**ISSN**: 1231-4005
**e-ISSN:** 2354-0133
**DOI:** 10.5604/01.3001.0010.2836

# STUDY OF THE NONCOMMUTATIVITY OF SUBMITTING AIRCRAFT ROTATION IN HIGH-MANOEUVRING FLIGHTS

**Andrzej Szelmanowski, Jerzy Borowski**

*Air Force Institute of Technology*
*Ksiecia Boleslawa Street 6, 01-494 Warsaw, Poland*
*tel.: +48 261 851 603, +48 261 851 469, fax: +48 261 851 646*
*e-mail: andrzej.szelmanowski@itwl.pl, jerzy.borowski@itwl.pl*

**Mariusz Garbacz**

*Polish Air Force Academy*
*Dywizjonu 303 Street 28, 08-521 Deblin, Poland*
*tel.: +48 261 518 271, fax: +48 261 518 270*
*e-mail: mariusz.garbacz@wsosp.pl*

***Abstract***

*The article presents selected results of the analytical work carried out in the Air Force Institute of Technology and the Polish Air Force Academy by means of a computer simulation of noncommutativity phenomena submitting aircraft turnover on high-manoeuvrability flights. The results of these simulations allowed to define the guidelines for the method of "production" of noncommutativity phenomenon in specialized rotating positions for strapdown inertial navigation systems (to evaluate the errors of determining spatial orientation).*

*Also, it analysed the possibility of "production" noncommutativity rotational movement in mobile flight simulator used for testing pilots and candidates for pilots (to test the sensitivity of the pilot's vestibular system in terms of feeling the impact of this phenomenon). The results of the assessment of "greatness" noncommutativity angular velocity vector, occurring in some parts of the high-manoeuvrability flight of aircraft on the example of a fighter-bomber airplane Su-22 were discussed as well.*

***Keywords:*** *transport, aviation, inertial navigation systems, coning motion, noncommutativity rate vector*

## 1. Introduction

One of the most interesting phenomena occurring during the aircraft high-manoeuvring flight (in particular, military fighter aircraft and aerobatic aircraft) is noncommutativity while submitting the finite rotations in the three-dimensional space. In addition to the sensors' drift out, it is the main source of errors of determining the lowering and tilting angles and course in the strapdown spatial orientation and inertial navigation systems [3, 8]. This effect includes both the classical systems (with gyroscopic sensors of angular velocity), and the systems with optoelectronic sensors (optical fibre and laser).

As it was presented [2, 3], if the aircraft instantaneous rotation axis changes its direction, its spatial orientation cannot be determined with the use of simple integration of the measured angular velocities of the performed rotation. While performing rotations in the three-dimensional space, in addition to the angular velocity measured by inertial methods (with the use of gyroscopic or optoelectronic sensors), there is the so-called noncommutativity rate vector, the value of which depends not only on the value itself, but also on the order of submitting the rotations. Differential equations of the spatial orientation vector transformation from the measuring coordinate system (associated with the aircraft) to the horizontal reference system should use the noncommutativity correction algorithms of submitting the finite rotations in the three-dimensional space. It is particularly important for the strapdown inertial navigation systems, in which the noncommutativity rate vector causes calculation errors at numerical determination of the matrix coefficients of directional cosines. A detailed formula of determining the value of the spatial orientation vector, provided in [6], allows specifying the noncommutativity rotation vector and the correction of the aircraft spatial orientation calculation errors, which it caused.

## 2. Generation of noncommutative motion and its impact on the spatial orientation systems

In order to test "sensitivity" of the spatial orientation and inertial navigation systems on the occurrence of noncommutativity errors (and "efficiency" of the used noncommutativity impact correction algorithms), the specialised multi-axis rotary stations (Fig. 1), which generate complex forms of spatial motion, were used [4, 9].

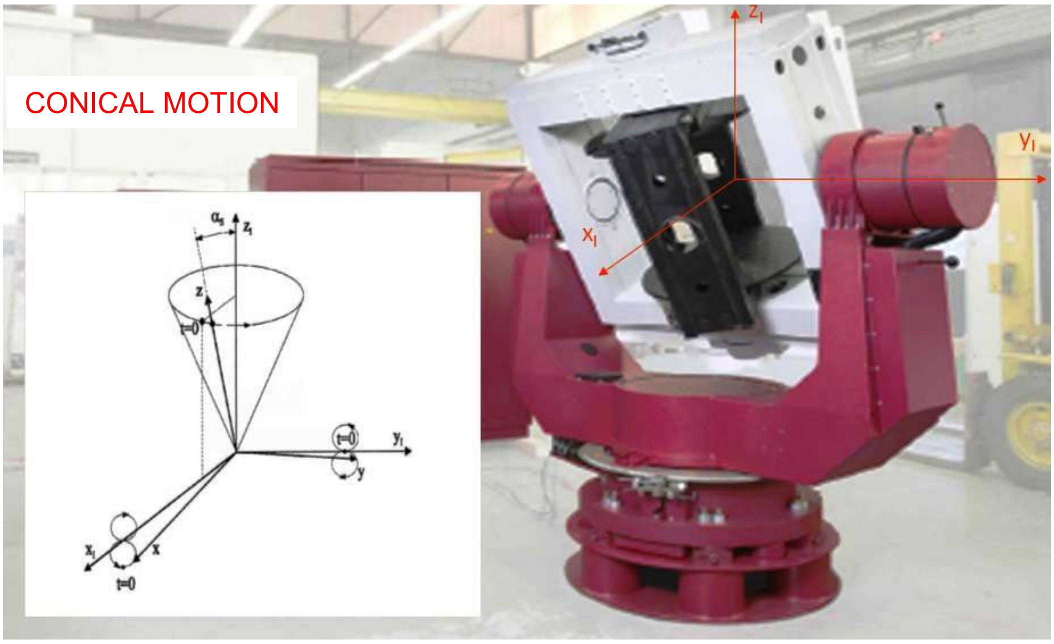


*Fig. 1. View of the multi-axis rotary station for the noncommutative motion generation*

One of such extortions is the classical conical motion [2, 8], in which the "impact" of noncommutativity results in "the fastest" accumulation of errors determining the aircraft spatial orientation. It currently constitutes the rotary motion standard extortion in testing of modern strapdown inertial navigation systems [5, 6, 8, 9].

The noncommutativity of rotations in the conical motion directly influences the error values of determining the spatial orientation angles in the strapdown inertial navigation systems (Fig. 2), which use optoelectronic angular velocity sensors and the matrix of directional cosines. The matrix coefficients are updated on the basis of the information of the measured angular velocity, but they can be also determined from the conical motion parameters [3, 8].

Similar errors occur in the spatial orientation systems using analogue angular velocity sensors (e.g. dynamically tuned gyroscopes). These sensors are located on the gyroscopic platform in the Cardan suspension (Fig. 3), and the signals of the angular velocity are introduced to tracking systems positioning the gyroscopic platform. The noncommutativity errors cause accumulation of inaccuracy of setting the measuring axes of inertial sensors and accumulation of errors of determining the aircraft spatial location.

During the tests carried out in the Air Force Institute of Technology [6], it was found that the noncommutativity phenomenon of submitting the rotations has an impact on errors of the determined matrix of directional cosines, updated on the basis of signal on the aircraft current angular velocity or the correction quaternion components (used in modern strapdown systems). It also affects the way of feeling the rotary motion by the pilot's sense of balance during performing the advanced flight manoeuvres, e.g. in the Su-22 military aircraft high-manoeuvring flights.

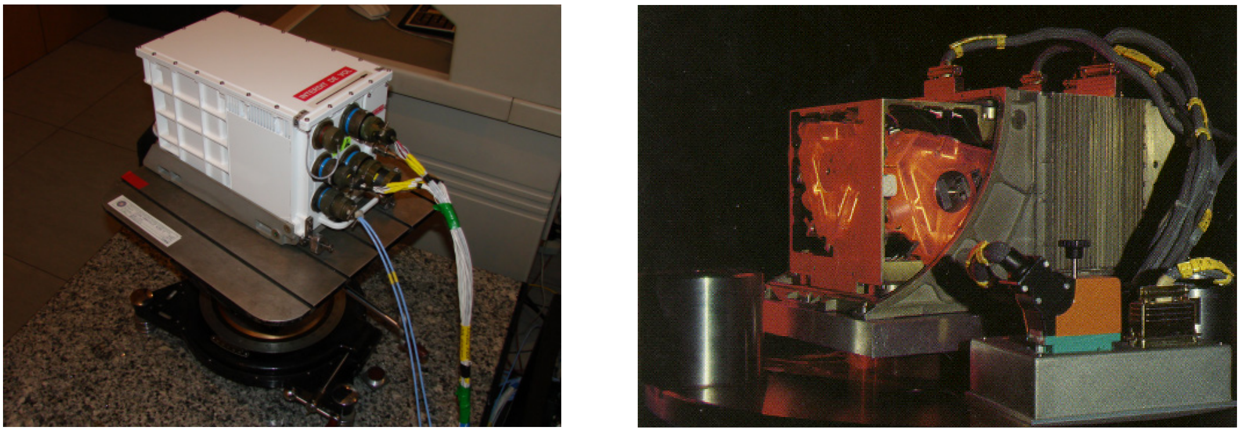

*Fig. 2. View of the inertial navigation system (left) and optoelectronic sensors (right) of the W-3PL helicopter*

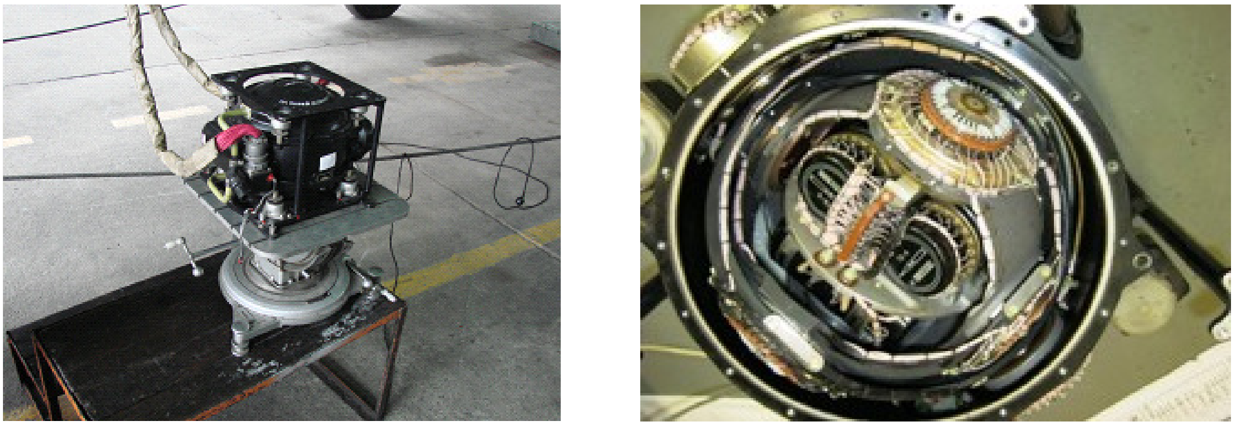

*Fig. 3. View of the inertial navigation system (left) and gyroscopic sensors (right) of the Su-22 aircraft*

In the algorithms used in the strapdown inertial navigation systems, among others [3, 5], approximation, which assumes the constancy of coefficients in the rotation equation in the sampling period, is used, and the matrix is updated on the basis of the following relationship:

$$[C(t)] = \frac{[\phi(t)][\phi(t)]^T}{(\phi(t))^2}[1-\cos(\phi(t))] + [I]\cos(\phi(t)) + \frac{\sin(\phi(t))}{(\phi(t))}[\phi(t)], \tag{1}$$

where:
$[C(t)]$ – matrix of directional cosines updated on the basis of the measured rotation angle vector,
$[\phi(t)]$ – actual rotation angle vector implemented in the three-dimensional space,
$\phi(t)$ – actual rotation angle vector module implemented in the three-dimensional space,
$[I]$ – identity matrix (with the size of 3×3).

**3. Computer simulation of noncommutativity for submitting the rotations in conical motion**

In the strapdown inertial navigation and spatial orientation systems, the frequency of updating the matrix coefficients of directional cosines is in the range from 10 Hz to 200 Hz (sampling time from 0.1 s to 0.005 s). For the assumed maximum, update time of 0.1 s (algorithm of updating the matrix coefficients of directional cosines) and the maximum angular velocity of the executed manoeuvre of 360 deg/s (rotation in the tilt channel), the rotation angle increase amounts to 36 deg. It was assumed that submitting of the finite rotations in the three-dimensional space of such values should provide the noncommutativity symptoms not only in conical motion, but also during the high-manoeuvring flight performance in the form of the advanced flight manoeuvre [7].

In the Avionics Division of the Air Force Institute of Technology, a mathematical model of determining the noncommutative rate vector components, which constitutes an error in defining the aircraft actual angular velocity, which in the general form is described by the relationship [6]:

$$[\sigma'(t)] = [\phi'(t)] - [\omega(t)] = 0.5\left[[\phi(t)]x[\omega(t)]\right] + A(\phi(t))\left[[\phi(t)]x([\phi(t)]x[\omega(t)])\right], \qquad (2)$$

where:

[σ'(t)] – noncommutative angular velocity vector, non-measurable by inertial sensors,
[ϕ'(t)] – actual angular velocity vector, implemented in the three-dimensional space,
[ϕ(t)] – actual rotation angle vector, implemented in the three-dimensional space,
[ω(t)] – angular velocity vector measured by inertial sensors (e.g. gyroscopic ones),
A(ϕ(t)) – coefficient dependent on the modulus value of the rotation angle vector ϕ(t).

In case of the adopted method of calculation, the general diagram of the algorithm, given in [2], of determination and correction of the noncommutativity error was used (Fig. 4). The tests of the noncommutativity error value were conducted for extortion in the form of classical conical motion of the selected cone-opening angle in the range of 90 deg/s, and the angular velocity and its circulation to 90 deg/s.

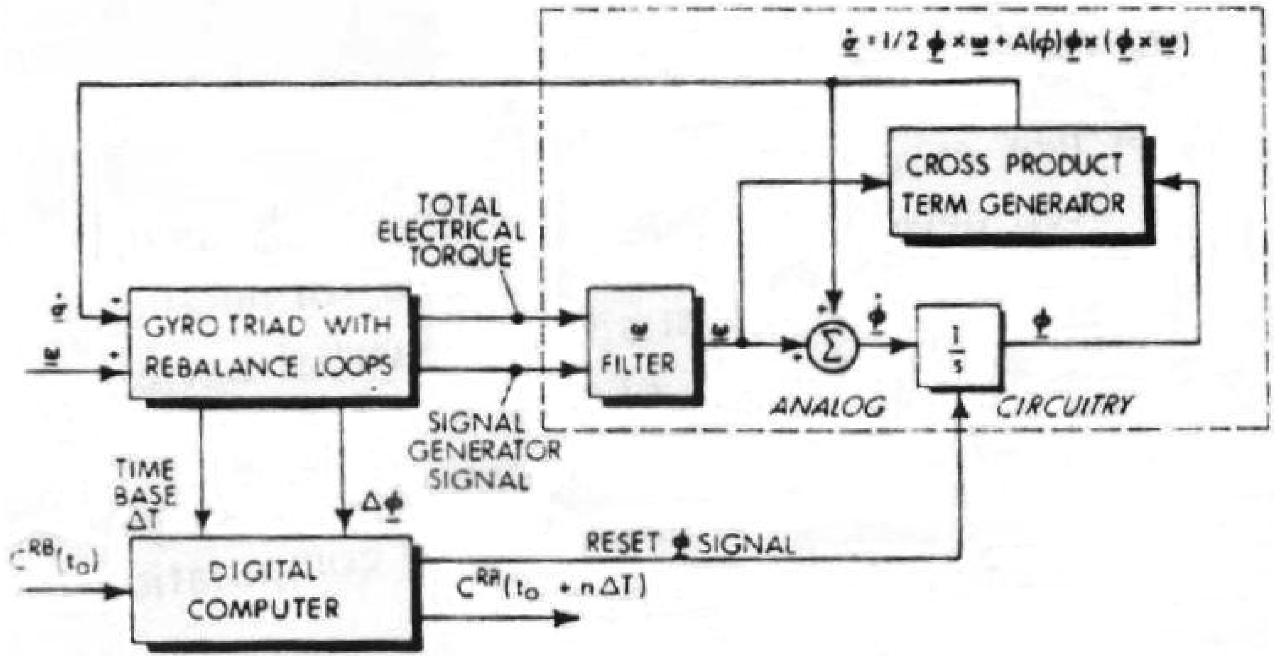


*Fig. 4. General diagram of the calculation algorithm and correction of noncommutativity errors in the complex rotary motion [2]*

The example values of components of the angular velocity measured by inertial sensors, and the noncommutative angular velocity, for extortion in the form of conical motion with the cone-opening angle of 45 deg and its circulation angular velocity of 45 deg/s, were presented in Tab. 1.

For the set conical motion parameters, noncommutative rate vector components in the OX and OY axes achieve the level of 10% of the value of the measured velocity components, however, in the OZ axis; the noncommutative velocity reaches the level of 30% of the cone circulation velocity value. The courses of the vector components of the real, measured and noncommutative angular velocity in the conical motion have the oscillating nature (Fig. 5 and 6), with the exception in the OZ axis (values dependent on the cone opening angle and cone circulation angular velocity).

*Tab. 1. Values of the measured and noncommutative angular velocity components in conical motion*

| Time | Measurity angle velocity | | | Noncommutativity angle velocity | | |
|---|---|---|---|---|---|---|
| t | OX – axis | OY – axis | OZ – axis | OX – axis | OY – axis | OZ – axis |
| [s] | [deg/s] | [deg/s] | [deg/s] | [deg/s] | [deg/s] | [deg/s] |
| 0.0 | 31.820 | 0 | 13.179 | 3.523 | 0 | –13.179 |
| 0.5 | 29.398 | –12.177 | 13.179 | 3.255 | –1.348 | –13.179 |
| 1.0 | 22.500 | –22.500 | 13.179 | 2.491 | –2.491 | –13.179 |
| 1.5 | 12.177 | –29.398 | 13.179 | 1.348 | –3.255 | –13.179 |
| 2.0 | 0 | –31.820 | 13.179 | 0 | –3.523 | –13.179 |
| 2.5 | –12.177 | –29.398 | 13.179 | –1.348 | –3.255 | –13.179 |
| 3.0 | –22.500 | –22.500 | 13.179 | –2.491 | –2.491 | –13.179 |
| 3.5 | –29.398 | –12.177 | 13.179 | –3.255 | –1.348 | –13.179 |
| 4.0 | –31.820 | 0 | 13.179 | –3.523 | 0 | –13.179 |
| 4.5 | –29.398 | 12.177 | 13.179 | –3.255 | 1.348 | –13.179 |
| 5.0 | –22.500 | 22.500 | 13.179 | –2.491 | 2.491 | –13.179 |
| 5.5 | –12.177 | 29.398 | 13.179 | –1.348 | 3.255 | –13.179 |
| 6.0 | 0 | 31.820 | 13.179 | 0 | 3.523 | –13.179 |
| 6.5 | 12.177 | 29.398 | 13.179 | 1.348 | 3.255 | –13.179 |
| 7.0 | 22.500 | 22.500 | 13.179 | 2.491 | 2.491 | –13.179 |
| 7.5 | 29.398 | 12.177 | 13.179 | 3.255 | 1.348 | –13.179 |
| 8.0 | 31.820 | 0 | 13.179 | 3.523 | 0 | –13.179 |

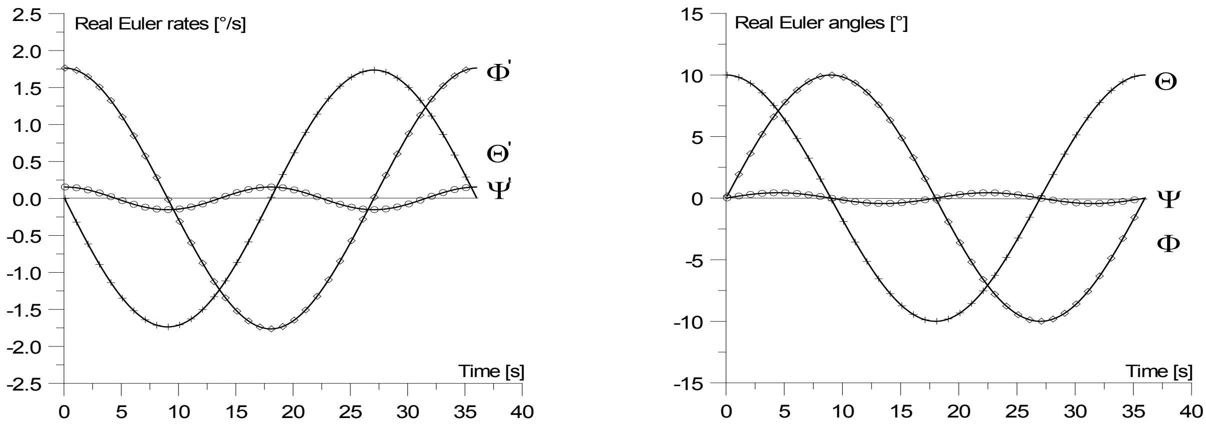


*Fig. 5. Real Euler velocities (left) and real Euler angles (right) found in the conical motion*

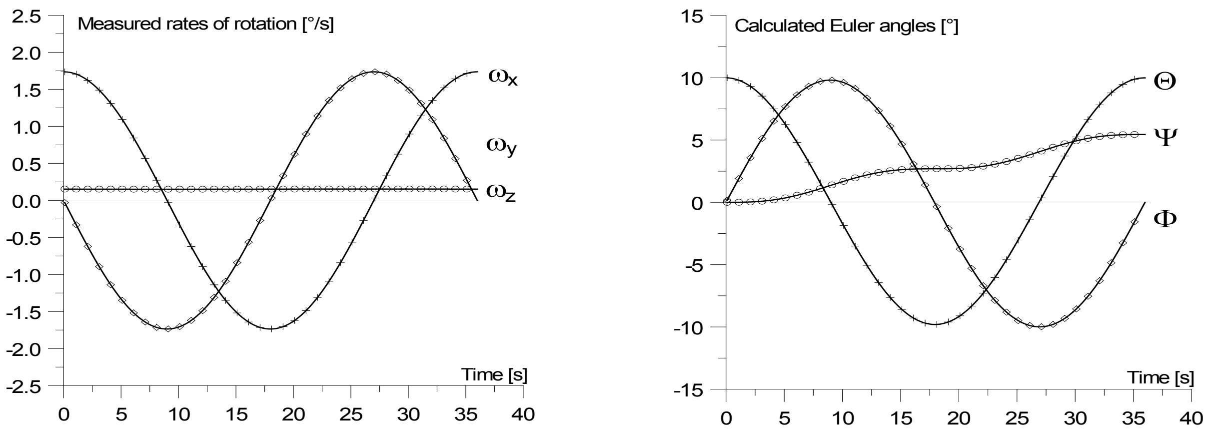


*Fig. 6. Measured angular velocities (left) and calculated Euler angles (right) found in the conical motion*

The simulation tests of a model for determining the spatial orientation errors (Tab. 2) showed that the impact of the phenomenon of noncommutativity is more evident while increasing the cone-opening angle than increasing the angular velocity of its envelope in the performed conical motion.

*Tab. 2. Noncommutative angular velocity values depending on the conical motion parameters*

| Angle of conning | Angle velocity of conning motion | | | | | |
|---|---|---|---|---|---|---|
| [deg] | [deg/s] | [deg] | [deg/s] | [deg] | [deg/s] | [deg] |
| 0 | 1 | 10 | 30 | 45 | 60 | 90 |
| 1 | 0.000152 | 0.001537 | 0.004611 | 0.007121 | 0.009234 | 0.014750 |
| 10 | 0.015230 | 0.154234 | 0.462702 | 0.711209 | 0.925404 | 1.474323 |
| 30 | 0.153238 | 1.372781 | 4.117134 | 6.356689 | 8.234268 | 13.16684 |
| 45 | 0.305811 | 3.058434 | 9.175302 | 13.64243 | 18.35061 | 29.27633 |
| 60 | 0.536285 | 5.362854 | 16.08856 | 24.80416 | 32.17712 | 51.27939 |
| 90 | 1.160212 | 11.60334 | 34.81002 | 53.53022 | 69.62006 | 110.2003 |

The obtained results allowed specifying guidelines for the construction of specialised low-speed positions for testing the operation algorithms of the inertial navigation and spatial orientation systems. They also make it possible to specify the assumptions to develop and test the selected types of the assumed spatial movement, including noncommutative motion, resulting in spatial disorientation states in the flight mobile simulators.

## 4. Research of noncommutativity of rotations during the Su-22 aircraft manoeuvring flight

On the basis of the obtained results of analytical works in the Air Force Institute of Technology and the Polish Air Force Academy, the tests of parameters of the spatial movement generated during the fighter and bomber aircraft actual flight (Fig. 7). The research methodology is based on the information of current attitude values generated from the TESTER on-board flight parameter recorder (data recording in the catastrophic version of the sampling high frequency).

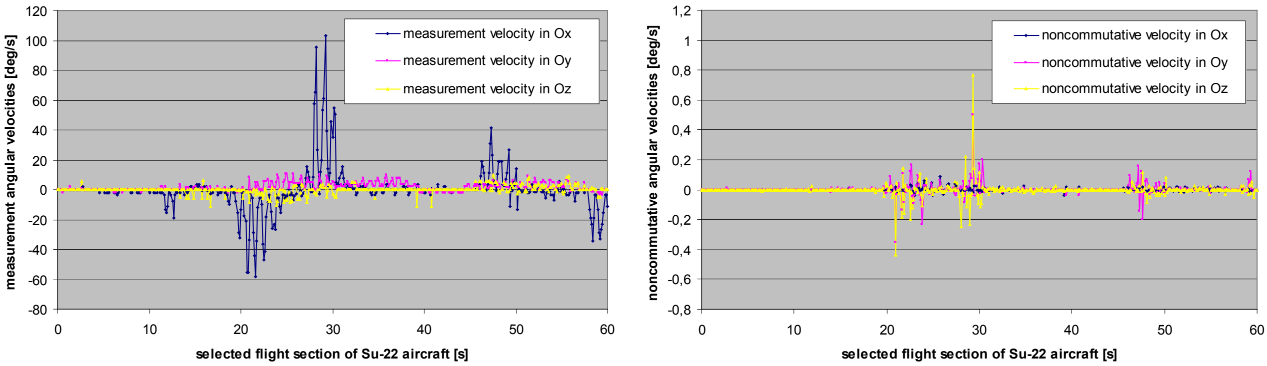


*Fig. 7. Courses of the measured (left) and noncommutative (right) velocities in the Su-22 aircraft manoeuvring flight*

The parameters noncommutativity occurring in the selected manoeuvring flight sections as a complex rotation with noncommutativity elements (among others, noncommutative velocities and angular accelerations) were determined with the use of general mathematical relationships [3], which were brought to the specified form, based on the noncommutative characteristics of the vector product [6]. The purpose of the research was to determine the level of noncommutative rotation velocity in relation to the measured angular velocities by inertial sensors in the inertial navigation and spatial orientation systems in the selected high-manoeuvring flight sections [7].

The research of the rotational motion parameters for the Su-22 aircraft selected flight sections demonstrated that while performing the flight manoeuvres, the angular velocity values, which are not measurable with the use of inertial methods, can reach the values of several deg/s as a result of the occurrence of the noncommutativity phenomenon in submitting the rotations.

The high-manoeuvring flights performed in the Su-22 fighter and bomber aircraft, among others, in terms of making rotations while performing the advanced flight manoeuvres [7], are similar to the trajectory implemented in the "rollercoaster" maze (Fig. 8).

*Fig. 8. Example of the noncommutative rotary motion occurring in the flight (left) and maze (right)*

The attempts to recover these types of manoeuvres can be implemented also using flight mobile simulators (Fig. 9) and specialised control and measuring stations, including [1, 9]. It is assumed that in order to produce the states of noncommutativity by semi-circular canals of the pilot's sense of balance, the elliptical conical motion should be used [3, 8]. The change of the cone opening angle instantaneous value in elliptical motion results in the noncommutative velocity oscillating changes. These changes as noncommutative angular accelerations may interfere spatial orientation felt by the pilot, and influence correctness of the process of the aircraft control in the manoeuvring flight, which was performed by the pilot [7].

*Fig. 9. Example of the ETC-PZL flight mobile simulator (left) and control panel station (right)*

## 4. Summary

The analysis of noncommutativity errors of making rotations in the high-manoeuvring flights is essential to assess the required accuracy of the spatial orientation strapdown inertial systems. The research on the conical motion extortion also allows to quickly assessing the correction methods used in numerical processing algorithms of the measured angular velocity vector in the tested system of inertial navigation and spatial orientation.

The simulation tests of the numerical models of the spatial orientation and inertial navigation strapdown systems, developed in the Air Force Institute of Technology, allow to determine the conditions that can emphasise the phenomena of the rotation making noncommutativity and parameters of its impact for certain types of spatial movement (among others, classical and elliptical conical motion).

The computer-aided test of this phenomenon can also be used to design specialised rotary stations and mobile flight simulators intended for generating the selected states of spatial disorientation caused by noncommutativity errors while testing the pilots and candidates for pilots of high-manoeuvring aircraft in terms of their resistance to this phenomenon effect.

## References


[1] Bartnik, R., Grenda, B., Galej, P., *Symulatory lotu oraz symulatory kontroli ruchu lotniczego w szkoleniu lotniczym*, Wydawnictwo Akademii Obrony Narodowej, Warszawa 2014.

[2] Bortz, J., *A New Mathematical Formulation for Strapdown Inertial Navigation*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 7, Iss. 1, pp. 61-66, 1971.

[3] Gosiewski, Z., Ortyl, A., *Algorytmy inercjalnego bezkardanowego systemu orientacji i położenia obiektu o ruchu przestrzennym. Awionika*, Wydawnictwo Instytutu Lotnictwa, Warszawa 1999.

[4] Gruszecki, J., Bociek, S., *Układy sterowania automatycznego samolotem*, Oficyna Wydawnicza Politechniki Rzeszowskiej, Rzeszów 1999.

[5] Huang, L., Liu, J., Zeng, Q., Xiong Z., *New High-Precision Strapdown Navigation Attitude Algorithm under Angular-Rate Input Condition*, Applied Mathematics & Information Sciences, International Journal, Vol. 9, No. 2L, pp. 283-290, 2015.

[6] Michalak, S., Szelmanowski, A., Raczyński, E., *Badania symulacyjne modeli numerycznych systemów orientacji przestrzennej i nawigacji inercjalnej*, Biblioteka Techniczna Instytutu Technicznego Wojsk Lotniczych, Warszawa 2000.

[7] Milkiewicz, A., *Praktyczna aerodynamika i mechanika lotu samolotu odrzutowego, w tym wysokomanewrowego*, Wydawnictwo Instytutu Technicznego Wojsk Lotniczych, Warszawa 2009.

[8] Ortyl, A., *Synteza i badanie algorytmów inercjalnego bezkardanowego systemu orientacji przestrzennej statku powietrznego*, Wydawnictwo Wojskowej Akademii Technicznej, Warszawa 1995.

[9] Tomczyk, A., *Testing of the attitude and heading reference system*, Aircraft Engineering and Aerospace Technology, International Journal, Vol. 74, pp. 154-160, 2002.