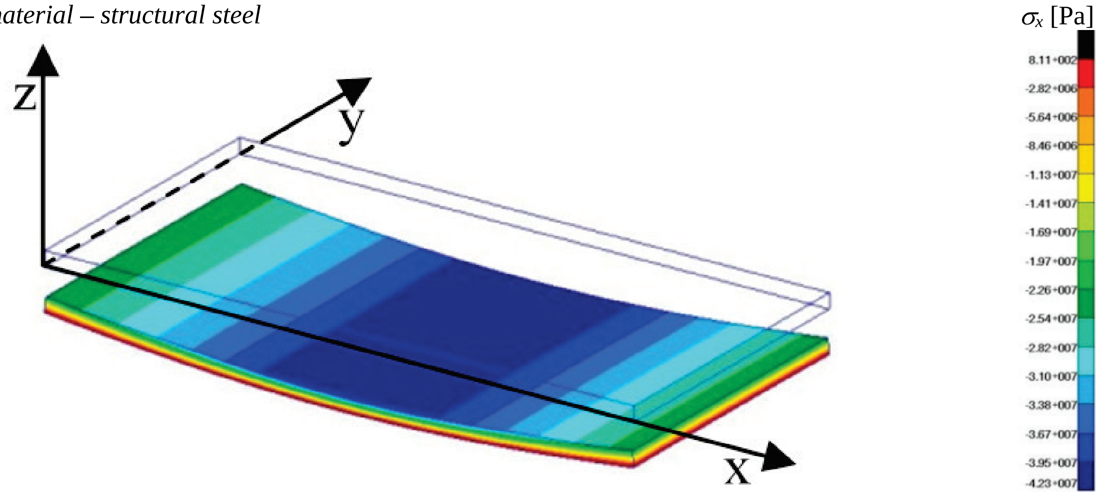
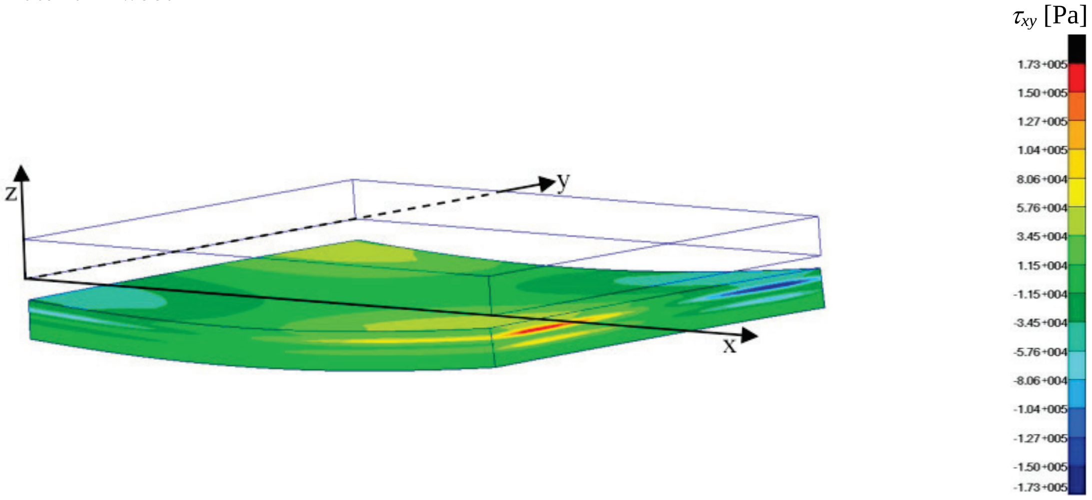


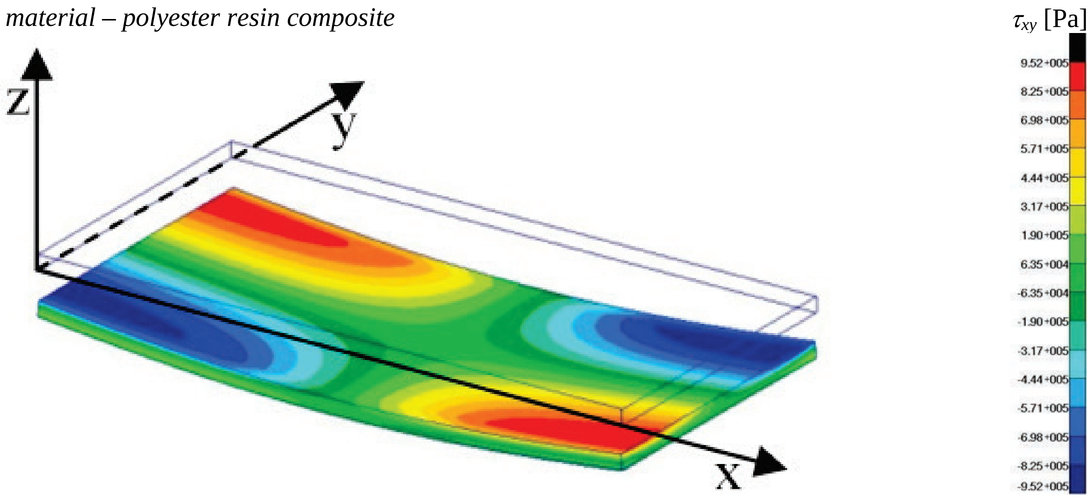
Fig. 5. Stresses distribution σ_x in the central part of the beam subjected to bending a) wood, b) composite, c) steel

Figure 6 presents shear stress distribution τ_{xy} in the beams of examined materials. The character of shear stresses distribution τ_{xy} in wood is different from the other two materials. It results from the layered properties of wood, composite and steel. Steel is an isotropic material and the layers forming a polyester resin composite also show isotropic character. On the contrary, wood layers have the properties of a monotropic material [7].

a) orthotropic material – wood



b) monotropic material – polyester resin composite



c) isotropic material – construction steel

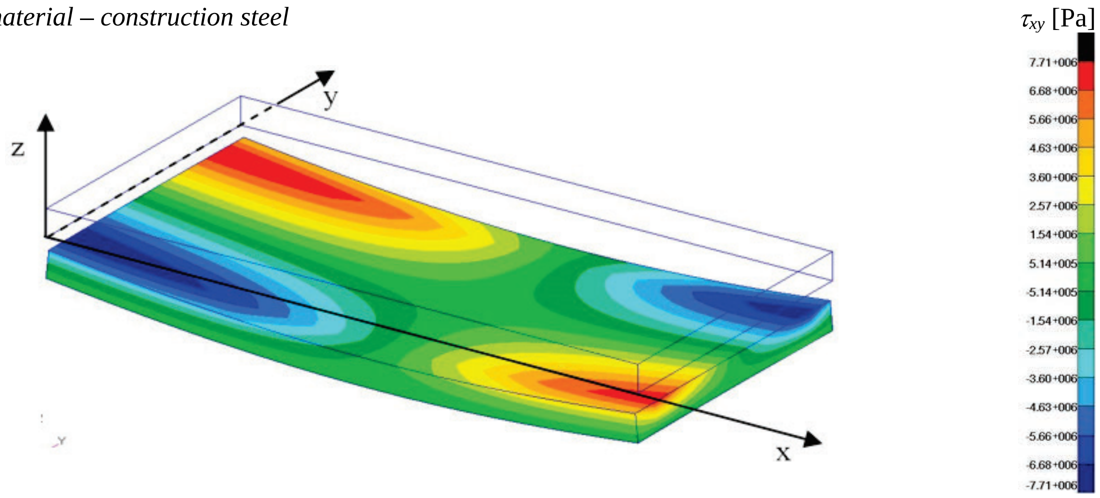


Fig. 6. Distribution of the shear stresses τ_{xy} in the central part of a beam subjected to bending in a) wood, b) composite, c) steel

4. Conclusions

The stress in the beams subjected to outer loads was calculated by the method of finite elements utilizing the Patran-Nastran software. Owing to that, it was possible to illustrate the distribution of stresses, in detail, in any given cross-section of a beam, which allowed observing very precise distribution of stresses especially in layered materials.

The visualisation of normal stress σ_3 and shear stress τ_{xy} permitted the authors to distinguish diverse stress values in a beam subjected to load, which resulted from its composition. In case of layered materials, the load is generally carried away by stronger layers. Therefore, in case of wood load is transmitted by hard wood layers, while in case of plastic it is transferred by polyester resin layers.

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