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MAGNETIC FIELD IMPACT ON THE TEMPERATURE AND PRESSURE DISTRIBUTION IN SLIDE JOURNAL BEARING

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Abstract

The topic of the presented article aims to demonstrate a new principle of hydrodynamic lubrication in mechanical, non-isothermal and electro-magnetic fields. In this article is presented hydrodynamic slide journal bearing in electromagnetic field. The aim of this article is a new general analytical and numerical solution determined the influence of the electro-magnetic field on the temperature distribution in the internal surfaces of the slide-bearing sleeve and pressure distribution in bearing gap. To the research methods and materials used in this article realization belong: the Mathcad 15 Professional Program and new semi-analytical methods applied for theory of hydrodynamic lubrication extended to the curvilinear orthogonal surface and coordinates. Particular solutions are introduced to the cylindrical coordinates. To the obtained results of lubrication of movable sleeve surface belong the increments of the bearing load carrying capacity during the presence and absence magnetic induction field in the case if non-Newtonian lubricant features and constant temperature are taken into account. Simultaneously are observed the increments of the bearing load carrying capacity during the presence magnetic field and Newtonian lubricant. Temperature increments (decrements) without and with magnetic field lead to the hydrodynamic pressure decrements (increments). Moreover are assumed simultaneously the temperature T and oil dynamic viscosity variations in length, width and bearing gap-height directions. From this assumption follows, that the energy equation must be solved simultaneously with the equations of motion i.e. consequently with pressure equation where viscosity depends on temperature and temperature depends on the coordinate in gap height direction.

Keywords: *slide journal bearing, magnetic induction field, non-Newtonian lubricant, conjugated fields, new general method, load carrying capacity, pressure decrements, pressure increments*

1. Introduction

This article presents a new principle of slide journal bearing hydrodynamic lubrication in mechanical, thermal and electro-magnetic fields. The deformations of bearing surfaces are not taken into account. Three conjugated fields were simultaneously considered as viscous fluid, elastic body, electro-magnetic and heat transfer fields. The description of mentioned problem requires the mutually connections between hydrodynamic equations of motion and conservation of energy equation with the Maxwell equations presenting the electro-magnetic field.

Above mentioned problem and contemporary achievements in the field of AFM and computer height memory calculation results, do not correspond with the assumption of constant temperature and viscosity in gap height direction of slide bearing gap. Unfortunately, this fact was not observed in M.M. Khonsari, O. Pincus, D. J. Wilcock and A. Gadomski papers [9-11]. In the mentioned papers, despite the real 3D temperature and lubricant viscosity fields, the temperature gradient changes and its influences on lubricant viscosity differences crosswise the film thickness were not taken into account.

The coupled mechanical and electro-magnetic field presented in this paper implies the fact, that obtained solutions are more realistic than the hitherto results with indirect interactions between pressure and temperature field.

The main conclusion for 3D hydro-electro-magnetic non-isothermal lubrication obtained in this paper constitutes the semi analytical solutions, which are presented by the direct influence of hydrodynamic pressure on the lubricant temperature, and simultaneously direct influence of the mentioned temperature on the hydrodynamic pressure.

2. Fundamental relations for mechanical, thermal and electro-magnetic fields

This paper presented a semi analytical method of solution of the asymmetrical, laminar, steady, non-Newtonian lubrication flow problem between two non-rotational and not deformable, curvilinear orthogonal movable surfaces in conjugated hydro-electro-magnetic fields. The parallel and longitudinal intersections of the mentioned surfaces are curvilinear and non-monotone in general. The solutions are made in local curvilinear and orthogonal coordinates ($\alpha_1, \alpha_2, \alpha_3$) connected with one of the movable surfaces, where α_2 denote the direction of gap height.

The fluid apparent viscosity η_p varies in ($\alpha_1, \alpha_2, \alpha_3$) directions and depends on pressure, temperature flow shear ratio, and magnetic induction field [2-3, 7-8, 12-18].

The lubricant flow between the two above-mentioned solid surfaces of journal bearing and sleeve in the electromagnetic field will be described by the 3 momentum equations of equilibrium in a vector form, a fluid continuity equation, and by equation of a conservation of energy equation in a scalar form, hence we obtain the following system [7-8, 19-20]:

$$Div\,\mathbf{S} + \mu_o(\mathbf{N}\nabla)\mathbf{H} + \frac{1}{2}\mu_o rot(\mathbf{N}\times\mathbf{H}) + \mathbf{J}\times\mathbf{B} = \rho\left(grad\,\frac{\mathbf{vv}}{2} - \mathbf{v}\times rot\ \mathbf{v}\right) + \rho\frac{\partial\mathbf{v}}{\partial t}, \tag{1}$$

$$\frac{\partial\rho}{\partial t} + div(\rho\mathbf{v}) = 0\,, \tag{2}$$

$$div(\kappa\ grad\ T) + \phi_F = \rho\frac{d}{dt}(c_v T) + \mu_o T\Xi(\mathbf{v}\nabla)\mathbf{H} + \mathbf{J}^2/\sigma\,. \tag{3}$$

Moreover, we add the Maxwell and Ohm equations for the liquid layer between two surfaces. Thus, we get such equations [19-20]:

$$\nabla\cdot\mathbf{B} = 0, \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}, \tag{4}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\ \ \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B})\,. \tag{5}$$

The relationship between stress tensor $\mathbf{S}$ and strain rate tensor $2\mathbf{T_d}=\mathbf{A_1}$ i.e. constitutive equations are assumed for the lubricant in the following form [8], [20]:

$$\mathbf{S} = -p\boldsymbol{\delta} + \eta_p\mathbf{A_1}\,, \tag{6}$$

whereas unit tensor $\boldsymbol{\delta}$, strain rare tensor $\mathbf{A}_1$ has the following components: δ_{ij}, Θ_{ij}. We assume the following notations: μ_o – magnetic permeability in vacuum H/m, T – fluid temperature in K, $\mathbf{B}$ – magnetic induction vector in T, $\mathbf{N}$ – magnetization vector A/m, $\mathbf{E}$ – electric intensity vector V/m,

H – magnetic intensity vector A/m, Ξ – first derivative of magnetization vector with respect to temperature A/mK, σ – electrical conductivity coefficient S/m, **J** – electric current density in A/m^2, **D** – electric induction vector As/m, ρ – fluid density kg/m^3, κ – thermal conductivity coefficient W/mK, **v** – fluid velocity vector in m/s, ϕ_F – dissipation of energy in W/m^3.

For power, law of compressible fluid the apparent viscosity η_p and dissipation have the form [8, 11, 12, 14]:

$$\eta_p \equiv \eta_{pr} = 2^{n-1} m(n) \left| \frac{1}{2}\mathbf{I}_1^2 - \mathbf{I}_2 \right|^{\frac{n-1}{2}}, \ \phi_F = 4\eta_p \left(\frac{1}{2}\mathbf{I}_1^2 - \mathbf{I}_2 \right) - p\mathbf{I}_1, \tag{7a}$$

whereas:

$$\mathbf{I}_1 = \Theta_{11} + \Theta_{22} + \Theta_{33} = div\mathbf{v}, \ \ \mathbf{I}_2 = \Theta_{11}\Theta_{22} - \Theta_{12}^2 + \Theta_{11}\Theta_{33} - \Theta_{13}^2 + \Theta_{22}\Theta_{33} - \Theta_{23}^2, \tag{7b}$$

We assume n – dimensionless flow index, $m = m(n)$ – fluid consistency coefficient in Pas^n. The solutions are made in the local curvilinear and orthogonal coordinate system (α_1, α_2, α_3) connected with one of the movable surfaces, where α_2 denotes the direction of gap height. The total distance ε_T between two surfaces is significantly smaller than other dimensions of the considered surfaces. We have assumed that the fluid velocity components in α_1, α_3 directions do not have the same order of greatness. For rotational surfaces with non-monotone generating line the Lame` coefficients are reduced to the form: $h_1 = h_1(\alpha_3)$, $h_2 = 1$, $h_3 = h_1(\alpha_3)$. Finally for rotational surfaces with monotone generating line we obtain $h_1 = h_1(\alpha_3)$, $h_2 = 1$, $h_3 = 1$.

3. Steady state lubrication in the relative rotation motion

After expanding equations (1)-(3) in α_i (i=1, 2, 3) directions, and taking into account layer boundary simplifications i.e. neglecting the negligibly small terms of order 0.001 presenting the quotient ψ (radial clearance) of characteristic gap height ε_0 to the radius R of journal, we obtain the system of non-linear basic partial differential equations describing the lubrication of two arbitrary curvilinear rotational deformable surfaces. We assume: non-Newtonian, unsymmetrical, incompressible (for invariant $I_1 = 0$), steady (which does not change in time), pseudo-plastic lubrication in magnetic field (M_i, $M_T \neq 0$) for power law model with flow index n and consistency coefficient $m(n)$ without viscous-elastic properties. We consider: independent oil apparent viscosity $\eta_p(\alpha_1,\alpha_2,\alpha_3)$ changes in length, width and gap- height directions, inertia forces and terms of energy convections are neglected, pressure is constant in gap height direction and bearing rotational, deformed surfaces have non-monotone generating lines. Putting Maxwell equations (4),(5) and physical dependencies (6), (7), into expanded equations (1)-(3)we obtain the system of equations of conservation of momentum, continuity, energy in presence of magnetic field in the following form of curvilinear orthogonal co-ordinates (α_1, α_2, α_3):

$$0 = -\frac{1}{h_1}\frac{\partial p}{\partial \alpha_1} + \frac{\partial}{\partial \alpha_2}\left(m \left| \left(\frac{\partial v_1}{\partial \alpha_2}\right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2}\right)^2 \right|^{\frac{n-1}{2}} \frac{\partial v_1}{\partial \alpha_2} \right) + M_1, \tag{8}$$

$$0 = \frac{\partial p}{\partial \alpha_2}, \tag{9}$$

$$0 = -\frac{1}{h_3}\frac{\partial p}{\partial \alpha_3} + \frac{\partial}{\partial \alpha_2}\left(m \left| \left(\frac{\partial v_1}{\partial \alpha_2}\right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2}\right)^2 \right|^{\frac{n-1}{2}} \frac{\partial v_3}{\partial \alpha_2} \right) + M_3, \tag{10}$$

$$0=\frac{1}{h_1}\frac{\partial v_1}{\partial \alpha_1}+\frac{\partial v_2}{\partial \alpha_2}+\frac{1}{h_1 h_3}\frac{\partial}{\partial \alpha_3}\left(h_1 v_3\right), \tag{11}$$

$$\frac{\partial}{\partial \alpha_2}\left(\kappa\frac{\partial T}{\partial \alpha_2}\right)+m\left|\left(\frac{\partial v_1}{\partial \alpha_2}\right)^2+\left(\frac{\partial v_3}{\partial \alpha_2}\right)^2\right|^{\frac{n+1}{2}}=M_T. \tag{12}$$

$$M_T\left(\alpha_1,\alpha_3\right)\equiv \mu_0 T\Xi(\mathbf{v}\nabla)\mathbf{H}+\mathbf{J}^2/\sigma,\quad M_i\left(\alpha_1,\alpha_3\right)\equiv \mu_0(\mathbf{N}\nabla)H_i+\frac{1}{2}\mu_o rot(\mathbf{N}\times\mathbf{H})_i+(\mathbf{J}\times\mathbf{B})_i\,, \tag{13}$$

where the length, width and gap- height directions, are limited respectively: $0 < \alpha_1 \le 2\pi$, $-b_m \le \alpha_3 \le b_s$, $0\le \alpha_2 \le \varepsilon$ and $i = 1, 2, 3$.

The system of equation (8)–(12) contains the following dimensional unknown values: pressure $p(\alpha_1,\alpha_3)$, temperature in oil $T(\alpha_1,\alpha_2,\alpha_3)$ and sleeve T^*, three oil velocity components $v_i(\alpha_1,\alpha_2,\alpha_3)$ for $i = 1, 2, 3$ in three curvilinear, orthogonal dimensional directions: α_1, α_2, α_3.

4. Boundary conditions

The lubricant flow in bearing gap is generated by the rotation of a rotational, curvilinear journal. Hence, the boundary conditions for lubricant velocity components have the form of:

$$v_1=\omega h_1 \text{ for } \alpha_2=0,\ v_1=0 \text{ for } \alpha_2=\varepsilon_T, \tag{14}$$

$$v_2=0 \text{ for } \alpha_2=0, \text{ and for steady motion } v_2=0 \text{ for } \alpha_2=\varepsilon_T, \tag{15}$$

$$v_3=0 \text{ for } \alpha_2=0,\ v_3=0 \text{ for } \alpha_2=\varepsilon_T, \tag{16}$$

where: h_1 – Lame coefficient in α_1 direction and symbol ω denotes angular velocity of rotational journal, in circumferential direction.

Decrements or increments that are above the characteristic environmental temperature T_0 have constant value f_c on the journal surface and variable unknown values $f_p(\alpha_1,\alpha_3)$ on the sleeve surface. Hence, the boundary temperature values are as follows:

$$\begin{aligned} &T(\alpha_1,\alpha_2,\alpha_3)= T_0+f_c \text{ for } \alpha_2=0,\\ &T(\alpha_1,\alpha_2,\alpha_3)= T_0+f_p(\alpha_1,\alpha_3) \text{ for } \alpha_2=\varepsilon_T. \end{aligned} \tag{17}$$

Heat flux is transferred from the rotational surface of bearing journal into lubricant; hence, we obtain the boundary condition in the following form:

$$\kappa\frac{\partial T}{\partial \alpha_2}=-q_c \quad for \quad \alpha_2=0. \tag{18}$$

For free heat exchange between the journal and lubricant and by the virtue of Newton-Fourier law, using heat transfer coefficient υ, and temperature difference Δf in oil across the film, the dimensional heat flux density q_c in oil on the journal surface takes the following form:

$$q_c\equiv \upsilon\,\Delta f\,. \tag{19}$$

5. Dependence of temperature and magnetic field on the oil viscosity

According to R. E. Rosensweig and Rayleigh, temperature decreases and magnetic induction increases the oil dynamic viscosity coefficient η. This phenomenon is described by the following formula:

$$\eta=\eta_{0T}\eta_{0B}\, exp\left[\delta_B\left(B-B_0\right)-\delta_T\left(T-T_0\right)\right],\ \eta_{0T}\equiv\eta_0\, exp\left[-\delta_T T_0\right],\ \eta_{0B}\equiv exp\left[\delta_B B_0\right], \tag{20}$$

with coefficient δ_B of magnetic influence on the oil viscosity and coefficient δ_T of temperature influence on the oil viscosity for the following relation $\eta = m\cdot(v_0/\varepsilon_T)^{n-1}$ between dynamic viscosity coefficient η and oil consistency coefficient m.

6. Numerical calculations

The numerical calculations in cylindrical coordinates $\alpha_1 = \varphi$, $\alpha_2 = r$, $\alpha_3 = z$ for Lame coefficients $h_1 = r$, $h_2 = h_3 = 1$ are in this section performed by virtue of the equations included in section 3 for lubricant dynamic viscosity constant cross-wise film thickness but dependent on 2D temperature, variable cross-wise film thickness but dependent on 3D temperature and dependent on temperature and magnetic induction field. For cylindrical coordinates, we assume the following dimensional/dimensionless relations:

$$r=\varepsilon_0 r_1,\ z=bz_1,\ h_1=R,\ h_3=1,\ L=b/R,\ \varepsilon_T=\varepsilon_0\varepsilon_{T1},\ g=(n-1)/2,\ f_c \equiv Ec\,Pr\,T_0 f_{c1},\ f_p \equiv Ec\,Pr\,T_0 f_{p1},$$

$$T_1 \equiv T/T_0 = T_1^{(0)} + gT_1^{(1)} + \ldots + g^k T_1^{(k)} + \ldots,\quad p_1 \equiv p/p_0 = p_1^{(0)} + gp_1^{(1)} + \ldots + g^k p_1^{(k)} + \ldots,$$

$$R_F \equiv \frac{N_0 B_0 \varepsilon_0^2}{\omega R^2 \eta_0},\ R_J \equiv \frac{J_0 \varepsilon_0^2}{\sigma\omega^2 R^2 \eta_0},\ p^{(0)} = p_0 p_1^{(0)},\ p_0 \equiv \frac{\omega R^2 \eta_0}{\varepsilon_0^2},\ Ec\,Pr \equiv \frac{\omega^2 R^2 \eta_0}{\kappa T_0},\ Nu \equiv \frac{q_c \varepsilon_0}{\kappa T_0},\quad (21)$$

The hydrodynamic pressure, load carrying capacity and temperature distributions in machine slide journal bearing are solved by means of the quasi-half numerical Frobenius small parameter method implemented by the finite difference method using Mathcad 15 Professional Program. The unknown calculated functions are expanded by the infinite functional series in neighbourhood of dimension small parameter g defined in dependence on dimensionless flow index n. For upper limit, value of n equals to 1.5 the value of small parameter $g = 0.25$ was synonymously and uniquely associated, to attain the uniform convergences of functional series.

The performed calculations are carried out under stationary, vibration-less, laminar, incompressible non-Newtonian pseudo-plastic oil flow conditions with Reynolds number from 10 to 20 in magnetic induction field, where the electric, and magneto-electro-striction effects are neglected but magnetization vector depends on magnetic susceptibility coefficient and where zero values of Hartmann number $Ha = 0$ were required. In consequence of relative small Reynolds number, the oil inertia forces are negligibly small. Gap height between the considered, smooth (without roughness) cylindrical bearing surfaces, being independent of bearing length coordinate for non-skewing journal takes the following classical form:

$$\varepsilon_T=\varepsilon_0(1+\lambda\cos\varphi).\quad (22)$$

The classical non-deformable cylindrical bearing gap is restricted by the smooth motionless sleeve surface and movable non-skewing journal surface without roughness. The round angle of the bearing is considered, whereas the 2D dimensionless lubrication regions are defined by inequalities: $0 \le \varphi \le \varphi_k$, $-1 \le z \le +1$ associated to $-b \le z \le +b$ and referring to the circumferential φ and bearing length i.e. z directions, where φ_k denotes the bearing coordinate of the film end, and 2b- denotes the total bearing length. On the abovementioned boundaries of the region considered, the pressure attains the atmospheric pressure value in the dimensionless form p_{at}/p_0 and the remaining pressure is obtained by imposing known L. Gümbel-Everling (1925) conditions [15] on the pressure general solution.

In calculations are assumed the following data namely: radius of the journal $R = 0.03$ m, dimensionless bearing length $L = b/R = 1/2$, characteristic value of bearing gap height $\varepsilon_0 = 0.0000020$ m, eccentricity ratio values $\lambda = 0.4$; 0.6, characteristic value of dynamic viscosity $\eta_0 = 0.02$ Pas, angular velocity of the journal $\omega = 565.5$ s^{-1}, relative radial clearance $\psi = 0.00067$, two magnetic induction numbers $R_F = 0.5$, electric intensity number $R_J = 0$, magnetic susceptibility coefficient $\chi = 0.15$, vacuum magnetic permeability $\mu_o = 4\cdot\pi\cdot10^{-7}$ H/m. Brinkman number attains small value equal to $EcPr = 0.108705$ and $\delta_{B1} = 0.375$, of thermal conductivity coefficient $\kappa_0 = 0.15$ W(mK)$^{-1}$, $Nu = 0.03022$. The calculated distribution values of the dimensionless hydrodynamic pressures and dimensional load carrying capacities represented by: Newtonian oils only, by corrections from the non-Newtonians oils and the sums of predicted results are depicted in Fig. 1-4 and in a, b, c pictures respectively.

Figures 1 (2) depict the dimensionless pressure values without (with) magnetic induction field influences. To obtain dimensional values of pressure occurring in each picture, we must multiply the presented dimensionless pressure value by characteristic dimensional pressure value $\omega\eta_0/\psi^2$ indicated on the vertical axis of each picture.

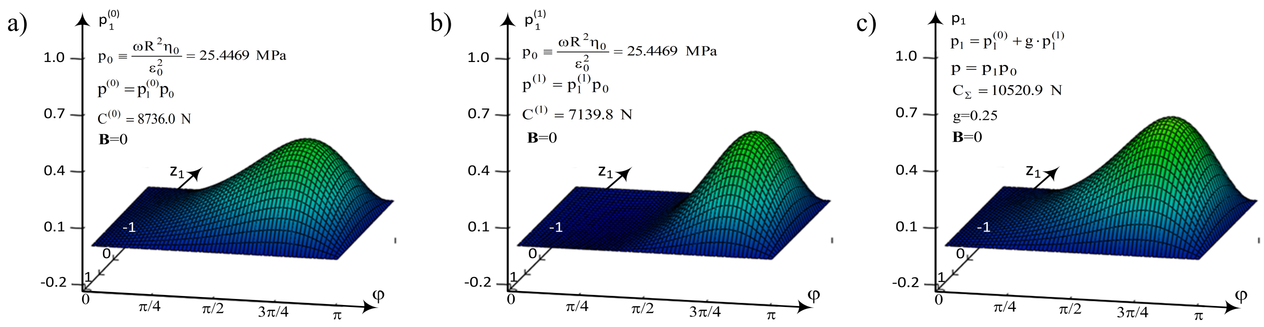

Fig. 1. Dimensionless pressure distribution and dimension load carrying value in cylindrical journal bearing for eccentricity ratio $\lambda = 0.4$ without the magnetic induction field influences on calculated pressure values: a) for Newtonian oils, b) pressure correction caused by the non-Newtonian oil properties, c) the final pressure value

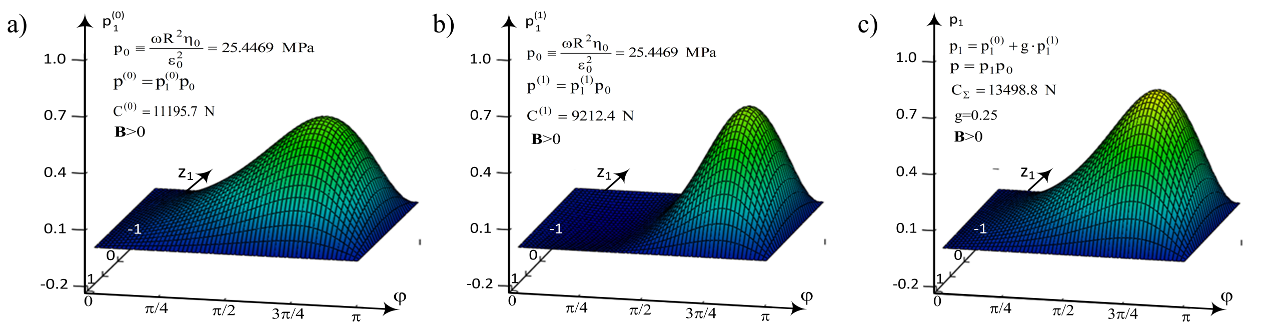

Fig. 2. Dimensionless pressure distribution and dimension load carrying values in cylindrical journal bearing for eccentricity ratio $\lambda = 0.4$ with the magnetic induction field influences on calculated pressure values: a) for Newtonian oils, b) pressure correction caused by the non-Newtonian oil properties, c) the final pressure value

The calculated dimensional distribution of temperature values on the internal sleeve surface represented by: temperatures produced by Newtonian oils only, as well as temperature corrections were obtained from the non-Newtonians oils and the sums of foreseen results are depicted in Fig. 3, 4 in a, b, c pictures respectively. Figures 3 (4) depict the temperature values without (with) magnetic induction field influences on the calculated values.

6. Conclusions

1. The numerical calculations that were performed both for 2D, 3D temperature field, indicate that the non-Newtonian properties of oil presented in calculation, change in 15 percent the pressure and load capacity in slide journal bearing in comparison with the pressure and capacity obtained for Newtonian oil lubrication independent of the presence or the absence of magnetic induction field.

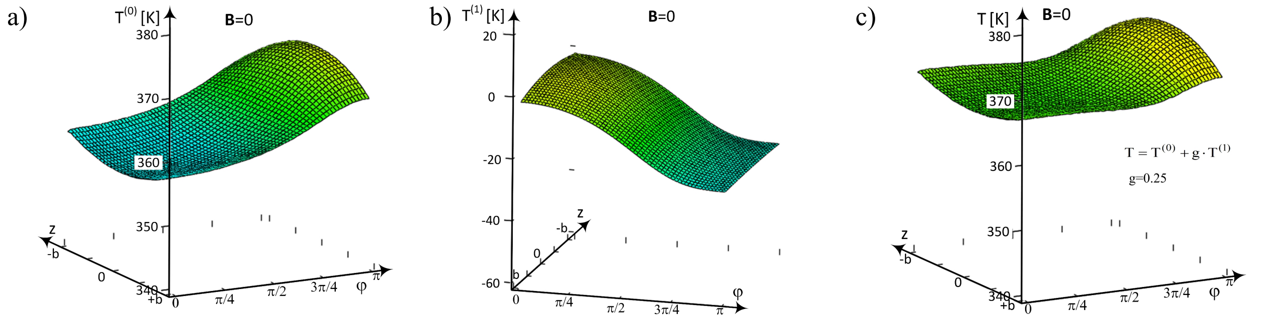

Fig. 3. Dimensional temperature distribution on the internal surface of cylindrical journal bearing sleeve for eccentricity ratio $\lambda = 0.4$ without the magnetic induction field influences ($B = 0$) on calculated temperature values: a) temperature produced by Newtonian oils, b) temperature correction caused by the non-Newtonian oil properties, c) the sum of values presented in picture a) added to the value corrections illustrated in b) multiplied by small parameter $g = 0.25$

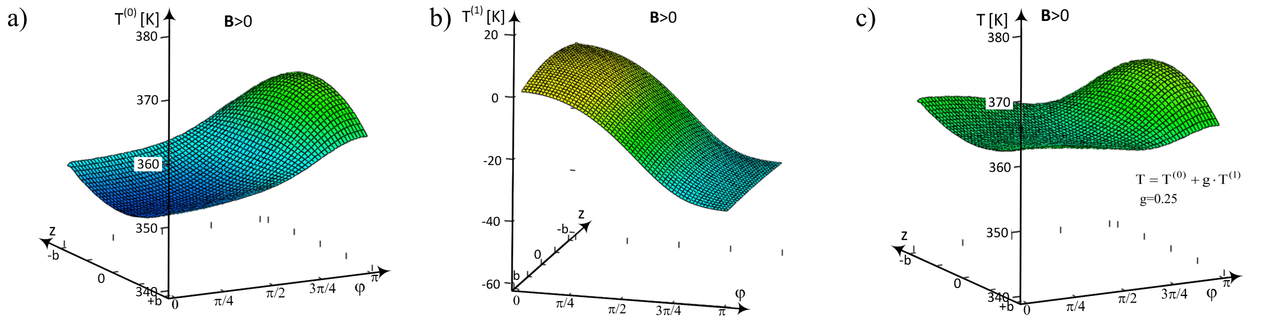

Fig. 4. Dimensional temperature distribution on the internal surface of cylindrical journal bearing sleeve for eccentricity ratio $\lambda = 0.4$ with the magnetic induction field influences ($B > 0$) on calculated temperature values: a) temperature produced by Newtonian oils, b) temperature

2. The numerical calculations that were performed enable to show that oil presented in calculation, in the presence of magnetic induction field, increase the pressure and load capacity in slide journal bearing for Newtonian and non-Newtonian oil in comparison with the pressure and capacity obtained without magnetic induction field.
3. If lubricant viscosity is dependent on 2D-temperature the pressure and temperature can be only indirectly connected or directly connected but in next step level of approximation step. Pressure decrements caused by temperature can attain from 5 to 10 percent. However, such restricted form of solutions is not acceptable for describing the lubrication process in micro-bearing calculations.
4. If lubricant dynamic viscosity is everywhere assumed as dependent on really existing 3D-temperature field (particularly in gap height direction),then the hydrodynamic solutions present the direct influence of hydrodynamic pressure on the lubricant temperature, and simultaneously the direct influences of the mentioned temperature on the hydrodynamic pressure decreases. The decreases mentioned are accepted for classical macro bearings, as well as for micro and nano bearings.
5. After initial numerical calculations it appears that the pressure obtained directly from equation using 3D temperature field influences, is different about 5 to 7 percent in comparison with the pressure values calculated indirectly on the grounds of pressure equation using 2D temperature influences on the viscosity in section 8.

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