ISSN: 1231-4005
e-ISSN: 23540133
DOI: 10.5604/01.3001.0010.3066

DESCRIPTION OF STRENGTH OF WOOD COMPOSITE IN COMPOUND STATE OF LOAD

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Abstract

All constructions are subject to the most compound loads and therefore a suitable effort hypothesis should be used for their calculation. For anisotropic materials, a hypothesis should be used to describe the properties of such materials. In the work have been shown the strength of the layers composite on the example of construction wood in a compound stress state. Wood composite is an orthotropic material. Wood is composed of alternating layers of soft wood and hard wood. Single layers are monotropic material. The use of a stress hypothesis, which describes the strength of an orthotropic material, requires will make an investigation. Studies is purposed to determine the tensile strength along and across the fibres. The compressive strengths along and across the fibres and the shear strength. Particularly the determination of shear strength requires special tooling so that in the case of flat samples in the measuring part it is possible to determine the shear stresses. Therefore, a research stand was designed and constructed. Known stress hypotheses for anisotropic materials have been analysed. The analysis showed that the strength of the wood composite could be described by the Tai-Wu stress hypothesis. Based on the results of the research, numerical calculations were performed. Calculations allowed determining the distribution of stresses in the sample measuring part. The results tests and numerical calculations have shown that obtaining a homogeneous stress (shear) condition for anisotropic materials is very difficult. Wood belongs to materials whose mechanical properties depend on many parameters, so the description of the effort of this material is a compound issue. Studies have shown that wood reinforcement by polymer saturation is best suited to the compressed loaded structures.

Keywords: *compound state of stress, modified wood, effort criterion, strengthening of the material*

1. Tsai-Wu criterion to describe the strength of anisotropic materials

The knowledge of materials strength is required to design structural elements. The structures made of composites, even under uniform loads, are characterized by a compound stress state.

The elaborated criteria for anisotropic materials in compound stress states are based on a mathematical model, which describes experimental results without consideration of the material's behaviour mechanism. Appropriate criteria for a given group of anisotropic materials are assumed by their experimental verification. For this purpose, composite tests in compound stress conditions should be performed.

There are many hypotheses about the effort of anisotropic materials in the literature [4]. Effort hypotheses, presented in the form of series, are specific cases of the Tsai-Wu criterion [14]. Effort criteria for anisotropic materials in spatial stress states present a boundary surface that should be convex. For flat stress states, flat curves being the cross-sections of this surface are obtained. The theory of strength of materials, developed by Malmaistra [11], which is known in the literature as the Tsai-Wu theory [14], found wider application than other methods [16].

On the basis of the literature, it was assumed that the best method for describing the compound condition of the load on wooden structures is the Tsai-Wu hypothesis in the form of infinite tensor series:

$$P_{ij}\sigma_{ij} + P_{ijkl}\sigma_{ij}\sigma_{kl} + P_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn} + ... \leq 1, \quad (1)$$

where:

σ_{ij}, σ_{kl}, σ_{mn} – average values of stress components (stress tensor components),

P_{ij} ; P_{ijkl} ; P_{ijklmn} – coordinates of the surface equation (1).

Due to the lack of possibilities experimentally to determine the expressions of higher orders than four, the first two terms of the tensor series are taken.

For the flat state of stress:

$$P_{ij}\sigma_{ij} + P_{ijkl}\sigma_{ij}\sigma_{kl} \le 1, \tag{2}$$

Coordinates of the surface equation P_{ij}, P_{ijkl}. The coordinates of the surface equation present the material constants and are determined experimentally, in particular:

– stretching in directions R_{11}, R_{22}, R_{33},
– compression in directions S_{11}, S_{22}, S_{33},
– twisting with respect to the axis of symmetry of the material T_{12}, T_{23}, T_{31},
– twisting with respect to the axis rotated by an angle 45^0: $T_{12}^{45}, T_{23}^{45}, T_{31}^{45}$ or stretching (compression) with simultaneous pressure inside the tube [12].

On the basis of the on dependence (2) is obtained

$$P_{11} = \frac{1}{R_{11}} - \frac{1}{S_{11}}, \quad P_{22} = \frac{1}{R_{22}} - \frac{1}{S_{22}}, \quad P_{33} = \frac{1}{R_{33}} - \frac{1}{S_{33}},$$

$$P_{1111} = \frac{1}{R_{11}S_{11}}, \quad P_{2222} = \frac{1}{R_{22}S_{22}}, \quad P_{3333} = \frac{1}{R_{33}S_{33}}, \tag{3}$$

$$P_{1212} = \frac{1}{4T_{12}^2}, \quad P_{2323} = \frac{1}{4T_{23}^2}, \quad P_{3131} = \frac{1}{4T_{31}^2}.$$

The remaining expressions of the matrix can be determined with some approximation by the dependence:

$$P_{1122} \approx \frac{1}{R_{11}R_{22} + S_{11}S_{22}}, \quad P_{2233} \approx \frac{1}{R_{22}R_{33} + S_{22}S_{33}}, \quad P_{3311} \approx \frac{1}{R_{33}R_{11} + S_{33}S_{11}}$$

2. Own research of natural and modified wood

The strength tests were carried out on natural pinewood K0.0 and on modified K0.56. The digits standing after the "K" letter define the amount of kg of polymethylmethacrylate per 1 kg of dry wood. A detailed description of surface-modified wood technology is presented in the paper [6-9].

To determine the strength of the wood composite according to the Tsai-Wu hypothesis, it is necessary to designate:

– $R_{m_{//}}$ – tensile strength along the fibres,
– $R_{c_{//}}$ – compression strength along the fibres,
– $R_{m_{\perp}}$ – tensile strength across the fibres,
– $R_{c_{\perp}}$ – compression strength across the fibres,
– R_t – shear strength.

It is especially important to determine the strength of wood K0.0 and K0.56 on shear, which is troublesome because it requires special equipment. When examining flat samples, clean shear should be produced. An important feature of the sample is the homogeneity of the strain in the test area of the sample.

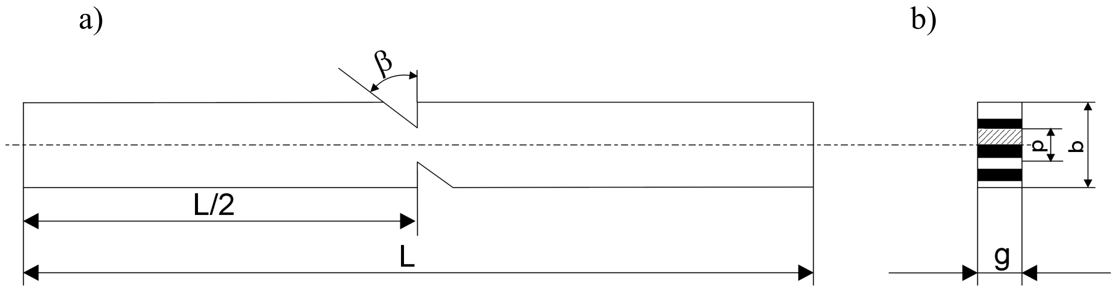

Fig. 1. a). Sample for shear test, b) outer layer hardwood, c) outer layer softwood: β = 45⁰, g = 5 mm – sample thickness, b = 20 mm – sample width, p = 5 mm – measuring section length, L = 80 mm – sample length

Based on multiple research and numerical calculations a sample, which final shape and dimensions are given in Fig. 1, was prepared [5]. The strength tests for the flat state of stress were carried out on the appropriate tooling. For this purpose, a prototype position was shown in Fig. 2 was developed and constructed [5]. In comparison to the traditional sample testing instruments [3, 15], the position of the sample axis relative to the load direction F can vary.

Thus, depending on the angle of rotation of the sample relative to the loading direction, in the mid-section of the sample, apart from shear, tensile or compression is implemented. The operating principle and the method of performing the research were fully described in the paper [5].

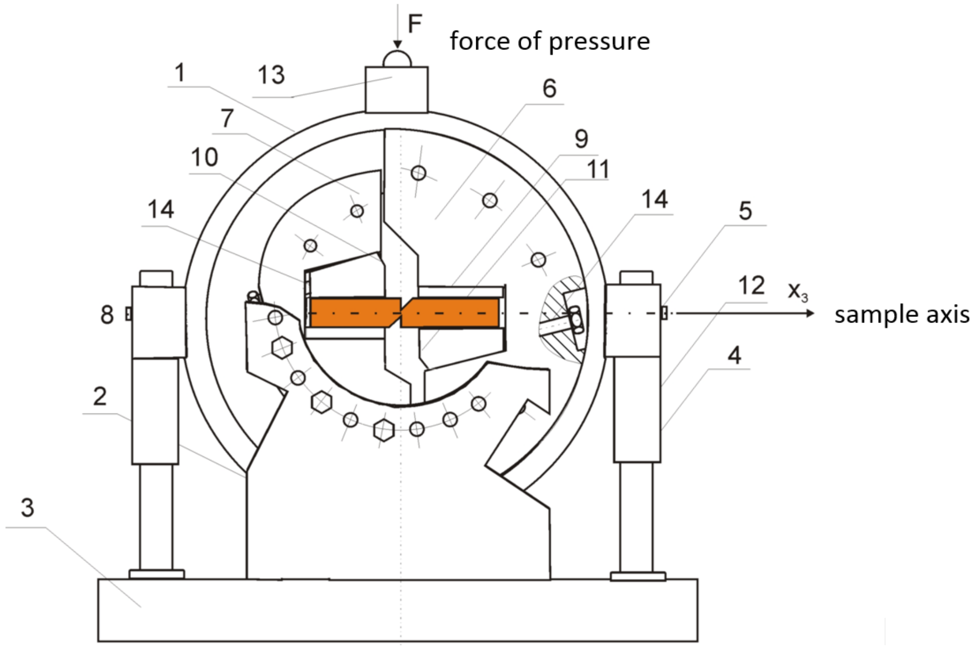

Fig. 2. Tool for producing flat state of stress: 1 – housing; 2 – bracket; 3 – base; 4 – guide; 5 – guide sleeve; 6 – right handle; 7 – left handle; 8 – insert for the jaw of the left handle; 9 – insert for right handle jaw; 10 – left retaining block; 11 – right-hand block; 12 – guide sleeve; 13 – fixing ball and ball f12; 14 – screw M6

The sample can be rotated every 15^0 in the range of -45^0 to $+45^0$. The rotation of the sample from the horizontal position in the counterclockwise direction causes shear and compression of the central portion of the sample, whereas clockwise rotation causes shear and tensile (extension/stretching) of the sample. The shear in implemented in the horizontal position of the sample (as in Fig. 2). Numerical calculations have been carried out for which purpose elastic constants of the layers of early wood and latewood were used. [1, 2].

On the basis of the elastic constants, the susceptibility matrix of layers of early wood and latewood of the natural wood K0.0 and modified wood K0.43 were determined.

The thickness ratio of hardwood layers to softwood layers was 1/2.

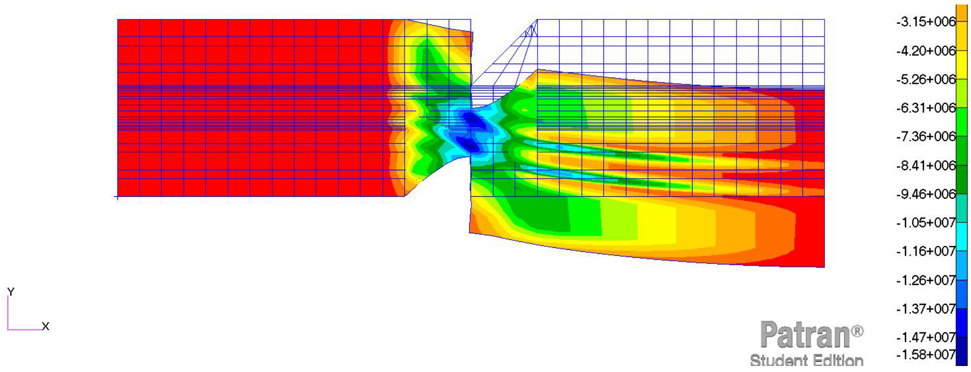

Fig. 3. Distribution of shear stresses σ_{31} in the measuring cross-section of sample K0.0 (20: 1 deformation) [10]

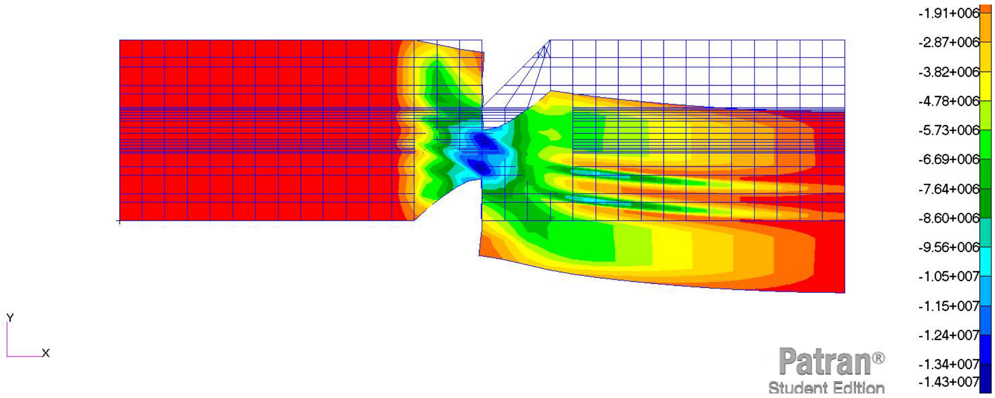

Fig. 4. Distribution of shear stresses σ_{31} in the measuring cross-section of sample K0.43 (20: 1 deformation) [10]

Figure 3 and 4 present the distributions of stress in the analysed cross section of the samples for natural wood K0.0 and modified wood K0.43 for "pure shear" (δ = 0^0). In the measuring section of the sample of natural and modified wood, shear strength and stress distribution in complex load, conditions (shear / stretch and shear / compression) were determined.

If in the measuring cross-section the outer layer was a hardwood layer, the stress was greater than in case of a softwood layer. Tensile strength and compression along and across the fibres were determined on samples taken according to norms [4, 13, 14, 18].

The results of the tensile, compression and shear tests are shown in Tab. 1.

Tab. 1. Tensile, compression and shear strength of natural and modified wood

Type of material	$R_{m_{II}}$, MPa	$R_{c_{II}}$, MPa	$R_{m_\perp}$ MPa	$R_{c_\perp}$, MPa	R_t, MPa
K0.0	95	55	4.5	8.5	22.10
K0.35	102	70	7	24	25.68
K0.43	110	80	8	26	26.50
K0.48	112	88	8.5	29	27.80
K0.56	118	98	9	32	30.20

where: $R_{m_{//}}$ – tensile strength along the fibres; $R_{c_{//}}$ – compression strength along the fibres; $R_{m_\perp}$ – tensile strength across the fibres; $R_{c_\perp}$ – compression strength across the fibres; R_t – shear strength

Filling the wood with polymer makes the material homogeneous; therefore, the modulus of softwood is similar to the modulus of hardwood. However, the material is still anisotropic; hence there are differences in its strength at the opposite load angles.

The use of numerical methods allows determining the distribution of stresses in the analysed measuring section of samples being subjected to stretching or compression. The analysis showed that the stretching / compression of the sample in a parallel direction to the fibres x_3 cause the stretching / compression of the fibres in the transverse direction x_1.

3. Results of wood strength tests in complex stress states

The Tsai-Wu criterion was used to describe the effort of natural and modified wood. After expanding the series (2), the following equation was obtained:

$$P_{11}\sigma_{11} + P_{1111}\sigma_{11}^2 + 4P_{1212}\sigma_{12}^2 + P_{22}\sigma_{22} + P_{2222}\sigma_{22}^2 + 2P_{1122}\sigma_{11}\sigma_{22} \le 1, \tag{4}$$

To simplify the notation, it was introduced:

$$P_{11} = A,\ P_{1111} = B,\ 4P_{1212} = C,\ P_{22} = D,\ P_{2222} = E,\ 2P_{1122} = G,$$

therefore, the expression (4) will take the form (assuming the stress sign for the anatomical directions of the wood):

$$A\sigma_{33} + B\sigma_{33}^2 + C\sigma_{31}^2 + D\sigma_{11} + E\sigma_{11}^2 + G\sigma_{33}\sigma_{11} \le 1. \tag{5}$$

The coefficients of the criterion can be expressed by dependencies:

$$B = \frac{1}{R_{m_{II}} \cdot R_{c_{II}}},\ A = \frac{R_{c_{II}} - R_{m_{II}}}{R_{m_{II}} \cdot R_{c_{II}}},\ E = \frac{1}{R_{m_\perp} \cdot R_{c_\perp}},\ D = \frac{R_{c_\perp} - R_{m_\perp}}{R_{m_\perp} \cdot R_{c_\perp}},$$

$$C = \frac{1}{S_{31}^2},\ G = \frac{G^*}{\sqrt{R_{m_{II}} \cdot R_{c_{II}} \cdot R_{m_\perp} \cdot R_{c_\perp}}},$$

where:
G^* – interaction term.

This term is difficult to determine, therefore for the purpose of calculations its value is set to be equally -0.5 on the basis of R. Mises's criterion [13], where:
$R_{m_{II}}$, $R_{c_{II}}$ – tensile and compression strength along the fibres,
$R_{m_\perp}$, $R_{c_\perp}$ – tensile and compression strength across the fibres.

S_{31} shear strength in plane (x_1, x_3) (data for calculation in Tab. 1). Tsai – Wu's effort criterion for a flat state of stress represents a boundary surface, which is an ellipse. The effort hypothesis is determined on the basis of (5) equation which factors A, B, C, D, E, G were assigned on the basis of the experiment. This hypothesis can be graphically presented by the ellipse equation:

$$\frac{(u - x_0)}{a^2} + \frac{(v - y_0)}{b^2} = 1, \tag{6}$$

rotated by an angle ψ and shifted by a value x_0, y_0, such that:

$$u = x_0 + x\cos\psi - y\sin\psi \qquad x_0 = (R_{m_{II}} - R_{c_{II}})/2$$
$$v = y_0 + x\sin\psi + y\cos\psi \quad \text{and} \quad y_0 = (R_{m_\perp} - R_{c_\perp})/2,$$

after substituting and solving the equation and comparing its corresponding expressions with the expressions from the hypothesis equation (5), it can be noted:

$$x\left(\frac{2x_0\cos\psi}{a^2} + \frac{2y_0\sin\psi}{b^2}\right) + x^2\left(\frac{\cos\psi^2}{a^2} + \frac{\sin\psi^2}{b^2}\right) + y\left(\frac{-2x_0\sin\psi}{a^2} + \frac{2y_0\cos\psi}{b^2}\right) +$$
$$+ y^2\left(\frac{\sin\psi^2}{a^2} + \frac{\cos\psi^2}{b^2}\right) + xy\left(\frac{-2\cos\psi\sin\psi}{a^2} + \frac{2\cos\psi\sin\psi}{b^2}\right) + \left(\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}\right),$$

where: $A=\frac{2x_0\cos\psi}{a^2}+\frac{2y_0\sin\psi}{b^2},\ B=\frac{\cos^2\psi}{a^2}+\frac{\sin^2\psi}{b^2},\ D=\frac{-2x_0\sin\psi}{a^2}+\frac{2y_0\cos\psi}{b^2},$
$E=\frac{\sin^2\psi}{a^2}+\frac{\cos^2\psi}{b^2},\ G=\frac{-2\cos\psi\sin\psi}{a^2}+\frac{2\cos\psi\sin\psi}{b^2},\ C=\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}$.

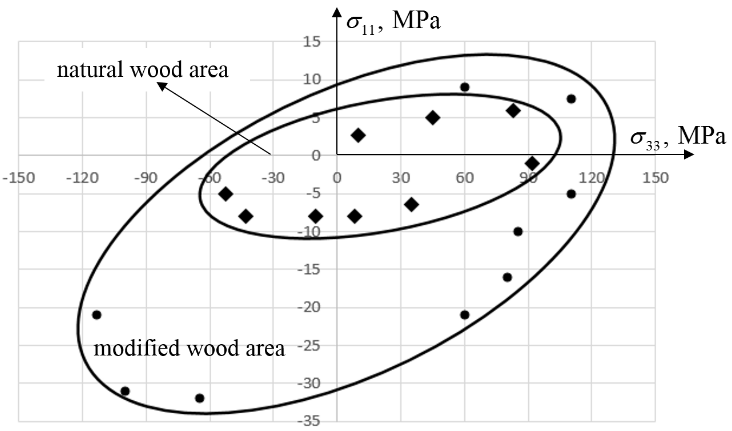

Fig. 5. Contours of effort of natural K0.0 and modified K0.56 wood using the Tsai-Wu hypothesis

Figure 5 shows the ellipses in the main stress system. The ellipses form the contours of the effort the natural wood K0.0 and modified K0.56 using the Tsai-Wu hypothesis (5). The area bounded by the ellipses is an area corresponding to the effort of natural and modified natural wood. At the same time, the area of effort the modified wood is much larger than that of natural wood.

As a result of wood modification with methylmethacrylate polymer, modified wood has been significantly strengthened. Tensile strength and shear increased significantly. However, the compression strength increased the most. The structure of the wood being filled with polymer became harder and more resistant to the load.

4. Analyse and final conclusions

In order to choose the right hypothesis, it should be verified with the experimental results of the particular material [5]. Analysis of stress distribution in the cross-section sample revealed that for each case of loading the sample, there is a non-homogeneous state of stress in the sample.

The common drawing shows the areas of effort the natural and modified wood in order to compare them. The surfaces of these ellipses present the load range of two different effort states created in natural and modified wood. The effort of the modified material at the compression along the fibres may be almost twice as large and at the compression for the transverse fibre system four times as large in comparison to the natural wood.

Hence, a practical conclusion for constructors is that mainly modified wood should be applied to structures subjected to compression. Also, when stretching, there is a noticeable increase in the strength of modified wood. The stress values corresponding to the loads either coincided with the ellipses or were included in the area determined by the ellipses.

The effort hypothesis applied to the composites describes the strength properties of the tested materials in a proper way.

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