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METHOD FOR CALCULATING FUNCTIONAL READINESS OF VEHICLES SUPPLYING FUELS

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Abstract

Readiness as a measure can be used for describing technical objects, such as vehicles, workstations or operation systems, which are on call and implement the tasks occurring at random time moments. The fact that a low level of readiness may cause various types of losses (human, material, financial, etc.) is particularly important.

The article presents a method that allows calculating readiness of vehicles supplying aviation fuels to aircraft during the performance of flights. The presented model was supported by a numerical example together with interpretation of the obtained results.

In particular, the following elements were presented: oriented graph of the operation process of the vehicle supplying aviation fuel, n_{ij} average numbers between states for ten vehicles' tests, ω_{ij} empirical frequencies of transitions between states for ten vehicles' tests, values of $p_j(n)$ limit probabilities for the Markov chain, values of $p_j(t)$ limit probabilities for the Markov process, as well as comparison of the values of probabilities for the Markov $p_j(n)$ chain $p_j(t)$ and process

Keywords: *means of transport, vehicles, supply, optimisation*

1. **Introduction**

By analysing the concept of readiness, it should be stated that individual authors interpret this term ambiguously and subjectively adapting it to own needs. In general, readiness [5, 10] is understood as a feature of the technical object, which positions its capability in terms of timely undertaking the task at random moment t and/or its implementation in a given period $(t, t + \Delta t)$. It has significant importance in the intervention systems, which perform tasks in the on-call systems (fire service, the army, police, health service), and also in the systems associated with the means of transport (e.g. urban) or in the broadly understood sector of services.

In the subject literature, it is possible to distinguish its following types [9, 10, 11]:

1) *task readiness* – a set of states that make it possible to perform the task or operation within the required time interval $(t, t + \Delta t)$;
2) *functional readiness* – a set of the technical object's operating states that allow to start the task implementation at the "random moment" (without the forecast concerning the task implementation);
3) *operational readiness* – means a set of the technical object's operating states that allow for the task start at the random moment and proper operation in the required time interval $(t, t + \Delta t)$; in practice, it is a combination of functional readiness and task readiness;
4) *initial readiness* – a set of states that allow for proper operation (task start) before the passage of the specified time reserve t;
5) *potential readiness* – a set of states that allow to undertake the task before the time reserve passage and its implementation (or proper functioning in a given time interval); in practice, it is equal to initial readiness and task readiness.

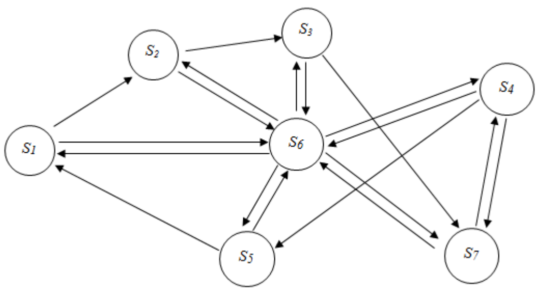
In the article, the attempt to calculate functional readiness for the vehicles supplying fuel to aircraft was undertaken.

2. **Calculation model of vehicles' functional readiness for Markov chain**

On the basis of the analysis of the actual operation process of vehicles during the performance of flights, a seven-state model, in which the following indications were adopted, was distinguished:

S_1 – vehicle access to the airport apron;
S_2 – fuel left to stand;
S_3 – fuel purity control in the vehicle;
S_4 – aircraft refuelling (including the vehicle access to the aircraft, appropriate refuelling and return to the airport apron);
S_5 – vehicle refuelling cycle (access to the storage, vehicle refuelling);
S_6 – vehicle unfitness (replacement to the technically fit vehicle);
S_7 – vehicle waiting for refuelling (dependent on the table of flights, number of vehicles, intensity, type, and length of flights, etc.).

The image of tasks performed by the vehicle includes an operation graph (Fig. 1) and $P = [p_{ij}]_{7x7}$ matrix of transitions described by the relationship (1).



S_1 – vehicle access to the airport apron,
S_2 – fuel left to stand,
S_3 – fuel purity control in the vehicle,
S_4 – aircraft refuelling,
S_5 – vehicle refuelling cycle,
S_6 – vehicle unfitness (replacement to the technically fit one),
S_7 – vehicle waiting for the aircraft refuelling.

Fig. 1. Oriented graph of the operation process of the vehicle supplying aviation fuel

$$P = [p_{ij}]_{7\times 7} = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 \\ 0 & 0 & p_{23} & 0 & 0 & p_{26} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{36} & p_{37} \\ 0 & 0 & 0 & 0 & p_{45} & p_{46} & p_{47} \\ p_{51} & 0 & 0 & 0 & 0 & p_{56} & 0 \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & 0 & p_{67} \\ 0 & 0 & 0 & p_{74} & 0 & p_{76} & 0 \end{bmatrix}. \quad (1)$$

For the actual operation process tests, the following numbers of transitions between the states and empirical frequencies of transitions between states in the test of 10 vehicles (Tab. 1 and 2) were obtained.

Tab. 1. Average numbers n_{ij} between states for the test of ten vehicles

i/j	S_1	S_2	S_3	S_4	S_5	S_6	S_7	$\sum$
n_{1j}	0	30	0	0	0	0.16	0	30.16
n_{2j}	0	0	32	0	0	0	0	32
n_{3j}	0	0	0	0	0	0	37	37
n_{3j}	0	0	0	0	32	0	25	57
n_{3j}	36	0	0	0	0	0	0	36
n_{6j}	0.16	0	0	0	0	0	0	0.16
n_{7j}	0	0	0	55	0	0	0	55

Tab. 2. Empirical frequencies ω_{ij} of transitions between states for the test of ten vehicles

i/j	S_1	S_2	S_3	S_4	S_5	S_6	S_7	$\sum$
ω_{1j}	0	0.99	0	0	0	0.01	0	1
ω_{2j}	0	0	1	0	0	0	0	1
ω_{3j}	0	0	0	0	0	0	1	1
ω_{4j}	0	0	0	0	0.56	0	0.44	1
ω_{5j}	1	0	0	0	0	0	0	1
ω_{6j}	1	0	0	0	0	0	0	1
ω_{7j}	0	0	0	1	0	0	0	1

According to the theory [2, 3, 7, 8] on Markov processes with discrete time, the limit probabilities are calculated as:

$$P * [p_j] = p_j \tag{2}$$

with the system standardisation condition at the same time $\sum_{j \in S} p_j = 1$,
where:
$P-$ a stochastic matrix of transitions, where $P = [p_j, i, j \in S]$; S – a phase space of the process.
The standardisation condition is an additional and essential equation, because it excludes a zero solution of systems (2).
After inserting data from Tab. 2 for discrete time, the following system of equations of the limit probabilities were obtained:

$$\begin{cases} p_5 + p_6 - p_1 = 0, \\ 0.99p_1 - p_2 = 0, \\ p_2 - p_3 = 0, \\ p_7 - p_4 = 0 \\ 0{,}56p_4 - p_5 = 0, \\ 0{,}01p_1 - p_6 = 0, \\ p_3 + 0{,}44p_4 - p_7 = 0, \end{cases} \tag{3}$$

together with the system standardisation condition $\sum_{j=1}^{7} p_j = 1$.

The limit probabilities $p_j(n)$ constituting the solutions of systems (3) were presented in Fig. 2.

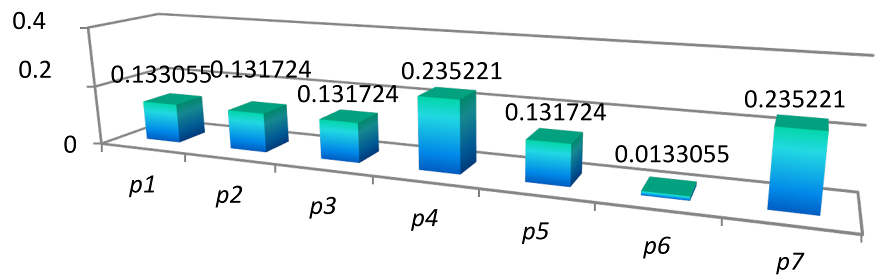


Fig. 2. Values of $p_j(n)$ limit probabilities for the Markov chain

The obtained results (Fig. 2) show that there is the greatest probability of the vehicle entry into the states of refuelling p_4 and waiting for refuelling p_7. This interpretation applies to the number limit of the vehicle occurrence in individual states to the sum of the number of all the transitions (discrete time) of the Markov chain. It means that calculated $p_j(n)$ is standardised in the set of all the process states, and not within the actual time. Therefore, they cannot be interpreted in the quality sense to the readiness assessment. The functional readiness indicator of the vehicle can be determined after taking into account the continuous time, which refers to the actual phase trajectories of the process. Therefore, it is important to convert P matrix to the standardised form in the set of times (Λ intensity matrix of process transitions), i.e. transitions from discrete time to the actual one.

3. Calculation model of vehicles' functional readiness for Markov process

The transition from the discrete time to the actual one is done by the intensity matrix of the process transitions, which was presented below for the described process (equation 4).

$$\Lambda_{7X7} = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & 0 \\ 0 & -\lambda_{22} & \lambda_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_{33} & 0 & 0 & 0 & \lambda_{37} \\ 0 & 0 & 0 & -\lambda_{44} & \lambda_{45} & 0 & \lambda_{47} \\ \lambda_{51} & 0 & 0 & 0 & -\lambda_{55} & 0 & 0 \\ \lambda_{61} & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 \\ 0 & 0 & 0 & \lambda_{74} & 0 & 0 & -\lambda_{77} \end{bmatrix}. \tag{4}$$

For the stochastic process being the Markov process $X(t)$, off-diagonal intensities [1, 2, 3, 4, 7, 8] are calculated according to the following formula:

$$\lambda_{ij} = \frac{1}{\bar{t}_{ij}}, \tag{5}$$

where: $i,j \in \{1, \dots 7\}$, however, $\bar{t}_{ij}$ is the average time $X(t)$ process staying in i state before transition to j state calculated according to the relationship (6):

$$\bar{t}_{ij} = \sum_{k=1}^{n} {}_n\bar{t}_{ij} / N, \tag{6}$$

where:

${}_n\bar{t}_{ij}$ – the average time of staying in i state before transition to j state for the vehicle No. n;

N – number of vehicles in $N \in \{1, \ldots, 10\}$ studied test.

After substituting Λ matrix into $\Lambda^T * [p_j] = 0$ equation, for the tested operation process, the following equation in the matrix form was obtained (7):

$$\Lambda^T{}_{7X7} = \begin{bmatrix} -\lambda_{11} & 0 & 0 & 0 & \lambda_{51} & \lambda_{61} & 0 \\ \lambda_{12} & -\lambda_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_{23} & -\lambda_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_{44} & 0 & 0 & \lambda_{74} \\ 0 & 0 & 0 & \lambda_{45} & -\lambda_{55} & 0 & 0 \\ \lambda_{16} & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 \\ 0 & 0 & \lambda_{37} & \lambda_{47} & 0 & 0 & -\lambda_{77} \end{bmatrix} * \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

or in the form of linear equations as the relationship (8):

$$\begin{cases} -\lambda_{11}p_1 + \lambda_{51}p_5 + \lambda_{61}p_6 = 0, \\ \lambda_{12}p_1 - \lambda_{22}p_2 = 0, \\ \lambda_{23}p_2 - \lambda_{33}p_3 = 0, \\ -\lambda_{44}p_4 - \lambda_{74}p_7 = 0, \\ \lambda_{45}p_4 - \lambda_{55}p_5 = 0, \\ \lambda_{16}p_1 - \lambda_{66}p_6 = 0, \\ \lambda_{37}p_3 - \lambda_{47}p_4 - \lambda_{77}p_7 = 0. \end{cases} \tag{8}$$

For data obtained from tests of the operation process implemented in the actual logistic system, λ_{ij} and λ_{ii} intensity values listed in Tab. were calculated. 1.

Tab. 3. Intensity matrix Λ of the process transitions

λ_{ii} / λ_{ij}	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
λ_1	-0.076136364	0.000378788	0	0	0	0.075757576	0
λ_2	0	-0.001	0.001	0	0	0	0
λ_3	0	0	-0.000333333	0	0	0	-0.000333333
λ_4	0	0	0	-0.001042425	0.000727273	0	0.000315152
λ_5	0.000309091	0	0	0	-0.000309091	0	0
λ_6	0.03607634	0	0	0	0	-0.03607634	0
λ_7	0	0	0	0.0001022735	0	0	-0.0001022735

Off-diagonal intensities λ_{ij} were calculated according to the formula [5], however, λ_{ii} diagonal intensities as:

$$\lambda_{ii} = -\sum_{j \in S} \lambda_{ij}, i \neq j, \tag{9}$$

where:
$\sum_{j \in S} \lambda_{ij}$ – the sum of intensities of transitions from i state to j state, in individual rows of Λ matrix, with $S \in \{1, \dots 7\}$.

The probabilities $p_j(t)$, normalised in the actual time, of the vehicle staying in different states were presented in Fig. 3.

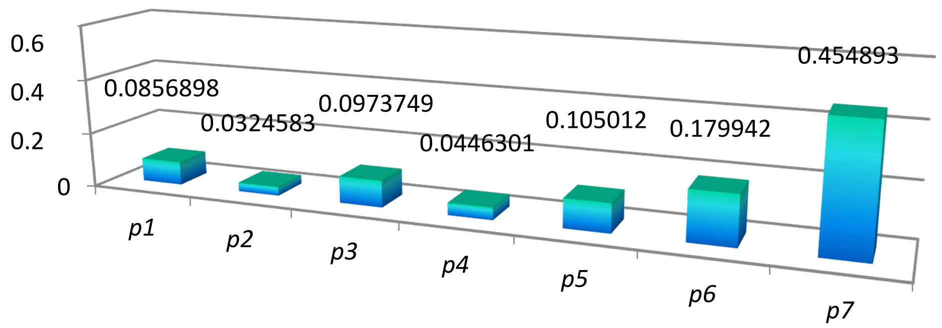


Fig. 3. Values of $p_j(t)$ limit probabilities of the Markov process for $T_0 = 8[h]$

By analysing the results presented in Fig. 3, it should be stated that the vehicle, taking into account $T_0 = 8$ h, on average, stays in the state of readiness for refuelling (p_4+p_7) for approx. 50% of time (functional readiness indicator), the remaining time, i.e. approx. 50%, is intended for necessary organisational measures, such as the vehicle access to the airport apron, fuel left to stand, purity control, refuelling cycle or damage. The above activities, from the perspective of the tested operation process, are organisationally necessary, and they must be implemented in accordance with the adopted procedures.
The comparison of the values of p_j limit probabilities for the Markov process and chain was demonstrated in Fig. 4.

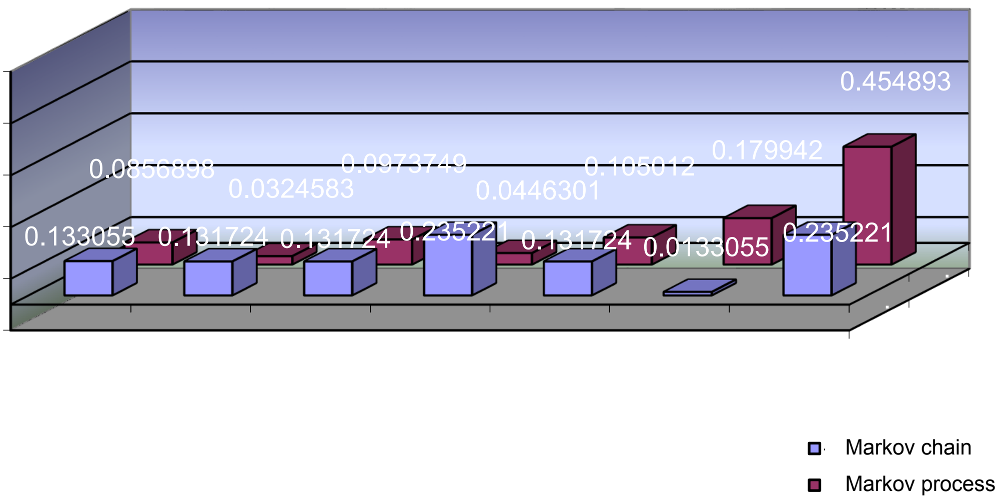


Fig. 4. Comparison of the probability values for the Markov chain $p_j(n)$ – front and $p_j(t)$ process – back

4. Final conclusions

The article presents the method for calculating functional readiness of vehicles supplying aviation fuel for aircraft. The studied test included 10 vehicles of the tank-distributor type, with the capacity of 7.5 [m^3], supplying Su-22 aircraft performing the flights. For calculation, the Markov processes with discrete and continuous time were used.

By analysing the results on p_j limit probabilities, referring to discrete time (Markov chain) and continuous time (Markov process), the following conclusions can be formulated:

a) for discrete time:
  - the greatest entry probability was observed for S_4 (aircraft refuelling) and S_7 (vehicle waiting for refuelling) states, and it is a proper phenomenon from the perspective of the fundamental purpose of the tested process;
  - the same entry probabilities of $p_2, p_3, p_5 = 0.131724$ were obtained for S_2 (fuel left to stand), S_3 (fuel purity control in the vehicle) and S_5 (vehicle refuelling cycle) states, which is consistent with the analysed operation process organisation. The above−mentioned states are positively correlated in parallel and in case of the occurrence of one of them; the other must be implemented;
  - slightly higher entry probability (compared to p_2, p_3, p_5) was observed for S_1 state (vehicle access to the airport apron). It is associated with the first access of the refuelled vehicle to the airport apron.
  - the lowest entry probability was observed for S_6 state (vehicle unfitness), for which the damage in the studied test occurred on average every 6 years.

b) for continuous time:
  - the calculated functional readiness indicator of the vehicle supplying aviation fuel is 0.5 ($p_4 + p_7 = 0.49952$), however, it would seem to be too low. It should be noted that this indicator is understood as the vehicle capability to perform tasks at the randomly selected moment. Having regard to the fact that the structure of flights is a process completely covered by a plan (the so-called planned table of flights), it should be considered that the calculated indicator value fully protects the supply of fuel for aircraft;
  - probability of staying in the state of unreadiness is also 0.5, and it is justified by organisational activities and those necessary to implement the vehicle refuelling cycle, i.e. the state of fuel left to stand and purity control, the state of access to the airport apron and damage, which statistically occur very rarely, but in case of the studied test, they are a long-lasting state from the perspective of time.

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