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SIMULATION OF A THERMOELECTRIC GENERATOR WITH RADIOISOTOPE HEAT SOURCE AS A LONG-LIVED ENERGY SOURCE

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Abstract

Long-lived and sustainable energy sources that are able to supply power in an uninterrupted way without need of maintenance for many years are a topic of interest for many researchers. Thermoelectric generators using radioisotope heat source are currently the longest-lived sources of energy that require no refuelling or other maintenance. Such energy system is also safe and clean. In this article, a mathematical model developed for simulation of a radioisotope thermoelectric generator is presented. The model was prepared using MATLAB/SIMULINK software. It allows performing dynamic simulations of radioisotope thermoelectric generator behaviour with varying load conditions as well as in-time simulations to investigate the effect of aging. A brief theoretical introduction to the topic of thermoelectric generators with radioisotope heat source is presented. Other energy sources typically used in space and remote applications such as fuel cells and photovoltaic systems with auxiliary battery storage are also mentioned and briefly compared. Two cases of radioisotope thermoelectric generator behaviours were simulated. In the first case, the effect of radioisotope decay was investigated, while in the second case the simulations were done at various time instances with varying load conditions. Data received from the simulations was organized and presented. The developed model allows investigating the impact of several thermoelectric generator-working parameters and it is easily modifiable, what could allow performing simulations that are more detailed.

Keywords: *radioisotope thermoelectric generator, thermoelectric module*

1. Introduction

Power sources in some remote systems such as space probes, satellites as well as several off-grid facilities like Arctic stations are a topic of interest of many researchers because of the specific requirements that the power system must fulfil. Such systems that operate in space may not depend on continuous consumption of large amounts of fuel and they require constant power input for extended period, often lasting for years. During that time no maintenance can be performed. In most cases, it is also preferable if the power source would be able to generate as large as possible amounts of power when compared to its size and weight (power and energy density).

As stated before, there are several requirements that must be fulfilled for an independent energy source to operate properly, these requirements are:
- extended lifetime,
- ability to operate for long periods of time without maintenance,
- little or no need of fuel,
- reasonable power and energy density.

Currently, there are several power sources used for spacecraft and other remote applications. Systems used for such power generators are as following:
- radioisotope thermoelectric generator is a device that directly converts heat into electrical current through thermoelectric Seebeck effect,
- a PV system consist of photovoltaic cells that are semiconductor elements where sun radiation energy is transformed into electricity through photoelectric effect,

- fuel cell is a device that generates electric current through reaction of oxidizing fuel, which is constantly provided into it from outside.

In this article, brief comparison of these energy sources is presented. In the second part of the work, a mathematical model of a radioisotope thermoelectric generator is presented. The model was made with MATLAB/SIMULINK software and it allows performing generic calculations of several radioisotope thermoelectric generator parameters. All mathematical dependences and structure of the model are presented. Several simulations that were done with this model are described. Selected results of these simulations are shown.

2. Comparison of energy sources

All energy systems stated before have completely different characteristics. While all of them in theory could be used to supply power in most applications, for every application one of the systems has many advantages when compared to other. In this chapter, a brief comparison of systems characteristics, advantages and disadvantages is conducted.

Tab. 1. Brief comparison of presented energy systems

System	Radioisotope thermoelectric generator	Fuel cell	Solar photovoltaic system	Batteries
Total system electrical power	Up to several hundred W	1kW up to 2MW	1kW up to several hundred MW	Several W up to several hundred MW
Efficiency	5-7%	40%-70% (up to 85% with cogeneration)	10%-46%	60%-90%
Cost (rough estimates)	~12000$/W	~50-250$/kW	~1500-2000$/kW	~500$/kW
Applicability for off-grid applications	Totally impractical	Yes	Yes	Yes
Applicability for space applications	Yes, even for deep space missions	Yes, except deep space missions	Yes, except deep space missions	Yes

Radioisotope thermoelectric generators are definitely the most rarely used energy source from sources presented in this work. Their main characteristics are [1]:

- longest operation time without any need of maintenance,
- extremely huge cost,
- very low efficiency,
- radioisotope thermoelectric generators are only applicable in small spacecraft such as probes that requires up to several hundred watts of electrical powers.

In conclusion – radioisotope thermoelectric generators are only applicable for small spacecraft that does not exceed several hundred watts in energy needs and only for systems that cannot generate power through the sun irradiation – these are deep space missions or missions taking place in a constant eclipse.

Solar photovoltaic systems in terrestrial systems are getting more popular due to increased concern about global warming and environment protection. Their main characteristics are:

- as an energy source in space they are ideal due to the constant solar irradiation that is only affected by distance to the sun and they work is only interrupted during an eclipse. However, they are not applicable in deep space missions due to decreasing solar irradiation,
- their main problems in terrestrial applications are lack of their operation predictability and intermittent operation due to day – night cycle.

- all applications (off-grid and space) require auxiliary energy system such as battery or a regenerative fuel cell to maintain steady system operation.

Fuel cells are also an energy source that is recently getting more and more attention. They are characterized in a different way than other sources but also find application in both terrestrial and space systems.

- Fuel cells can be perceived as well as a power generation system and as energy storage system in case of regenerative fuel cells.
- As a power generation devices in remote off-grid systems they are much more reliable than systems based only on photovoltaic cells and batteries.
- Possibility to build regenerative fuel cells and combine them with normal fuel cells allows steady operation when connected to a photovoltaic system.

Batteries are the basic method of electrical energy storage. Nowadays they are commonly used in a very wide range of applications varying from small portable devices up to several megawatts grid energy storage.

- Batteries come in a variety of types where every type is suitable for different type of application.
- Batteries are also a point of interest for many researchers what results in improved performance, mostly in terms of lifetime and production cost.
- Construction of a battery is much simpler than a regenerative fuel cell that requires additional fuel cell and hydrogen storage tank.

In a very short summary, every energy source presented has its own particular field of application where it is the most suitable solution.

3. Model of a radioisotope thermoelectric generator

A mathematical model of a radioisotope thermoelectric generator was developed for purpose of this work. It is capable of determining several operating conditions of a thermoelectric module. The model was prepared using MATLAB/SIMULINK computational software. The model is capable of computing operational parameters such as radioisotope generator's power, voltage, current, heat input and efficiency for any given set of material, source and load properties. Similar models used for simulation of thermoelectric generator behaviours have been used as a reference [2-4]. In the model, it is assumed that the whole system is an infinite cylinder with internal heat generated inside by the radioactive isotope of Pu-238. The geometry of the model is presented on the Fig. 1.

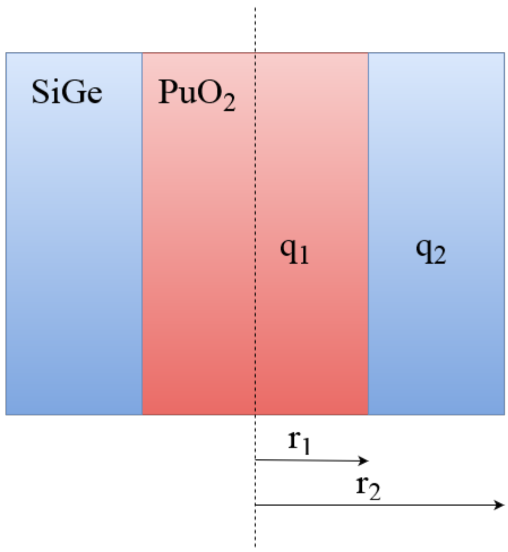

Fig. 1. Graphical representation of the system

On the Fig. 1 r1 and r2 stand for radii of isotope heat source and thermoelectric module on the outer side of the cylinder, q1 and q2 represent heat generated by isotope and consumed by the module.

The model itself consists of four parts.

1. The first part is used to determine the isotope source heat generation output in time. Initial value of heat generation and half-life time are given as input values.
2. The second part is capable of determining radioisotope thermoelectric generator's hot and cold end temperatures by using an analytical solution to the one dimensional heat equation with heat generation and consumption.
3. In the third part, several operational parameters governing thermoelectric generator's work are determined.
4. Fourth part, consisting of all previous parts, determines the value of heat consumption by making an initial assumption and adjusting the heat sink in an iterative way.

Each part is described more precisely in the following sections. There are given exact mathematical formulas performed by the model in each section.

Determination of heat source output:

First part performs a simple calculation using the isotope's half-life time to determine the heat source output at a given time.

Given input:

Q_0 (W/m^3) – Initial heat generation rate

$t_{1/2}$ (y) – Half-life time

Heat source output:

$$Q(t) = Q_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}. \tag{1}$$

Determination of thermoelectric generator hot and cold leg temperature:

One-dimensional approximation of a heat source (isotope) placed next to another negative heat source (thermoelectric element) is made with assumed heat transfer on the outer wall. This model is suitable for heat transfer inside a cylinder with an internal heat source and sink. Total heat transfer coefficient α is adjusted to take into account the impact of thermal radiation.

Given input:

Q (W/m^3) – heat source (from previous section),

r_1 (m) – radius of isotope heat source,

r_2 (m) – total radius of the device (length of thermoelectric element is equal to $R_2 – R_1$),

λ_1 (W/mK) – thermal conductivity of the isotope,

λ_2 (W/mK) –thermal conductivity of thermoelectric material,

α (W/m^{2}K) – total heat transfer coefficient on outer wall,

T_{inf} (K) – cooling medium temperature.

Formulas used to determine the temperatures at hot and cold legs are as following:

$$\frac{d}{dr}\left(\lambda_1 r\frac{dT_1}{dr}\right) + q_1 r = 0, \tag{2}$$

$$\frac{d}{dr}\left(\lambda_2 r\frac{dT_2}{dr}\right) + q_2 r = 0. \tag{3}$$

Solving these differential equations leads to:

$$T_1(r) = \frac{q_1 r_1^2 - q_1 r^2}{4\lambda_1} + \frac{q_1 r_1^2 + q_2 r_2^2 - q_2 r_1^2}{2\alpha r_2} + \left(ln\frac{r_2}{r_1}\right)\frac{q_1 r_1^2 + q_2 r_1^2}{2\lambda_2} + \frac{q_2 r_2^2 - q_2 r_1^2}{4\lambda_2} + T_{inf}, \tag{4}$$

$$\mathrm{T_2(r)} = \frac{\mathrm{q_1 r_1^2 + q_2 r_2^2 - q_2 r_1^2}}{2\alpha \mathrm{r_2}} + \left(\ln\frac{\mathrm{r_2}}{\mathrm{r}}\right)\frac{\mathrm{q_2 r_1^2 + q_1 r_1^2}}{2\lambda_2} + \frac{\mathrm{q_2 r^2 + q_2 r_2^2}}{4\lambda_2} + \mathrm{T_{inf}}, \tag{5}$$

where:

r – radius of the system,

q_1 – internal heat generation of the isotope,
q_2 – internal heat consumption of the thermoelectric element ($q_2 < 0$),
T_1 – temperature of the isotope,
T_2 – temperature of the thermoelectric element,
T_{inf} – temperature of the surroundings,
λ_1 – thermal conductivity of the heat source,
λ_2 – thermal conductivity of the thermoelectric element.

In the model the equation is solved for $T_2(r_1)$ and $T_2(r_2)$ to obtain temperatures at both hot and cold ends. These are the output values of the subsystem denoted as T_h and T_c.

Calculation of thermoelectric generator parameters:

In the next step, the operating values of the radioisotope thermoelectric generator are determined. Prepared model is able to calculate electric parameters such as voltage, current and power as well as total heat input and overall efficiency. It plots the output as a function of time. It is also capable of performing simulations with varying load resistance.

Given input:

T_h (K) – hot end temperature (from previous part),
T_c (K) – cold end temperature (from previous part),
S (V/K) – Seebeck coefficient of thermoelectric material,
R (Ω) – internal resistance of thermoelectric generator,
n – number of thermoelectric modules,
N – number of thermoelectric junctions in a module,
m – load to internal resistance ratio.

Governing equations [3]:

$$I = \frac{nS(T_h - T_c)}{R(1+m)}, \quad (6)$$

$$V = N(nS(T_h - T_c) - IR), \quad (7)$$

$$P = IV, \quad (8)$$

$$Q_h = N(SIT_h + \lambda(T_h - T_c) - 0{,}5RI^2), \quad (10)$$

where:

I – electric current,
V – generated voltage,
P – Thermoelectric generator power,
Qh – Heat input at electric generator hot end,
H – TEG efficiency.

Determining the value of heat sink:

The last part, makes use of all previous parts is used to adjust the value of heat consumed by the thermoelectric element. The value of q_2 is calculated in an iterative way. At first, an initial efficiency is assumed to be at 5% (typical value for a radioisotope thermoelectric generator). The q_2 is calculated according to cross section areas of the isotope and thermoelectric elements. Cross section areas comparison is shown in the Fig. 2.

Final formula that is used to calculate q_2 is following:

$$q_2 = -\eta q_1 \frac{r_1^2}{r_2^2 - r_1^2}. \quad (11)$$

All computations used to obtain thermoelectric generator efficiency are done. At next iteration new efficiency and new value of q_2 is used to make calculations again. The iterations stop when convergence of the efficiency is reached i.e. change of the efficiency is smaller than the convergence criterion:

$$|\eta_{n+1} - \eta_n| < 10^{-6}, \quad (12)$$

where:
n – number of iterations.

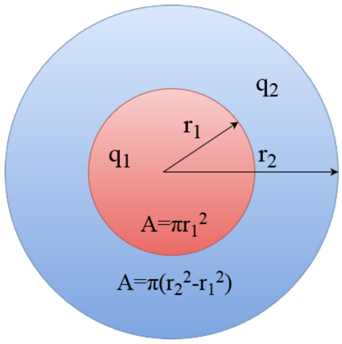

Fig. 2. Cross section areas of the source and thermoelectric module

Assumptions made in the model:

- heat transfer occurring only in one direction,
- material properties are constant,
- system is an infinite cylinder,
- radiation heat transfer is neglected, adjusted heat transfer coefficient is introduced,
- uniform and isotropic materials,
- adjusted thermal conductivity of the thermoelectric material due to real thermoelectric element shape.

5. Simulation results

In this section there are presented results obtained from several simulations. Material data for the simulations was taken from literature [1, 5]. Simulations were performed for time instants at the origin, after 10 years and after 20 years of operation. The load to internal resistance was varying in range from 0 to 15. I-P and I-V characteristic dependence on material properties and the effects of aging are presented in Fig. 3 and 4.

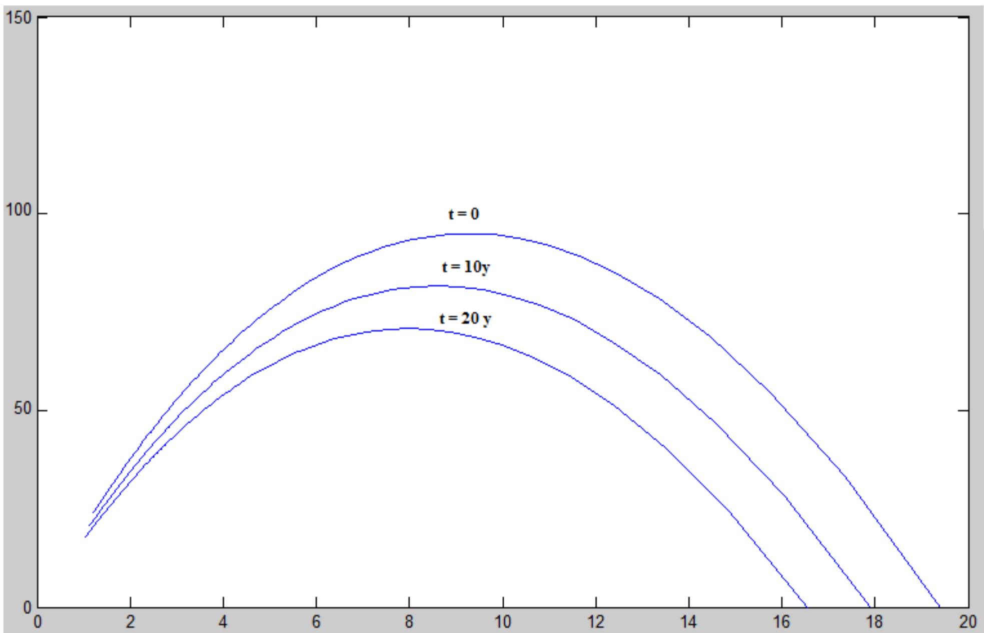

Fig. 3. I-P Characteristics of RTG (S=400 μV/K, t=0, 10y, 20y)

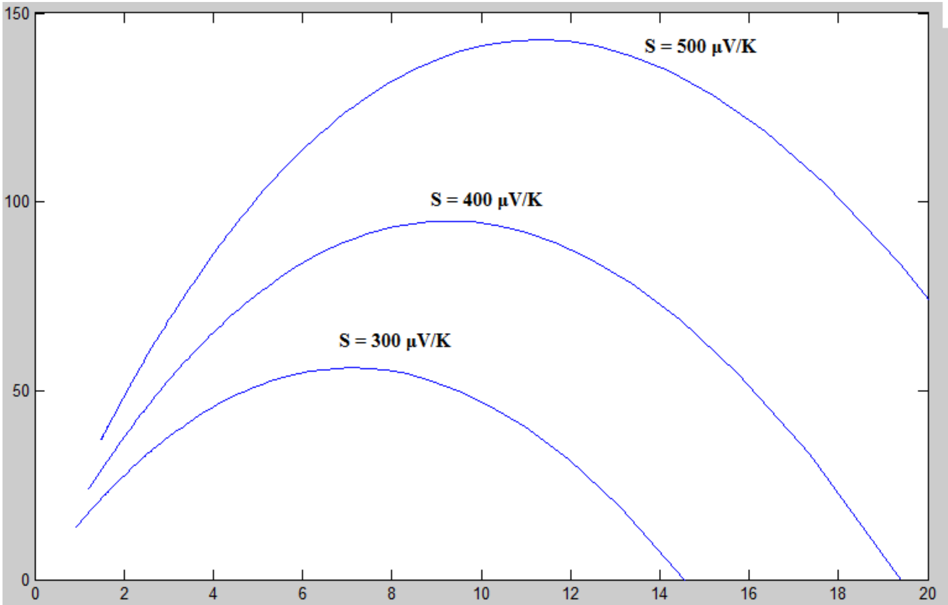

Fig. 4. I-P Characteristics of RTG (S=300, 400, 500 µV/K, t=0)

Simulations show great impact of material properties, values of Seebeck coefficient from about 300 up to 500 µV/K that are presented in literature results in very large difference in thermoelectric generator performance. Impact of aging is quite small. Basing on the simulation result one can draw a conclusion, that a radioisotope thermoelectric generator using plutonium oxide isotope, as all RTGs used in space missions, can steadily operate for several decades.

6. Conclusions

The article presents a generic simulation tool designed in MATLAB/SIMULINK software that allows performing simulations of radioisotope thermoelectric generator's behaviour. It is to be noted that the model is suitable for simulating thermoelectric generator systems with any heat source, not necessarily isotope source. The developed model can be used as an assistance tool in design phase of thermoelectric generator systems.

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