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OPTIMISATION OF AIRCRAFT MAINTENANCES IN THE POLISH ARMED FORCES

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Abstract

Maintaining the aircrafts (AC) of the Polish Armed Forces in operating condition is determined by adequate resources to secure maintenance and repairs possessed by the companies implementing these tasks on them. The article characterises and then describes formally the Aircrafts Maintenance System in the Polish Armed Forces. The assumptions of the aircraft maintenance system model, proposed model structure, and mapping of aircraft maintenance performance were described. The most important elements (such as data, decision variables, boundary conditions and assessment criterion function) of an optimisation task aimed at minimising the executed aircraft maintenances' costs were formulated. For this purpose, the record of evaluation criterion functions of the companies performing aircraft maintenance was shown. The article was completed with a conclusion, which underlined the importance of the proposed optimisation model.

The optimisation model proposed in the article allows evaluating the aircraft maintenances' organisation and their parameters. The formal record concerns both in the case of one-dimensional maintenance (one aircraft type) and multi-dimensional one (various aircraft types).

Keywords: *aircraft, maintenance system, optimisation task*

1. Introduction

Logistics has always accompanied combat operations and played a role supporting the armed forces according to the intentions of their commanders. One of the components of the military logistics is the correct organisation of technical equipment operation process, which is used by forces within the structure of the Polish Armed Forces. It is obvious that external companies involved in the proper organisation of this process within outsourcing have an effect on it. One of the determinants of the companies implementing the operational services of technical devices is the rational use of held resources. The resources are understood both as the means of production (the appropriate process, documentation and personnel level), as well as an organisational system. The combination of these resources into an appropriate system determines its proper functioning. The article assumed that maintenance system management is the effective implementation of functions involving quantitative and structural selection of resources in accordance with their intended purpose and their continuous maintenance of state of readiness. While the term of technical equipment maintenance means the team of intentionally organisational-technical and economic actions aimed at maintaining or restoring of equipment usability for further operation. In the technical devices maintenance system, an executive subsystem responsible for maintenance performance and a planning subsystem responsible for proper planning the executed maintenance tasks can be distinguished. The maintenance and use subsystems are included in the technical operation system. This article presents the issue of aircrafts operation, without which it is hard to imagine its operation these days.

2. Aircraft maintenance system

Any system can be presented as a set of items associated with each other and the environment in such a way enabling to achieve a certain purpose. In a technical devices maintenance system, three subsystems can be distinguished in general: operation (use), planning and executive ones. Elements of these subsystems and the relationships between them make up the maintenance system.

Therefore, the Aircraft Maintenance System (AMS) consists of:

- operation (use) subsystem elements presented in the form of collections of: aircraft types, aviation-technical personnel, technical operation means, regulations and standards governing the selection and maintenance of appropriate scopes of aviation technique operation and maintaining and restoring operation usability,
- maintenance executive subsystem elements presented in the form of collections of: aircraft types, engineering and aviation personnel, maintenance means and maintenance programmes and methods of operation. In this subsystem, primarily maintenances aimed at, among others, supplying the aircrafts with fuel, electricity, operation gases and liquids and, additionally, special equipment to control the technical condition, prevention activities, repairs are performed,
- planning subsystem elements presented in the form of collections of: aircraft types, aviation technical personnel, technical operation means and historical data and resulting forecast data on use of the above mentioned collections' elements.

As mentioned earlier, the competent logistics security of the Polish Armed Forces is affected by the Aircraft Manufacture System. Aircraft maintenance at military airports involves among others: supplying (delivering) spare parts, fuel, oxygen and other utilities, airport power engineering, individual ground equipment, etc. [4, 9].

Organisations deal with maintenances of aircrafts used at military airports in two ways: using specialised service groups at the user's airport or, if this is impossible, after transporting at the organisation's headquarters. Of course, the implementation of the aforementioned tasks is determined by the possession of specific resources in the form of service groups, process lines by the company. At the same time, the activity of any company is based on maximising the use of the abovementioned resources at minimising involvement of the company's personnel and maintenance means.

Therefore, the implementation of aircraft maintenances requires specified resources in the form of service groups, process lines, personnel trained in the field of AC maintenance and an appropriate spare parts supply. Evaluation of AMS's activities should take into account – apart from minimising the maintenances costs, personnel engagement costs – also costs related with maintenance means.

Because the actual aircraft maintenance system (due to the large number of elements) is complicated for its analysis, it is advisable to develop the model, which should reflect the complexity and interdependence of phenomena taking place during the actual operation.

3. Aircraft maintenance system model

3.1. Assumptions for the model construction

For the purpose of constructing the model, it is necessary to map its basic properties, i.e.: elements of the system and their characteristics. For the purpose of a formal description of maintenances system modelling, the models input data must be identified [2, 3].

The Aircraft Maintenance System **MSOSP** is shown as an ordered three in the form of:

$$\mathbf{MSOSP} = \langle \mathbf{G},\ \boldsymbol{F},\ \boldsymbol{O} \rangle, \tag{1}$$

where:

MSOSP – aircraft maintenance system model,
G – **MSOSP** model structure,
F – a set of **MSOSP** elements' characteristics,
O – organisation of implementing aircraft maintenances.

The **MSOSP** model structure is presented in an ordered dyad:

$$\mathbf{G} = \langle \boldsymbol{A},\ \boldsymbol{R} \rangle, \tag{2}$$

where:

A – a set of elements belonging to the modelled system (aircraft types, airports, companies providing maintenances, etc.),
R – a set of the relationships between the aircraft maintenance system's elements.

Optimal organisation of aircraft maintenances allows the best (in terms of the adopted criteria) evaluation of the implementation of these maintenances. Optimisation tasks formulated adequately to the contemplated decision situation require the definition of decision variables, constraints and criterion function, which is a measure of the problem solution quality.

3.2. Model structure

For the purpose of AMS's model development, it is assumed that different aircraft (AC) types will be numbered with index s. Therefore, the set $\boldsymbol{S}$ will be a set of numbers of aircraft types, i.e. a set in the form of:

$$\boldsymbol{S} = \{1, 2, ..., s, ..., S\}, \tag{3}$$

whereas S is the set $\boldsymbol{S}$ size (the number of different AC types).

In addition, for each element of the set $\boldsymbol{S}$, an aircraft type, the $\boldsymbol{Re}(s)$ of numbers service life types of the s-type aircrafts to be regenerated, i.e.:

$$\forall s \in \boldsymbol{S}, \qquad \boldsymbol{Re}(\boldsymbol{s}) = \{(s, re): \quad re = 1, 2, ..., Re(s)\}, \tag{4}$$

where: $Re(s)$ means the number of different service lives of the s-type aircrafts.

Since, from the operational point of view, individual aircrafts are important, for each element of the set $\boldsymbol{S}$ an aircraft type, a set $\boldsymbol{L}(s)$ of type-s aircraft number was specified, i.e.:

$$\forall s \in \boldsymbol{S}, \qquad \boldsymbol{L}(\boldsymbol{s}) = \{(s, k): \quad k = 1, 2, ..., L(s)\}, \tag{5}$$

where: $L(s)$ means the number of different s-type aircrafts.

Types of maintenances performed on the aircrafts were numbered with an index p. The maintenance numbers set was recorded in the form of:

$$\boldsymbol{P} = \{1, 2, ..., p, ..., P\}, \tag{6}$$

where: P means the number of all maintenances types.

We assume that function ψ was set on the Cartesian product $\boldsymbol{S}\times\boldsymbol{P}\times\boldsymbol{Re}(s)$. The function assigns the following numbers from the set {0, 1} to the product elements, i.e.:

$$\psi: \ \boldsymbol{S} \times \boldsymbol{P} \times \boldsymbol{Re}(\boldsymbol{s}) \longrightarrow \{0, 1\}, \tag{7}$$

whereas $\psi(s,p,(s,re))=1$, when a p-type maintenance is executed on the s-type aircraft in order to reconstitute the service life with the number re, otherwise $\psi(s,p,(s,re))=0$.

For each aircraft, type of number s, a set $\boldsymbol{LP}(s)$ of maintenance numbers was specified, on which they are performed, i.e. a set in the form of:

$$\forall s \in \boldsymbol{S}, \quad \boldsymbol{LP}(s)=\{p: \ \psi(s,p,(s,re))=1,\ (s,re)\in \boldsymbol{Re}(s)\}, \\ \boldsymbol{LP}(s)\subset \boldsymbol{P}. \tag{8}$$

Since the maintenances may consist of various activities (depending on the technical condition of the aircraft). They can be implemented in various ways, therefore, in order to distinguish them, different processes were marked with an index r. The set of process numbers is, therefore, in the form of:

$$\boldsymbol{R}=\{1,2,...,r,...,R\}. \tag{9}$$

It was assumed that the α function was set on the Cartesian product $\boldsymbol{R}\times\boldsymbol{P}\times\boldsymbol{Re}(s)$:

$$\alpha: \ \boldsymbol{R}\times\boldsymbol{P}\times\boldsymbol{Re}(\boldsymbol{s})\longrightarrow\{0,1\}. \tag{10}$$

whereas $\alpha(r,p,(s,re))=1$, when there are the type of the p-type maintenance for renewal of a service life of the number re is implemented with the rth process, otherwise $\alpha(r,p,((s,re))=0$.

Therefore, for each type of service life and maintenance for renewal this aircraft service life with the number s, a set $\boldsymbol{R}(p,(s,re))$ of process numbers that are being performed on it was specified, i.e. a set in the form:

$$\forall p\in \boldsymbol{P},\ \forall s\in \boldsymbol{S},\ \forall(s,re)\in \boldsymbol{Re}(\boldsymbol{s}), \quad \boldsymbol{R}(p,(s,re))=\{r: \ \alpha(r,p,(s,re))=1,\ r\in \boldsymbol{R}\}, \\ \boldsymbol{R}(p,(s,re))\subset \boldsymbol{R}. \tag{11}$$

For the purposes of deliberation, a set $\boldsymbol{R}1(p,(s,re))$ with the elements defined as follows was defined:

$$\forall p\in \boldsymbol{P}, \forall s\in \boldsymbol{S},\ \forall(s,re)\in \boldsymbol{Re}(s),\ \boldsymbol{R}1(p,(s,re))=\{(r,p,(s,re)): \ \alpha(r,p,(s,re))=1\}. \tag{12}$$

On the set $\boldsymbol{R}1(p,(s,re))$, the function τ was set, which assigns positive real numbers to the elements of this set, $\Re^+$ i.e.:

$$\tau: \ \boldsymbol{R}1(p,(s,re))\longrightarrow\Re^+,$$

whereas the size of the $\tau(r,p,(s,re))\in\Re^+$ is interpreted as the time of the p-type maintenance execution in order to renew the service life re on the s-type aircraft with rth process.

On the set $\boldsymbol{R}1(p,(s,re))$, the function χ was set, which assigns positive numbers to the elements of this set, $\Re^+$ i.e.:

$$\chi: \ \boldsymbol{R}1(p,(s,re))\longrightarrow\Re^+,$$

whereas the size of the $\chi(r,p,(s,re))\in\Re^+$ is interpreted as the cost of the p-type maintenance execution in order to renew the service life re on the s-type aircraft with rth process.

Therefore, the characteristics of all types of manufactures implemented on the *s-th* AC type can be presented in the form of three matrices:

$\boldsymbol{\Psi}=[\psi(p,(s,re))]$ – assignment matrix of maintenance types to the aircraft types,

$\mathbf{W}=[\tau(r,p,(s,re))]$ – matrix of the pth operation execution times on the *s-th* AC type using the rth technology. Later in the article, matrices **W1** and **W2** of maintenance time taking into account the renewal of the service life, service groups and process lines were defined,

$\mathbf{H} = \left[\chi\left(r, p, (s,re)\right)\right]$ – matrix of maintenance costs of the system implementing the maintenance number p on the *s-th* AC type using the rth technology. Later in the article, matrices **H1** and **H2** of maintenance times taking into account the renewal of the service life, service groups and process lines were defined.

Whereas single aircrafts are also characterised:

– intensity of use of the s-th type aircraft number k in an adopted time unit $\lambda(s,k)$,

– maintenance frequency of an aircraft $\Delta\mathbf{E}\ (s,k)$.

In order to describe the intensity of the aircraft use, it is assumed that the function λ is set on the set $\boldsymbol{L}(s)$, which assigns positive number to the elements of this set, i.e.:

$$\lambda :\ \boldsymbol{L}(s) \longrightarrow \Re^{+},$$

whereas $\lambda(s,k) \in \Re^{+}$ and has use intensity interpretation of the kth aircraft specimen of the s-type aircraft.

Operation of the aircrafts does not affect their maintenance's condition. In the case of intensive use, periodic inspections are performed more often. The inspections are executed "after spending" a specified number of hours in the air or a certain period of the aircraft's operation. Therefore, the aircraft service life is determined not only in years, but also in hours. After flying a "critical" number of hours, the airplane is out of service or its service life is renewed. In this way, for example, the trainer planes PZL-130 Orlik designed and manufactured in the early 90s of the last century have been renovated in recent years and modified by the PZL "Warszawa – Okecie" S.A. company to the standard of PZL-130 TC-II and can be operated for another at least 20 years. Within the parallel work, the Air Force Institute of Technology (AFIT) has developed the On-condition Maintenance System Programme (Polish: System Operation according to the Technical Condition, SEWST), which changed the way of service and repairs. It provides using the PZL-130 TC-II airplane without time-specified repair periods and based on real service life, assuming at least 300 hours of annual flying time. The necessary data are entered into the AFIT-developed "Computer System for Registration and Evaluation Process of Aircraft Operation – SAMANTA." The system will allow reducing operation costs by extending the technical service life up to 10 thousand hours (and more).

The disadvantage of intensive aircraft operation is increased frequency of failures and assemblies' wear. It happens that because of these – often-small – failures or procedural reviews, even the majority of the held machines are excluded from use.

The maintenance frequency is affected by normative aircrafts' service life consumption, beyond which the aircraft should be directed to the pth kind maintenance $p \in \boldsymbol{P}$.

Service life resources intervals $\boldsymbol{E}_n(p,(s,k),re)$ shall be determined as follows:

$$\begin{gathered}\forall(s,k) \in \boldsymbol{L}(s),\ \ \forall(s,re) \in \boldsymbol{Re}(s),\ \forall p \in P,\\ \Delta E_n\left(p,(s,k),(s,re)\right) = E_n\left(p,(s,k),(s,re)\right) - E_{n-1}\left(p,(s,k),(s,re)\right)\ \ for\ \ \ n = 1, 2, ..., N,\\ assume\ that\ for\ \ \ \forall(\ \ k) \in \boldsymbol{L}(s),\ \wedge \forall p \in \boldsymbol{P},\ \ \ \ E_0\left(p,(s,k),(s,re)\right) = 0.\end{gathered} \tag{13}$$

where: $\boldsymbol{E}_n(p,(s,k))$ – means the size of the s-th type service life of an aircraft number k after n-th maintenance.

Next, we assume that for each n value – the number of the next maintenance, the maintenance type p is specified so that the function is specified, whose values are numbers of highlighted maintenance kinds.

We assume that we know time $\theta_0(s,k)$ of efficient operation (flying time) of the *s-th* type aircraft number k, and the number of pth kind maintenances $L\left(p,(s,k),(s,re)\right)$ of this service life, to which the aircraft is subjected during operation. We designate operation time $TE(s,k)$ of an aircraft from the dependence:

$$\forall (s,k) \in \boldsymbol{L}(s),$$
$$TE(s,k) = \theta_0(s,k) + \sum_{p \in P} \sum_{r \in \boldsymbol{R}(p,(s,re)))} L(p,(s,k),(s,re)) \cdot t(r,p,(s,k),(s,re)). \tag{14}$$

The graphical image of aircraft time operation is shown in Fig. 1.

The duration $t(r,p,(s,k),(s,re))$ of the pth maintenance made on the kth specimen of the s-th aircraft type using the rth service life renewal process number re is the sum of the aircraft's waiting times $\gamma(p,(s,k),(s,re))$ for the p-type maintenance and the time $\tau(r,p,(s,re))$ of this maintenance's actual implementation.

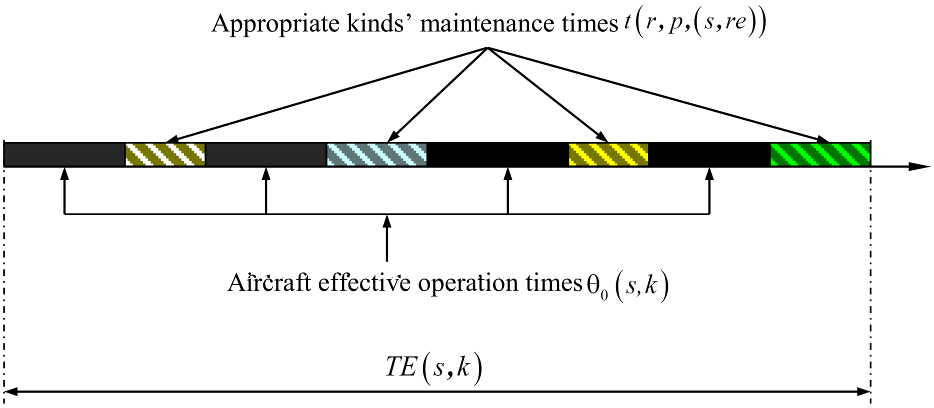

Fig. 1. Illustration of the aircraft operation time

The time $t(r,p,(s,k),(s,re))$ of maintenance duration therefore can be presented in the form of:

$$\forall (s,k) \in \boldsymbol{L}(s), \qquad t(r,p,(s,k),(s,re)) = \gamma(p,(s,k),(s,re)) + \tau(r,p,(s,re)), \tag{15}$$

where:

$t(r,p,(s,k),(s,re))$ – duration time of the pth maintenance executed with the rth process of the kth specimen of the s-type aircraft, whose goal is the renewal of the service life re,

$\gamma(p,(s,k),(s,re))$ – duration time of the pth maintenance of the kth specimen of the s-type aircraft, whose goal is the renewal of the service life re,

$\tau(r,p,(s,re))$ – duration time (actual execution) of the pth maintenance with the rth process of the s-type aircraft, whose goal is the renewal of the service life re.

3.3. Representation of the aircraft maintenances execution characteristics

Assuming that the organisations providing services associated with the AC maintenance are a system, whose elements are process lines (maintenance on the company's premises), numbered with an index i, and service groups (maintenances performed at airports, at which the ACs are located) numbered with an index j. Therefore, it was assumed that $\boldsymbol{I}$ is a set of technological line numbers in the form of:

$$\boldsymbol{I} = \{1, 2, \ldots, i, \ldots., I\}, \tag{16}$$

and that ***J*** is the set of service group numbers in the form of:

$$\boldsymbol{J} = \{1, 2, \ldots, j, \ldots., J\}. \tag{17}$$

Analysis of the maintenance system components' availability requires the passing time in the model. Because of that, time was divided into segments of fixed time length (one second), which were numbered with an index t:

$$\boldsymbol{T} = \{1, 2, \ldots, t, \ldots., T\}. \tag{18}$$

Elements of aircraft maintenance system MSOSP are also airports, at which the aircrafts will be maintained. The airports were numbered with an index *a*, therefore, the set of supported airports number is:

$$\boldsymbol{A} = \{1, 2, \ldots, a, \ldots, A\} \tag{19}$$

whereas A is a size of the set A (numbers of airports, where there are the aircrafts).

Among the airports characteristics, we can distinguish:

1) Type of executed maintenance place

For the identification of the place of performed maintenance on the Cartesian product of sets ***P***, ***A***, ***L***(*s*), ***Re***(*s*), the function δ is set, which assigns the numbers of the two-element set {1,2} to the elements of this product, i.e.:

$$\delta: \ \boldsymbol{P} \times \boldsymbol{A} \times \boldsymbol{L}(s) \times \boldsymbol{Re}(s) \longrightarrow \{1, 2\}. \tag{20}$$

If $\delta\big(p, a, (s,k), (s,re)\big) = 1$ this means that at the airport number *a*, the pth maintenance can be performed on the *k*th specimen of the *s-th* aircraft type, renewing the service life number re.

If $\delta\big(p, a, (s,k), (s,re)\big) = 2$ this means that at the airport number *a*, the pth maintenance cannot be performed on the *k*th specimen of the *s-th* aircraft type, renewing the service life number re.

The function δ can be presented in the form of a matrix Δ:

$$\Delta = \Big[\delta\big(p, a, (s,k), (s,re)\big)\Big]_{P \times A \times K(s) \times Re(s)}. \tag{21}$$

2) Travel time of a service group to the airport:

We assume that the function *t* was set on the Cartesian product of sets ***J*** and ***A***, which assigns the positive real numbers to the elements of this product, i.e.

$$t: \ \boldsymbol{J} \times \boldsymbol{A} \longrightarrow \Re^{+}, \tag{22}$$

whereas the size $t(j,a) \in \Re^{+}$ is interpreted as the travel time of the *j*th service group to the airport number *a*.

Assigning the travel time to the service group to particular airports can be presented in the form of a matrix **TA**:

$$\mathbf{TA} = \big[t(p,a)\big]_{P \times A}. \tag{23}$$

3) Maintenance capabilities of the service groups:

For the identification of the maintenance service groups' capabilities, it was assumed that the function τ1 is set on the Cartesian product of sets ***J***, ***R***, ***P***, ***A***, ***Re***(*s*), which assigns the positive numbers to the elements of this set $\Re^{+}$, i.e.:

$$\tau 1: \ \boldsymbol{J} \times \boldsymbol{R} \times \boldsymbol{P} \times \boldsymbol{A} \times \boldsymbol{Re}(s) \longrightarrow \Re^{+}, \tag{24}$$

whereas the size $\tau 1\left(j,r,p,a,\left(s,re\right)\right)$ is interpreted as the time of providing the *p*th maintenance of the service life number *re* with the *r*th process on the *s-th* aircraft type by the *j*th service group.

The function $\tau 1$ can be presented in the form of a matrix **W1**:

$$\mathbf{W1}=\left[\tau 1\left(j,r,p,a,\left(s,re\right)\right)\right]_{J\times R\times P\times A\times Re(s)}. \tag{25}$$

4) Maintenance capabilities of process lines:

For the identification of the maintenance process lines' capabilities, it was assumed that the function $\tau 2$ is set on the Cartesian product of sets ***I***, ***R***, ***P***, ***A***, ***Re***(*s*), which assigns the positive numbers to the elements of this set $\Re^{+}$, i.e.:

$$\tau 2:\ \boldsymbol{I}\times\boldsymbol{R}\times\boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{Re}\left(s\right)\longrightarrow\Re^{+}, \tag{26}$$

whereas the size $\tau 2\left(i,r,p,a,\left(s,re\right)\right)$ is interpreted as the time of providing the *p*th maintenance of the service life number *re* with the *r*th process on the *s*-th aircraft type, which was at the airport number *a*, with the *i*th process line.

The function $\tau 2$ can be presented in the form of a matrix **W2**:

$$\mathbf{W2}=\left[\tau 2\left(i,r,p,a,\left(s,re\right)\right)\right]_{I\times R\times P\times A\times Re(s)}. \tag{27}$$

5) Costs of the service group's maintenances:

For the identification of the costs of service groups' maintenances, it was assumed that the function $\chi 1$ is set on the Cartesian product of sets ***J***, ***R***, ***P***, ***A***, ***Re***(*s*), which assigns the positive numbers to the elements of this set $\Re^{+}$, i.e.:

$$\chi 1:\ \boldsymbol{J}\times\boldsymbol{R}\times\boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{Re}\left(s\right)\longrightarrow\Re^{+}, \tag{28}$$

whereas the size $\chi 1\left(j,r,p,a,\left(s,re\right)\right)$ is interpreted as the cost of providing the *p*th maintenance of the service life number *re* with the *r*th process on the *s-th* aircraft type by the *j*th service group.

The function $\chi 1$ can be presented in the form of a matrix **H1**:

$$\mathbf{H1}=\left[\chi 1\left(j,r,p,a,\left(s,re\right)\right)\right]_{J\times R\times P\times A\times Re(s)}. \tag{29}$$

6) Cost of using the process lines:

For the identification of the costs of using the process lines in maintenances, the function $\chi 2$ is set on the Cartesian product of sets ***I***, ***R***, ***P***, ***A***, ***Re***(*s*), which assigns the positive numbers to the elements of this set $\Re^{+}$, i.e.:

$$\chi 2:\ \boldsymbol{I}\times\boldsymbol{R}\times\boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{Re}\left(s\right)\longrightarrow\Re^{+}, \tag{30}$$

whereas the size $\chi 2\left(i,r,p,a,\left(s,re\right)\right)$ is interpreted as the cost of providing the *p*th maintenance of the service life number *re* with the *r*th process on the *s*-th aircraft type, which was at the airport number *a*, with the *i*th process line.

The function $\chi 2$ can be presented in the form of a matrix **H2**:

$$\mathbf{H2}=\left[\chi 2\left(i,r,p,a,\left(s,re\right)\right)\right]_{I\times R\times P\times A\times Re(s)}. \tag{31}$$

3.4. Decision variables and optimisation task's restrictions

We assume that the function x is set on the Cartesian product of $\boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{R}\times\boldsymbol{L}\left(s\right)\times\boldsymbol{Re}\left(s\right)\times\boldsymbol{I}\times\boldsymbol{T}$, which assigns numbers of the two-element set {0, 1} to the elements of this product, i.e.:

$$x:\quad \boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{R}\times\boldsymbol{L}\left(s\right)\times\boldsymbol{Re}\left(s\right)\times\boldsymbol{I}\times\boldsymbol{T}\longrightarrow\left\{0,1\right\}. \tag{32}$$

If $x\left(p,a,r,(s,k),\left(s,re\right),i,t\right)=1$ this means that the *p*th maintenance involving the renewal of the *re*th service life with the *r*th process is executed on the *k*th specimen of the *s-th* aircraft type at the airport number *a* in the time interval *t* on the process line number *i*. Otherwise, $x\left(p,a,r,(s,k),\left(s,re\right),i,t\right)=0$.

The function *x* can be presented in the form of a matrix **X**:

$$\mathbf{X}=\left[x\left(p,a,r,\left(s,k\right),\left(s,re\right),i,t\right)\right]_{P\times A\times R\times L(s)\times Re(s)\times I\times T}. \tag{33}$$

Moreover, we assume that the function *y* is set on the Cartesian product of $\boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{R}\times\boldsymbol{L}(s)\times\boldsymbol{Re}(s)\times\boldsymbol{J}\times\boldsymbol{T}$, which assigns numbers of the two-element set {0, 1} to the elements of this product, i.e.:

$$y:\quad \boldsymbol{P}\times\boldsymbol{A}\times\boldsymbol{R}\times\boldsymbol{L}(s)\times\boldsymbol{Re}(s)\times\boldsymbol{J}\times\boldsymbol{T}\longrightarrow\{0,1\}. \tag{34}$$

If $y\left(p,a,r,(s,k),\left(s,re\right),j,t\right)=1$ this means that the *p*th maintenance involving the renewal of the *re*th service life executed with the *r*th process is executed on the *k*th specimen of the *s-th* aircraft type at the airport number *a* in the time interval *t* by the service group number *j*. Otherwise, $y\left(p,a,r,(s,k),\left(s,re\right),j,t\right)=0$.

The function y can be presented in the form of a matrix **Y**:

$$\mathbf{Y}=\left[x\left(p,a,r,\left(s,k\right),\left(s,re\right),j,t\right)\right]_{P\times A\times R\times L(s)\times Re(s)\times I\times T}. \tag{35}$$

The task of optimising the aircraft maintenance system's functioning is characterised with numerous restrictions type, among which, among others, the following can be highlighted:

– restrictions concerning the decision variables' nature:

$$\begin{gathered}\forall p\in\boldsymbol{P},\ \forall a\in\boldsymbol{A},\ \forall r\in\boldsymbol{R},\ \forall\left(s,k\right)\in\boldsymbol{L}(\boldsymbol{s}),\forall\left(s,re\right)\in\boldsymbol{Re}(\boldsymbol{s}),\forall i\in\boldsymbol{I},\ \forall t\in\boldsymbol{T},\\ x\left(p,a,r,(s,k),\left(s,re\right),i,t\right)\in\{0,1\},\end{gathered} \tag{36}$$

$$\begin{gathered}\forall p\in\boldsymbol{P},\ \forall a\in\boldsymbol{A},\ \forall r\in\boldsymbol{R},\ \forall\left(s,k\right)\in\boldsymbol{L}(\boldsymbol{s}),\forall\left(s,re\right)\in\boldsymbol{Re}(\boldsymbol{s}),\forall i\in\boldsymbol{J},\ \forall t\in\boldsymbol{T},\\ y\left(p,a,r,(s,k),\left(s,re\right),j,t\right)\in\{0,1\},\end{gathered} \tag{37}$$

– restrictions concerning duration time of particular maintenance of the aircraft (is systems), among others:
  1) for maintenance type assignment

$$\begin{gathered}\forall a\in\boldsymbol{A},\ \forall\left(s,k\right)\in\boldsymbol{L}(\boldsymbol{s}),\forall\left(s,re\right)\in\boldsymbol{Re}(\boldsymbol{s}),\ \forall t\in\boldsymbol{T}\left((s,k),\left(s,re\right)\right),\\ \sum_{p\in\boldsymbol{P}}\sum_{r\in\boldsymbol{R}}\sum_{i\in\boldsymbol{I}}x\left(p,a,r,(s,k),\left(s,re\right),i,t\right)+\sum_{p\in\boldsymbol{P}}\sum_{r\in\boldsymbol{R}}\sum_{j\in\boldsymbol{J}}y\left(p,a,r,(s,k),\left(s,re\right),j,t\right)=1,\end{gathered} \tag{38}$$

where: $\boldsymbol{T}\left((s,k),\left(s,re\right)\right)$ – a set of moments, at which the service lives *re* are renewed on the *k*th specimen of the *s*-type aircraft.

  2) for maximum time, in which the service live is to be renewed

$$\begin{gathered}\forall a\in\boldsymbol{A},\ \forall\left(s,k\right)\in\boldsymbol{L}(\boldsymbol{s}),\forall\left(s,re\right)\in\boldsymbol{Re}(\boldsymbol{s}),\ \forall t\in\boldsymbol{T},\\ \sum_{p\in\boldsymbol{P}}\sum_{r\in\boldsymbol{R}}\sum_{i\in\boldsymbol{I}}x\left(p,a,r,(s,k),\left(s,re\right),i,t\right)\cdot\tau2\left(i,r,p,a,\left(s,re\right)\right)+\\ \sum_{p\in\boldsymbol{P}}\sum_{r\in\boldsymbol{R}}\sum_{j\in\boldsymbol{J}}y\left(p,a,r,(s,k),\left(s,re\right),j,t\right)\cdot\tau1\left(j,r,p,a,\left(s,re\right)\right)\le T\left(a,(s,k),\left(s,re\right)\right),\end{gathered} \tag{39}$$

where: $T\left(a,(s,k),\left(s,re\right)\right)$ – maximum time, in which the service life *re* is to be renewed on the

k-th specimen of the s-type aircraft at the airport a.

- restrictions concerning the number of resources possessed by the maintenances contractor (service groups, process lines),
- restrictions concerning the number of the possessed employees,
- restrictions concerning the maintenance execution's costs,
- other.

3.5. Quality evaluation – an optimisation task criterion

The criterion of aircraft maintenance companies' activities evaluation should reconcile the interests of both operation process partners from one, general point of view, which can be identified with economic benefits. Therefore, for an organisation involved in the aircraft operation, the issue of maintenance system optimisation comes down to minimising the function F criterion.

The function F, comprising an evaluation criterion, is the sum of two components:

$$F\left(X,Y\right)=\sum_{t\in T}\sum_{s\in S}\sum_{a\in A}\sum_{(s,k)\in L(s)}\sum_{(s,re)\in \mathrm{Re}(s)}\sum_{p\in P}\sum_{r\in R}\sum_{i\in I}x\left(p,a,r,(s,k),(s,re),i,t\right)\cdot\chi 2\left(i,r,p,a,(s,re)\right)+$$
$$+\sum_{t\in T}\sum_{s\in S}\sum_{a\in A}\sum_{(s,k)\in L(s)}\sum_{(s,re)\in \mathrm{Re}(s)}\sum_{p\in P}\sum_{r\in R}\sum_{i\in J}y\left(p,a,r,(s,k),(s,re),j,t\right)\cdot\chi 1\left(j,r,p,a,(s,re)\right)\to \min . \tag{40}$$

Conclusion

The dynamic development of air transport, both in the civil and military field, is related to the increased demand for aircraft maintenances. These maintenances should be executed not only professionally, but also quickly and cheaply. This goal requires:

- on the one hand, from the companies implementing aircraft maintenances, rational use of the possessed resources,
- on the other hand, an appropriate order of assigning the aircrafts to given maintenances.

The optimisation model proposed in the article allows evaluating the aircraft maintenances' organisation and their parameters. The formal record concerns both in the case of one-dimensional maintenance (one aircraft type) and multi-dimensional one (various aircraft types). Apart from the evaluation criterion presented in the article, the following can be applied as additional criteria:

- minimising the maintenances time,
- minimising the time losses resulting from waiting for maintenance,
- maximising the process lines and service groups' load.

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