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SEMI-MARKOV MODEL OF MULTI-MODAL TRANSPORT OPERATION

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Abstract

Multi-modal transport means the transport of the objects through at least two different carriers of any combination of simple tasks of transport carriers (by truck, by train, by ship or by plane). A Semi-Markov (SM) model of multi-modal transport operation is presented in the article. The SM process is defined by the renewal kernel of that one. In our model, time to failure of the operation is represented by a random variable that denotes the first passage time from the given state to the subset of states. The duration of one operation cycle is a random variable representing the return time to the initial state. The appropriate theorems of the Semi-Markov processes theory allow us to calculate characteristics and parameters of the transport operation model.

The article presents the example of the transport operation final part of container with cargo from Warsaw to Stockholm, where from Warsaw to Gdynia, the container is transported by lorry, from Gdynia to Karlscorona by ferry and from Karlscorona to Stockholm by truck.

Keywords: *Semi-Markov model, multistage transport operation, reliability function, mean time to failure*

1. Introduction

The tasks of transport are realized by the transport operation systems. Some of them realized by any one of the carriers (by truck, by train, by ship or by plane) we would contractually agree to call "the simple transport". The task of transport, realized by a combined means (carriers) of delivery we would contractually call "complex transport". It is a combination of the above-defined basic tasks of transport.

Multi-modal transport, the transport of the objects through at least two different carriers of any combination of simple tasks of transport carriers (by truck, by train, by ship or by plane). The cargo might change any provided container – it means that it might be repackaged to another type of container to suit the requirements of any given carrier, as it might be required or practised for the logistic reasons.

Inter-modal transport [5]: the transport of the objects through at least two different carriers of any combination of transport basic media (by truck, by train, by ship or by plane). The cargo might not change the provided container – it means that it might not be repackaged to any another type of container, and has to be shipped in the originators container. To suit the requirements of any Air transport carrier, (Air transport should be used), it must be required to be packaged in an special approved for Air transport specific specialized container, usually leased or loaned from the Air carrier. Bi-modal transport: the transport using adopted bi-modal (truck, train) special truck trailer hook up container built to be transported by truck trailer (tractor) or on the railroads on the railroad platform car. The cargo is never repackaged between destinations.

The term reliability of the transport operation at the given moment t, means the probability of ability of the transport tasks realization at the instant t by the complex transport system.

2. Semi-Markov model

We will construct a semi-Markov model of the multi-modal transport operation under assumptions that there are possible perturbations during execution of the elementary tasks and there is possible total failure in some stage of the multi-modal operation. The model constructed here is some modification of the model presented in monograph [4].

2.1. Basic concepts

To determine the semi-Markov process as a model we have to define its initial distribution and all elements of its kernel [2-4]. Recall that the semi-Markov kernel is the matrix of transition probabilities [3]:

$$Q(t) = [Q_{ij}(t)\colon\ i,j \in S], \tag{1}$$

$$Q_{ij}(t) = P(\tau_{n+1} - \tau_n \ \le t, X(\tau_{n+1}) = j \mid \ X(\tau_n) = i), \quad t \ge 0, \tag{2}$$

where $\tau_n, n = 0, 1, 2, \ldots$ denote instants of the process state changes.

The sequence $\{X(\tau_n)\colon\ n = 0,1,\ldots\}$ is homogeneous Markov chain with transition probabilities

$$p_{ij} = P(X(\tau_{n+1}) = j \mid X(\tau_n) = i) = \lim_{t\to\infty} Q_{ij}(t). \tag{3}$$

The function

$$G_i(t) = P(T_i \le t) = P(\tau_{n+1} - \tau_n \le \ t \mid X(\tau_n) = i) = \sum_{j\in S} Q_{ij}(t) \tag{4}$$

is the CDF of the random variable T_i denoting time spent in state i when the successor state is unknown. This random variable is called a *waiting time.* The function

$$F_{ij}(t) = P(\tau_{n+1} - \tau_n \le t \mid X(\tau_n) = i, X(\tau_{n+1}) = j) = \frac{Q_{ij}(t)}{p_{ij}} \tag{5}$$

is CDF of a random variable T_{ij} that is called a *holding time* of state i, if the next state will be j. From (5) we have

$$Q_{ij}(t) = p_{ij}F_{ij}(t). \tag{6}$$

2.2. Descriptions and assumptions

We assume, that a duration of an i-th stage of operation, is a non-negative random variable ξ_i, $i = 1, \cdots, n$ having any distribution given by CDF

$$F_{\xi_i}(t) = P(\xi_i \le t). \tag{7}$$

We also assume that the time to some perturbation of the operation on the i-th stage is the nonnegative random variable $\eta_i, i = 1, \ldots, n$ with the exponential distribution

$$P(\eta_i \ \le t) = 1 - e^{-\lambda_i t}, \quad i = 1, \ldots, n. \tag{8}$$

We also suppose that the operation interrupted due to the perturbation can be carried out from the beginning of the stage corresponding to this one. Time to resume the operation is a nonnegative random variable $\zeta_i,\ i = 1, \ldots, n$ with CDF

$$F_{\zeta_i}(t) = P(\zeta_i \le t), \ \ i = 1, \ldots, n. \tag{9}$$

Time to failure of the operation on the i-th stage is the nonnegative random variable ϑ_i, $i = 1, \cdots, n$ with the exponential distribution

$$F_{\vartheta_i}(t) = 1 - e^{-\alpha_i t}, \quad i = 1, \ldots, n. \tag{10}$$

Time to failure of the operation on the i-th perturbed stage is the nonnegative random variable

$\vartheta_{3+i},\ i = 1,\cdots,n$ with the exponential distribution

$$F_{\vartheta_{3+i}}(t) = 1 - e^{-\beta_i t}, \quad i = 1,\ldots,n.$$

Time to renewal of the whole operation is random variable κ with the distribution function given by any CDF $F_\kappa(t)$.

The operation is cyclical in the sense that after its completion the operation starts in the opposite direction.

2.3. General model

In this case, we suppose that the state space of the process is

$$S = \{0, 1, 2, \ldots, n, n+1, \ldots, 2n\}, \tag{11}$$

where $i \in \{\ 1, \ldots n\ \}$ denotes i-th stage of the operation, $i \in \{\,n+1, \ldots, 2n\}$ represents a state of a perturbation on i-th stage of the operation, state 0 denotes total failure of the operation. The possible state changes of the process are represented by a flow graph that is shown in Fig. 1.

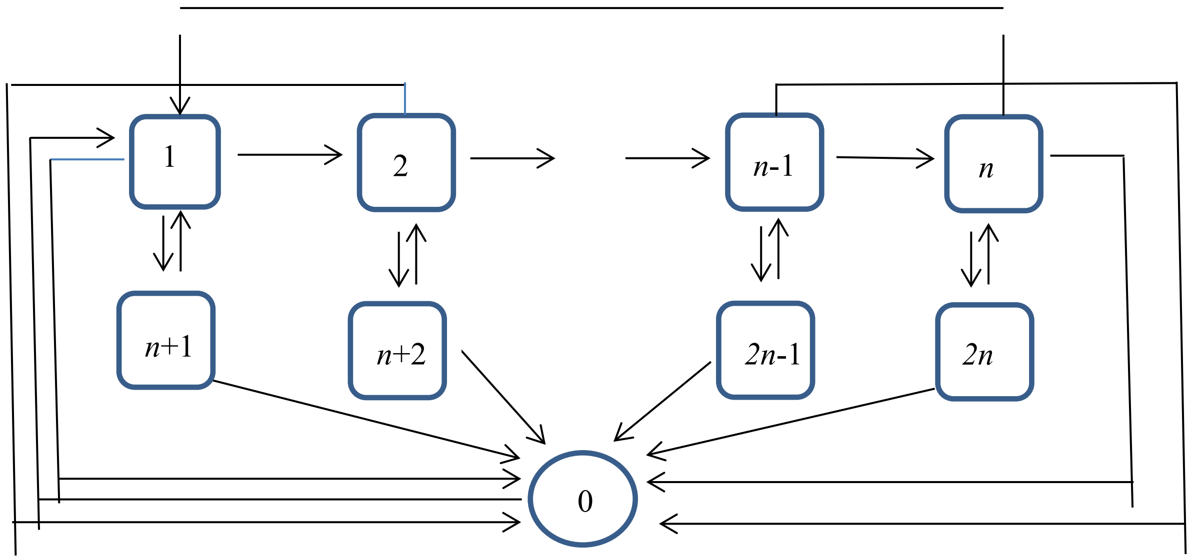

Fig.1. The graph of the state changes

The corresponding elements of Semi-Markov kernel are

$$Q_{i\,i+1}(t) = P(\xi_i \le t, \eta_i > \xi_i,\ \vartheta_i > \xi_i) = \int_0^t e^{-(\lambda_i + \alpha_i)x} dF_{\xi_i}(x), \quad i = 1, \ldots, n-1, \tag{12}$$

$$Q_{n\,1}(t) = P(\xi_n \le t, \eta_n > \xi_n,\ \vartheta_n > \xi_n) = \int_0^t e^{-(\lambda_n + \alpha_n)x} dF_{\xi_n}(x), \tag{13}$$

$$Q_{i\,n+i}(t) = P(\eta_i \le t, \xi_i > \eta_i, \vartheta_i > \eta_i) = \int_0^t \lambda_i e^{-(\lambda_i + \alpha_i)\,x}[1 - F_{\xi_i}(x)]dx,\ i = 1, 2, \ldots, \mathrm{n}, \tag{14}$$

$$Q_{i\,0}(t) = P(\vartheta_i \le t, \xi_i > \vartheta_i, \eta_i > \vartheta_i) = \int_0^t \alpha_i\, e^{-(\lambda_i + \alpha_i)\,x}[1 - F_{\xi_i}(x)]dx,\ i = 1, \ldots, n, \tag{15}$$

$$Q_{n+i\,i}(t) = P(\zeta_i \le t,\ ) \le t, \zeta_i < \vartheta_{n+i}) = \int_0^t e^{-\beta_i x}\, dF_{\zeta_i}(x),\ i = 1, \ldots, n, \tag{16}$$

$$Q_{n+i\,0}(t) = P(\vartheta_{n+i} \le t, \zeta_i > \vartheta_{n+i}) = \int_0^t \beta_i e^{-\beta_i x}[1 - F_{\zeta_i}(x)]dx,\ i = 1, \ldots, n, \tag{17}$$

$$Q_{01}(t) = P(\kappa \le t) = F_\kappa(t). \tag{18}$$

2.4. Model for *n* = 3

For *n* = 3 the set of states is $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and the semi-Markov kernel takes of the form:

$$Q(t) = \begin{bmatrix} 0 & Q_{01}(t) & 0 & 0 & 0 & 0 & 0 \\ Q_{10}(t) & 0 & Q_{12}(t) & 0 & Q_{14}(t) & 0 & 0 \\ Q_{20}(t) & 0 & 0 & Q_{23}(t) & 0 & Q_{25}(t) & 0 \\ Q_{30}(t) & Q_{31}(t) & 0 & 0 & 0 & 0 & Q_{36}(t) \\ Q_{40}(t) & Q_{41}(t) & 0 & 0 & 0 & 0 & 0 \\ Q_{50}(t) & 0 & Q_{52}(t) & 0 & 0 & 0 & 0 \\ Q_{60}(t) & 0 & 0 & Q_{63}(t) & 0 & 0 & 0 \\ & & & & & & \end{bmatrix}, \quad (19)$$

where:

$Q_{01}(t) = F_\kappa(t)$,

$Q_{i0}(t) = \int_0^t \propto_i e^{-(\lambda_i + \propto_i)\, x}\left[1 - F_{\xi_i}(x)\right] dx\,, \quad i = 1,\ 2,\ 3,$

$Q_{i\, i+1}(t) = \int_0^t e^{-(\lambda_i + \propto_i) x} dF_{\xi_i}(x)\,, \quad i = 1,\ 2, \qquad Q_{31}(t) = \int_0^t e^{-(\lambda_3 + \propto_3) x} dF_{\xi_3}(x),$

$Q_{3+i\; 0}(t) = \int_0^t \beta_i e^{-\beta_i x}\left[1 - F_{\zeta_i}(x)\right] dx\,,\ i = 1,\ 2,\ 3,$

$Q_{3+i\; \mathrm{i}}(t) = \int_0^t e^{-\beta_i x}\, dF_{\zeta_i}(x),\ i = 1,\ 2,\ 3,$

$Q_{i\; 3+\mathrm{i}}(t) = \int_0^t \lambda_i e^{-(\lambda_i + \propto_i)\, x}\left[1 - F_{\xi_i}(x)\right] dx\,,\ i = 1, 2,\ 3.$

3. Reliability and operation characteristics

Assume that evolution of a system reliability is describe by a finite state space semi-Markov process $\{X(t): t \geq 0\}$.

Suppose that a value of the random variable Θ_D denotes a first passage time to the subset D or the time of *a first arrival* at the set of states $D \subset S$ of semi-Markov process $\{X(t):\ t \geq 0\}$. A function:

$$\Phi_{iD}(t) = P(\Theta_D \leq t \mid X(0) = i), \qquad t \geq 0$$

is Cumulative Distribution Function (CDF) of the random variable Θ_{iD} denoting the first passage time from the state $i \in D'$ to the subset D.

If D' consists of the functioning states (up states) and D contains all failed states then $\Phi_{iD}(t), t \geq 0$ is CDF of the time to failure of an object with initial state $i \in D'$. For a finite state space semi-Markov, processes the distributions $\Phi_{iD}(t), i \in D'$ are proper and they are the only solutions of the equations system that Laplace-Stieltjes (L-S) transformation leads to the linear system of equations for (L-S) transforms:

$$\tilde{\varphi}_{iD}(s) = \sum_{j \in D} \tilde{q}_{ij}(s) + \sum_{k \in D'} \tilde{q}_{ik}(s) \tilde{\varphi}_{kD}(s), \quad (20)$$

where $\tilde{\varphi}_{iD}(s) = \int_0^\infty e^{-st} d\Phi_{iD}(t)$ are L-S transforms of the unknown CDF of the random variables Θ_{iA}, $i \in A'$ and $\tilde{q}_{ij}(s) = \int_0^\infty e^{-st} dQ_{ij}(t)$ are L-S transforms of the given functions $Q_{ij}(t),\ i, j \in S$. That linear system of equations is equivalent to the matrix equation:

$$(I - q_{D'}(s))\varphi_{D'}(s) = b(s), \quad (21)$$

where $I = [\delta_{ij}:\ i, j \in D']$ is the unit matrix,

$$q_{D'}(s) = [\tilde{q}_{ij}(s):\ i,j \in D'] \tag{22}$$

is the square sub-matrix of the L-S transforms of the matrix $q(s)$, while:

$$\varphi_{D'}(s) = [\tilde{\varphi}_{iD}(s):\ i \in A']^T, \quad b(s) = [\sum_{j\in D} \tilde{q}_{ij}(s):\ i \in D'] \tag{23}$$

are one-column matrices of the corresponding L-S transforms. The linear system of equations for the L-S transforms allows us to obtain the linear system of equations for the moments of the random variables $\Theta_{iD},\ i \in D'$.

In model (2.4) the states $1, 2, 3$ represent "up" reliability states, the states $4, 5, 6$ are the perturbed reliability states (partly "up"), the state 0 is a "down" reliability state. The random variable $\Theta_{iD}, D = \{0\}, i \in A' = \{1, \dots, 6\}$ denoting the first passage time from a state $i \in D'$ to the state 0 means the time to the total failure of the operation if the initial state is i. The Laplace-Stielties transform for the CDF of the random variables $\Theta_{iA},\ i \in A'$ we obtain from a matrix equation (21). In this case, we have $D' = \{1, 2, 3, 4, 5, 6\},\ D = \{0\}$.

From the solution of this equation, we get the Laplace-Stielties (L-S) transform of the density functions $\tilde{\varphi}_{i0}(s)$ of the random variable $\Theta_{i0},\ i = 1, \dots 6$. The Laplace transform of the reliability function is given by the formula

$$\tilde{R}(s) = \frac{1-\tilde{\varphi}_{10}(s)}{s}, \tag{24}$$

because the initial state is $1 \in A'$.

The expectations $E(\Theta_{iA}),\ i \in A'$ we can obtain by solving the equation:

$$(I - P_{D'})\overline{\Theta}_{D'} = \overline{T}_{A'}, \tag{25}$$

where:

$$P_{D'} = [p_{ij}:\ i,j \in D'],\ \ \overline{\Theta}_{D'} = [E(\Theta_{iD}):\ i \in D']^T, \qquad \overline{T}_{A'} = [E(T_i):\ i \in D']$$

and I is the unit matrix.

To find the second moments $E(\Theta_{iD}^2),\ i \in D'$ we have to solve the matrix equation:

$$(I - P_{D'})\overline{\Theta}_{D'}^2 = B_D, \tag{26}$$

where:

$$P_{A'} = [p_{ij}: i,j \in D'],\ \ \overline{\Theta}_{A'} = [E(\Theta_{iA}^2): i \in D']^T, \qquad B_D = [b_{iD}: i \in A']^T,$$

$$b_{iD} = E(T_i^2) + 2\sum_{k\in D'} p_{ik} E(T_{ik}) E(\Theta_{kD}).$$

A random variable $\Theta_{jj}, j \in S$ denotes the first return time to the state, j of the SM process. The CDF of the random variable Θ_{jj} is denoted $\Phi_{jj}(t) = P(\Theta_{jj} \le t)$. The L-S transform this function is:

$$\tilde{\varphi}_{jj}(s) = \tilde{q}_{ij}(s) + \sum_{k\in S-\{j\}} \tilde{q}_{jk}(s)\tilde{\varphi}_{kj}(s),\ \ j \in S. \tag{27}$$

The expectation and second moment of the random variable Θ_{jj} are given by the rules:

$$E\ (\Theta_{jj}) = E(T_j) + \sum_{k\in S-\{j\}} p_{jk} E(\Theta_{kj})\,,\ \ j \in S, \tag{28}$$

$$E(\ \Theta_{jj}^2) = E(T_j) + \sum_{k\in S-\{j\}} p_{jk} E(\Theta_{kj}^2) + 2\ \sum_{k\in S-\{j\}} p_{jk} E(T_{kj}) E(\Theta_{kj}),\ \ j \in S. \tag{29}$$

The random variable Θ_{11} in our model denotes duration of one cycle whole operation.

4. Example

A container with cargo is transported from Warsaw to Stockholm. From Warsaw to Gdynia, the container is transported by lorry, from Gdynia to Karlscorona by ferry and from Karlscorona to Stockholm by truck. The transport operation final part is unloading the container. To describe the transport operation we apply the model presented above, assuming $n = 3$. Random variables denoting duration of the operation stages are:

ξ_1 – duration of the container transport from Warsaw to Gdynia and loading on the ferry,
ξ_2 – duration of the container transport from Gdynia to Karlscorona and unloading,
ξ_3 – duration of the container loading on a truck, transport from Karlscorona to Stockholm and unloading at the destination

We assume the expectations and standard deviations of these random variables are:

$$E(\xi_1) = 6.5, \quad E(\xi_2) = 11.2, \quad E(\xi_3) = 8.2, \quad [\,h\,],$$

$$D(\xi_1) = 0, \quad D(\xi_2) = 11.2, \quad D(\xi_3) = 0, \quad [\,h\,].$$

It means that the duration of the stage i is determined, and it is equal to d_i for $i = 1,\ 2,\ 3$ and CDF of the random variables $\xi_i,\ i = 1, 2, 3$ are:

$$F_{\xi_i}(t) = \begin{cases} 0 & \text{for } t \le d_i, \\ 1 & \text{for } t > d_i, \end{cases} \quad i = 1, 2, 3.$$

In this case:

$$d_1 = E(\xi_1) = 6.5, \quad d_2 = E(\xi_2) = 11.2, \quad d_3 = E(\xi_3) = 8.2.$$

We also suppose that the failure rates on the transport stages are:

$$\propto_1 = 0.000028, \quad \propto_2 = 0.000014, \quad \propto_3 = 0.000022 \qquad \left[\frac{1}{\text{h}}\right],$$

the perturbation rates are:

$$\lambda_1 = 0.000086, \quad \lambda_2 = 0.000036, \quad \lambda_3 = 0.000066 \qquad \left[\frac{1}{\text{h}}\right]$$

and the failure rates on the transport stages during their perturbations are:

$$\beta_1 = 0.000038, \quad \beta_2 = 0.000024, \quad \beta_3 = 0.000032 \qquad \left[\frac{1}{\text{h}}\right].$$

We also assume that the random variable $\zeta_i, i = 1, 2, 3$ denoting time to resume the operation in stage i has CDF:

$$F_{\zeta_i}(t) = 1 - (1 + \gamma_i t)e^{-\gamma_i t} \quad \text{for} \ \ t \ge 0, \ \ i = 1,\ 2,\ 3,$$

with parameters:

$$\gamma_1 = 0.5, \ \gamma_2 = 0.8, \ \gamma_3 = 0.4 \qquad \left[\frac{1}{\text{h}}\right].$$

It means that:

$$E(\zeta_1) = \frac{2}{0.5} = 4, \quad E(\zeta_2) = \frac{2}{0.8} = 2.5, \quad E(\zeta_3) = \frac{2}{0.4} = 5 \ [\text{h}].$$

In this case the elements of the matrix $Q(t)$ are:

$$Q_{3\,1}(t) = \begin{cases} 0 & \text{for} \quad t \le d_3, \\ e^{-(\lambda_3 + \propto_3)\, d_3} & \text{for} \quad t > d_3, \end{cases}$$

$$Q_{i\,i+1}(t) = \begin{cases} 0 & \text{for} \quad t \le d_i, \\ e^{-(\lambda_i + \propto_i)\, d_i} & \text{for} \quad t > d_i, \end{cases} \quad i = 1, 2,$$

$$Q_{i0}(t) = \begin{cases} \frac{\alpha_i}{\lambda_i+\alpha_i}\left(1 - e^{-(\lambda_i+\alpha_i)t}\right) & \text{for } t \le d_i, \\ \frac{\alpha_i}{\lambda_i+\alpha_i}\left(1 - e^{-(\lambda_i+\alpha_i)d_i}\right) & \text{for } t > d_i, \end{cases} \quad i = 1, 2, 3,$$

$$Q_{i\,3+\mathrm{i}}(t) = \begin{cases} \frac{\lambda_i}{\lambda_i+\alpha_i}\left(1 - e^{-(\lambda_i+\alpha_i)t}\right) & \text{for } t \le d_i, \\ \frac{\lambda_i}{\lambda_i+\alpha_i}\left(1 - e^{-(\lambda_i+\alpha_i)d_i}\right) & \text{for } t > d_i, \end{cases} \quad i = 1,2,3,$$

$$Q_{3+i\,0}(t) = \frac{\beta_i(\beta_i+2\gamma_i - e^{-t(\beta_i+\gamma_i)}(\beta_i+(2+t\beta_i)\gamma_i+t\gamma_i^2))}{(\beta_i+\gamma_i)^2}, \quad i = 1, 2, 3,$$

$$Q_{3+i\,\mathrm{i}}(t) = \frac{\gamma_i^2(1-e^{-t(\beta_i+\gamma_i)}(1+t(\beta_i+\gamma_i)))}{(\beta_i+\gamma_i)^2}, \; i = 1, 2, 3,$$

$$Q_{01}(t) = 1 - (1 + \nu t)e^{-\nu t} \quad \text{for } t \ge 0.$$

The embedded Markov chain $\{X(\tau_n)$: $n = 0,1, \ldots$ of the semi-Markov process $\{X(t): t \ge 0\}$ has transtion probability matrix given by:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ p_{10} & 0 & p_{12} & 0 & p_{14} & 0 & 0 \\ p_{20} & 0 & 0 & p_{23} & 0 & p_{25} & 0 \\ p_{30} & p_{31} & 0 & 0 & 0 & 0 & p_{36} \\ p_{40} & p_{41} & 0 & 0 & 0 & 0 & 0 \\ p_{50} & 0 & p_{52} & 0 & 0 & 0 & 0 \\ p_{60} & 0 & 0 & p_{63} & 0 & 0 & 0 \end{bmatrix},$$

where:

$$p_{10} = \frac{\lambda_1}{\lambda_1 + \alpha_1}\left(1 - e^{-(\lambda_1+\alpha_1)\,d_1}\right), \quad p_{12} = e^{-(\lambda_1+\alpha_1)\,d_1}, \quad p_{14} = \frac{\alpha_1}{\lambda_1 + \alpha_i}\left(1 - e^{-(\lambda_1+\alpha_1)\,d_1}\right),$$

$$p_{20} = \frac{\lambda_2}{\lambda_2 + \alpha_2}\left(1 - e^{-(\lambda_2+\alpha_2)\,d_2}\right), \quad p_{23} = e^{-(\lambda_2+\alpha_2)\,d_2}, \quad p_{25} = \frac{\alpha_2}{\lambda_2 + \alpha_2}\left(1 - e^{-(\lambda_2+\alpha_2)\,d_2}\right),$$

$$p_{30} = \frac{\lambda_3}{\lambda_3 + \alpha_3}\left(1 - e^{-(\lambda_3+\alpha_3)\,d_3}\right), \quad p_{31} = e^{-(\lambda_3+\alpha_3)\,d_3}, \quad p_{36} = \frac{\alpha_3}{\lambda_3 + \alpha_3}\left(1 - e^{-(\lambda_3+\alpha_3)\,d_3}\right),$$

$$p_{3+\mathrm{i}\,0} = \frac{\beta_i^2+2\,\beta_i\,\gamma_i}{(\beta_i+\gamma_i)^2}, \qquad p_{3+\mathrm{i}\,\mathrm{i}} = \frac{\gamma_i^2}{(\beta_i+\gamma_i)^2}, \qquad i = 1, 2, 3.$$

$$G_0(t) = P(T_0 \le t) = Q_{01}(t) = 1 - (1 + \nu t)e^{-\nu t} \quad \text{for } t \ge 0,$$

$$G_1(t) = P(T_1 \le t) = Q_{10}(t) + Q_{12}(t) + Q_{14}(t) = 1 - e^{-(\lambda_1+\alpha_1)\,t}\, I_{[0,d_1)}(t),$$

$$G_2(t) = P(T_2 \le t) = Q_{20}(t) + Q_{23}(t) + Q_{25}(t) = 1 - e^{-(\lambda_2+\alpha_2)\,t}\, I_{[0,d_2)}(t)\,,$$

$$G_3(t) = P(T_3 \le t) = Q_{30}(t) + Q_{31}(t) + Q_{36}(t) = 1 - e^{-(\lambda_3+\alpha_3)\,t} I_{[0,d_3)}(t)\,,$$

$$G_{3+i}(t) = Q_{3+i\,\mathrm{i}}(t) + Q_{3+i\,0}\,(\mathrm{t}) = 1 - e^{-\beta_i t}(1 + \gamma_i t)e^{-\gamma_i t}, \quad i = 1, 2, 3.$$

The corresponding expected values are:

$$E(T_0) = E(\kappa) = \frac{2}{\nu},$$

$$E(T_i) = E(\min(\xi_i, \eta_i\,, \vartheta_i)) = \int_0^\infty e^{-(\lambda_i+\alpha_i)\,t} I_{[0,d_i)}(t)dt = \frac{1}{\lambda_i+\alpha_i}(1 - e^{-(\lambda_i+\alpha_i)\,d_i}, \quad i = 1, 2, 3,$$

$$E(T_{3+i}) = E(\min(\vartheta_{3+i}\,, \zeta_i)) = \int_0^\infty e^{-(\beta_i+\gamma_i)t}\,(1 + \gamma_i t)\,dt = \frac{\beta_i+2\gamma_i}{(\beta_i+\gamma_i)^2}, \quad i = 1, 2, 3.$$

The expected values of holding times are:

$$E(T_{3+i\,0})=\frac{\beta_i(\beta_i+3\gamma_i)}{(\beta_i+\gamma_i)(\beta_i^2+2\beta_i\gamma_i)}, \qquad E(T_{3+i\,i})=\frac{2}{\beta_i+\gamma_i}, \qquad i=1,2,3.$$

The first passage time from the state $i,\ i=1,2,3,4,5,6$ to the state 0, which is denoted as, Θ_{i0} represents a time to failure of the operation if the initial state is i. We will compute the expected values, the second moments and the standard deviations of these random variables. The equations (25) and (26) allow computing the expected values and second moments of these random variables. Using the own program in MATHEMATICA computer system we obtain:

$$\mathrm{E}(\Theta_{10})=17227.5, \qquad \mathrm{E}(\Theta_{10}^2)=593597442.5,$$
$$\mathrm{E}(\Theta_{20})=17230.6, \qquad \mathrm{E}(\Theta_{20}^2)=593705316.6,$$
$$\mathrm{E}(\Theta_{30})=17226.3, \qquad \mathrm{E}(\Theta_{30}^2)=593558730.6,$$
$$\mathrm{E}(\Theta_{40})=17228.9, \qquad \mathrm{E}(\Theta_{40}^2)=593645038.7,$$
$$\mathrm{E}(\Theta_{50})=17232.2, \qquad E(\Theta_{50}^2)=593755850.8,$$
$$\mathrm{E}(\Theta_{60})=17228.6, \qquad E(\Theta_{60}^2)=593636004.7.$$

Under assumption that the initial state of the operation is 1 the mean time to failure of the operation is:

$$E(T)=\bar{T}=17227.5 \ \text{[h]}.$$

In this case the standard deviation of the time to failure of the operation is:

$$D(T)=D(\Theta_{17})=17228.2 \ \text{[h]}.$$

Notice that the expected value and standard deviation are almost equal. Taking under consideration this fact and theorems of the perturbation theory [4] we can suppose that time to the transport operation failure is approximately exponentially distributed with parameter:

$$\lambda=\frac{1}{E(T)}.$$

Therefore the approximate reliability function of the transport operation is:

$$R(t)\approx e^{-0.000058\,t}.$$

A random variable Θ_{11} denotes the first return time to the initial state 1. The random variable Θ_{11} in our model denotes duration of the one cycle of whole operation. The expected value and second moment we can calculate using equations (28) and (27).

References

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