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# ANALYSIS OF THE LOAD CARRYING CAPACITIES AND THE FRICTION FORCE IN THE GAP OF THE SLIDE JOURNAL BEARING LUBRICATED WITH A NON-NEWTONIAN OIL DESCRIBED BY A POWER − LAW MODEL

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***Abstract***

*In this paper, the author presents the results of numerical calculations of load carrying capacities and friction forces in the gap of the slide journal bearing lubricated with an oil on the non-Newtonian's properties. In the studies, the power-law model has been assumed to describe the relationship between the stress tensors and shear rate tensors.*

*The analytical and numerical calculations have been performed for the plain bearing, non-porous with a full wrap angle. It has been assumed isothermal, laminar and steady flow of lubricant in the gap of a slide bearing. Numerical calculations have been performed for the Gumbel's boundary conditions and dimensionless lengths of the bearings like L=b/R =2;1.5;1; ½ and ¼. The flow-rate index and coefficient of consistency have been adopted based on the results of experimental studies of changes of dynamic viscosity in terms of a shear rate. It has been assumed that the apparent viscosity depends only on the shear rate. Dynamic viscosity of the engine oil, used in a gasoline engine with a capacity of 1800 $cm^3$, has been tested on the Haake Mars III rheometer.*

*The analytical solutions presented in the paper were based on more general derivations carried out by Professor K. Wierzcholski in his article: 'Non-linear hydrodynamic lubrication in conjugated fields' (publication in printing). In this paper, the key quantities such as components of vector of the velocity, hydrodynamic pressure and temperature were presented in the form of convergent power series.*

*The values of load carrying capacities and friction forces were determined and compared for the event where the oil has properties of Newtonian and non-Newtonian. Calculations have been made for the dimensionless quantities.*

***Keywords:*** *numerical calculation, load carrying capacities, friction forces, non-Newtonian oil, power-law model*

## 1. Introduction

Both own research and other researchers studies [4], [7] show that some engine oils are characterized by a non-Newtonian properties. Although the producers of engine oils are trying to make their products have properties similar to the Newtonian, as a result of operating in a variety of engine operating conditions, oil changes its properties on the non-Newtonian. A commonly used model of non-Newtonian fluid [2], [3], [5], [6], [8]-[11] is a power-low model [8], [10], [11], and for this particular model, the author conducted a numerical analysis of the load carrying capacities, friction force and friction coefficient in the gap of journal slide bearing lubricated with ferrofluid.

There are presented the numerical calculations for laminar, isothermal lubrication of journal slide bearings with the oil of non-Newtonian properties in this paper. The study assumed the apparent viscosity depending on the magnetic field, i.e. variables ($\alpha_1$, $\alpha_3$). There has been adopted the plain bushing bearing, non-porous with a full wrap angle in the consideration. In this paper has not been taken into account changes in viscosity of the ferrofluid on temperature and pressure due to appoint only the non-Newtonian influences. In the subsequent studies, the author intends to take into account the cross-influences of magnetic field, temperature and pressure changes on dynamic viscosity.

## 2. Analytical model

The fluid flow in the gap journal slide bearing is described by both the vector momentum equation (1) and the equation of continuity of the stream (2). There are included only magnetic forces in the momentum equation. Momentum equation and the equation of continuity of the stream are as follows [4], [7], [9], [10]:

$$\rho\frac{d\mathbf{v}}{dt} = \text{Div}\,\mathbf{S} + \mu_o(\mathbf{N}\nabla)\mathbf{H}, \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \text{div}(\rho\mathbf{v}) = 0, \tag{2}$$

where:
$\rho$ – fluid density kg/m$^3$,
$\mu_o$ – magnetic permeability in vacuum H/m,
$\mathbf{H}$ – magnetic intensity vector A/m,
$\mathbf{N}$ – magnetization vector A/m,
$\mathbf{v}$ – fluid velocity vector in m/s,
$\mathbf{S}$ – stress tensor in the fluid in Pa,
t – time in s.

Into equations of the momentum, we need to join Maxwell's equations for the ferrofluid in a constant magnetic field:

$$\nabla\cdot\mathbf{B} = 0,\ \ \nabla\times\mathbf{H} = \mathbf{0},\ \ \mathbf{B} = \mu_o(\mathbf{H}+\mathbf{N}). \tag{3}$$

Constitutive relationship between stress tensor **S** and rate of strain tensor $\mathbf{A_1}$ for liquids of a power-low model has the following form [4], [7], [9], [10]:

$$\mathbf{S} = -p\boldsymbol{\delta} + \eta_p\mathbf{A_1}, \tag{4}$$

where:
$\boldsymbol{\delta}$ – unit tensor,
$\eta_p$ – apparent dynamic viscosity of non-Newtonian fluid in Pas,
$\mathbf{A_1}$ – rate of strain tensor,
p – pressure in Pa.

For power-law model of the fluid, the apparent viscosity $\eta_p$ has the form:

$$\eta_p \equiv \eta_{pr} = 2^{n-1}m(n)\left|\frac{1}{2}\mathbf{I}_1^2 - \mathbf{I_2}\right|^{\frac{n-1}{2}},\quad \mathbf{I_1} = \Theta_{kk},\ \ \mathbf{I_2} = \frac{1}{2}e_{ijk}e_{imn}\Theta_{jm}\Theta_{kn}, \tag{5}$$

where:
$\mathbf{I_1}$, $\mathbf{I_2}$ – known invariants of displacement velocity tensor in s$^{-1}$, s$^{-2}$,
$\Theta_{ij}$ – displacement velocity tensor in s$^{-1}$,
*n* – dimensionless flow index,
m=m(n) – fluid consistency coefficient in Pas$^n$,
$e_{ijk}$ – tensor Levi-Civity.

For a cylindrical coordinate system, and the adoption of a steady flow in the considerations, as well as the omission of units of the relative radial clearance with order like $\psi = 10^{-3}$, we obtain the following equations:

$$0=-\frac{1}{r}\frac{\partial p}{\partial \phi}+\frac{\partial}{\partial r}\left(m\left|\left(\frac{\partial v_1}{\partial r}\right)^2+\left(\frac{\partial v_3}{\partial r}\right)^2\right|^{\frac{n-1}{2}}\frac{\partial v_1}{\partial r}\right), \tag{6}$$

$$0=\frac{\partial p}{\partial r}, \tag{7}$$

$$0=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial r}\left(m\left|\left(\frac{\partial v_1}{\partial r}\right)^2+\left(\frac{\partial v_3}{\partial r}\right)^2\right|^{\frac{n-1}{2}}\frac{\partial v_3}{\partial r}\right), \tag{8}$$

$$0=\frac{1}{r}\frac{\partial v_1}{\partial \phi}+\frac{\partial v_2}{\partial r}+\frac{\partial v_3}{\partial z}. \tag{9}$$

The system of differential equations (5)-(8) have analytical solutions in the form of infinite convergent power-low series written in the following form [4]:

$$\begin{gathered} v_i(\phi,r,z)=v_i^{(0)}+g\cdot v_i^{(1)}+\ldots+g^k\cdot v_i^{(k)}+\ldots \\ p(\phi,z)=p^{(0)}+g\cdot p^{(1)}+\ldots+g^k\cdot p^{(k)}+\ldots \\ C_\Sigma(\phi,z)=C^{(0)}+g\cdot C^{(1)}+\ldots+g^k\cdot C^{(k)}+\ldots \\ Fr_\Sigma(\phi,z)=Fr^{(0)}+g\cdot Fr^{(1)}+\ldots+g^k\cdot Fr^{(k)}+\ldots \\ \text{for } i=1,2,3;\ k=0,1,2,\ \ldots,\ g\equiv(n-1)/2, \end{gathered} \tag{10}$$

where:
g – small parameter is less then +1/4 and greater than −1/2 for $0<n\leq 3/2$.

Apparent viscosity is shown as a Taylor series in the neighbourhood of the point n=1 with respect to the small parameter of the form:

$$\begin{aligned} \eta_p(v_1,v_3,n,B)&=m(n,B)\left|\left(\frac{\partial v_1}{\partial r}\right)^2+\left(\frac{\partial v_3}{\partial r}\right)^2\right|^g= \\ &=\eta_0\eta_1(B)\left[1+g\cdot\eta_{pr1}+\ldots+g^k\cdot\eta_{prk}+\ldots\right], \end{aligned} \tag{11}$$

where:
$\eta_{prk}$ for k=1,2, …– dimensionless expansion coefficients of dynamic viscosity, for k=0 $\eta_{pr0}=1$.

Substituting the series (10) into equation (11) and receiving indications: $v_0$ – characteristic oil speed, $\varepsilon_0$ – characteristic height gap value, equation (11) is replaced as follows:

$$\begin{gathered} \eta_p(v_1,v_3,n,B)\equiv\eta_{pr}=\eta_0\eta_1(B)[f(n)]^g, \\ \eta_0\eta_1(B)\equiv m(n,B)\left(\frac{v_0}{\varepsilon_0}\right)^{n-1}, \end{gathered} \tag{12}$$

where:

$$f(n) \equiv \left(\frac{\varepsilon_0}{v_0}\right)^2 \left[ \left( \frac{\partial v_1^{(0)}}{\partial r} + \frac{n-1}{2}\frac{\partial v_1^{(0)}}{\partial r} + \ldots \right)^2 + \left( \frac{\partial v_3^{(0)}}{\partial r} + \frac{n-1}{2}\frac{\partial v_3^{(0)}}{\partial r} + \ldots \right)^2 \right]. \quad (13)$$

In order to present the system of equations (6)-(9) in the form of dimensionless there were used the following common known relationships between dimension of the dimensionless values [4], [7], [9], [10]:

$$r=\varepsilon_0 r_1,\ z=bz_1,\ L=b/R,\ \varepsilon_T=\varepsilon_0\varepsilon_{T1},\ s=r_1/\varepsilon_{T1},\ p^{(0)}=p_0 p_1^{(0)},\ p^{(1)}=p_0 p_1^{(1)},$$
$$p_0=\frac{\omega R^2 \eta_0}{\varepsilon_0^2},\ R_F \equiv \frac{N_0 B_0 \varepsilon_0^2}{\omega R^2 \eta_0}, \quad (14)$$

where:
$B_0$ – characteristics value of magnetic induction,
$L_1$ – dimensionless bearing length,
$N_0$ – characteristics value of magnetization effect,
$p_1$ – dimensionless pressure,
$p_0$ – characteristic value of hydrodynamic pressure,
R – radius of cylindrical journal,
$R_F$ – dimensionless magnetic pressure number,
$T_1$ – temperature changes on the journal and sleeve ($f_{c1}$, $f_{p1}$),
$\varepsilon_{T1}$ – dimensionless gap height,
$\eta_0$ – characteristic value of oil dynamic viscosity in average temperature of $T_0$,
$\omega$ – angular velocity of the journal.

Integrating twice the corresponding momentum equation (6)-(9) converted to a dimensionless form for zero phase of the approximation (Newtonian properties) and by imposing boundary conditions: $v_{11}^{(0)}=1,\ v_{21}^{(0)}=0,\ v_{31}^{(0)}=0,$ for $r_1=0$ and $v_{11}^{(0)}=0,\ v_{21}^{(0)}=0, v_{31}^{(0)}=0,$ for $r_1=\varepsilon_{T1}$ we obtain:

$$v_{11}^{(0)} = \left\{ (1-s) + \frac{\varepsilon_{T1}^2}{2\eta_1(\phi, z_1)} \left[ \frac{\partial p_1^{(0)}}{\partial \phi} - R_F M_{11} \right] s \cdot (s-1) \right\}, \quad (15)$$

$$v_{31}^{(0)} = \left\{ \frac{\varepsilon_{T1}^2}{2\eta_1(\phi, z_1)} \left[ \frac{1}{L}\frac{\partial p_1^{(0)}}{\partial z_1} - R_F M_{31} \right] s \cdot (s-1) \right\}, \quad (16)$$

$$\frac{\partial}{\partial \phi}\left( \frac{\varepsilon_{T1}^3}{\eta_1(\phi, z_1)} \left( \frac{\partial p_1^{(0)}}{\partial \phi} - R_F M_{11} \right) \right) + \frac{1}{L^2}\frac{\partial}{\partial z_1}\left( \frac{\varepsilon_{T1}^3}{\eta_1(\phi, z_1)} \left( \frac{\partial p_1^{(0)}}{\partial z_1} - R_F M_{31} \right) \right) = 6\frac{\partial \varepsilon_{T1}}{\partial \phi}, \quad (17)$$

for: $0<\phi<2\pi$, $0<r_1<\varepsilon_{T1}$, $-1<z_1<1$, $0<s<1$.

Integrating twice the corresponding momentum equation (6)-(9) converted to a dimensionless form for the first phase of the approximation (taking into account the properties of non-Newtonian) and by imposing boundary conditions: $v_{11}^{(k)}=0,\ v_{21}^{(k)}=0,\ v_{31}^{(k)}=0,$ for $r_1=0$ and $v_{11}^{(k)}=0,\ v_{21}^{(k)}=0,\ v_{31}^{(k)}=0$ for $r_1=\varepsilon_{T1}$, we obtain corrections of the components of velocity vector and pressure in the following form:

$$v_{11}^{(1)} \equiv \left[ \frac{1}{2\eta_1(\phi, z_1)} s(s-1)\varepsilon_{T1}^2 \frac{\partial p_1^{(1)}}{\partial \phi} - J_{1r} + sJ_{1\varepsilon} \right], \tag{18}$$

$$v_{13}^{(1)} \equiv \left[ \frac{1}{2\eta_1(\phi, z_1)} s(s-1)\varepsilon_{T1}^2 \frac{1}{L} \frac{\partial p_1^{(1)}}{\partial z_1} - J_{3r} + sJ_{3\varepsilon} \right], \tag{19}$$

$$\frac{\partial}{\partial \phi}\left[ \frac{\varepsilon_{T1}^3}{\eta_1(\phi, z_1)} \frac{\partial p_1^{(1)}}{\partial \phi} \right] + \frac{1}{L^2}\frac{\partial}{\partial z_1}\left[ \frac{\varepsilon_{T1}^3}{\eta_1(\phi, z_1)} \frac{\partial p_1^{(1)}}{\partial z_1} \right] = 12\frac{\partial}{\partial \phi}\left\{ \int_0^{\varepsilon_{T1}} (sJ_{1\varepsilon} - J_{1r}) dr_1 \right\} + 12\frac{1}{L}\frac{\partial}{\partial z_1}\left\{ \int_0^{\varepsilon_{T1}} (sJ_{3\varepsilon} - J_{3r}) dr_1 \right\}, \tag{20}$$

where for $i$=1,3 we have:

$$J_{i\varepsilon} \equiv \int_0^{\varepsilon_{T1}} \frac{\partial v_{i1}^{(0)}}{\partial r_1} \eta_{pr1} dr_1, \quad J_{ir} \equiv \int_0^{r_1} \frac{\partial v_{i1}^{(0)}}{\partial r_1} \eta_{pr1} dr_1, \quad \eta_{pr1} = \ln \Pi_{v1}, \quad \Pi_{v1} = \sum_{i=1,3} \left( \frac{\partial v_{i1}^{(0)}}{\partial r_1} \right)^2, \tag{21}$$

Dimensionless load carrying capacities $C_1^{(0)}$ and a dimensionless correction of the load carrying capacities $C_1^{(1)}$ were determined from the following relationships [4], [10]:

$$C_1^{(0)} = \sqrt{\left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(0)} \cos\gamma \sin\phi \, d\phi \right) dz_1 \right)^2 + \left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(0)} \cos\gamma \cos\phi \, d\phi \right) dz_1 \right)^2}, \tag{22}$$

$$C_1^{(1)} = \sqrt{\left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(1)} \cos\gamma \sin\phi \, d\phi \right) dz_1 \right)^2 + \left( \int_{-1}^{+1} \left( \int_0^{\phi_k} p_1^{(1)} \cos\gamma \cos\phi \, d\phi \right) dz_1 \right)^2}. \tag{23}$$

Dimensionless friction force $Fr_1^{(0)}$ and a dimensionless correction of the friction force $Fr_1^{(1)}$ were determined from the following relationships [4], [10]:

$$Fr_1^{(0)} = \int_{-1}^{+1} \left[ \int_0^{\phi} \left( \eta_1 \frac{\partial v_{11}^{(0)}}{\partial r_1} \right)_{r_1 = \varepsilon_{T1}} d\phi \right] dz_1 = \int_{-1}^{+1} \left[ \int_0^{2\pi} \left( \eta_1 \frac{\partial v_{11s}^{(0)}}{\partial r_1} \right)_{r_1 = \varepsilon_{T1}} d\phi \right] dz_1 + \int_{-1}^{+1} \left[ \int_0^{\phi_k} \left( \eta_1 \frac{\partial v_{11p}^{(0)}}{\partial r_1} \right)_{r_1 = \varepsilon_{1T}} d\phi \right] dz_1, \tag{24}$$

$$Fr_1^{(1)} = \int_{-1}^{+1} \left[ \int_0^{\phi} \left( \eta_1 \frac{\partial v_{11}^{(1)}}{\partial r_1} \right)_{r_1 = \varepsilon_{T1}} d\phi \right] dz_1 = \int_{-1}^{+1} \left[ \int_0^{2\pi} \left( \eta_1 \frac{\partial v_{11s}^{(1)}}{\partial r_1} \right)_{r_1 = \varepsilon_{T1}} d\phi \right] dz_1 + \int_{-1}^{+1} \left[ \int_0^{\phi_k} \left( \eta_1 \frac{\partial v_{11p}^{(1)}}{\partial r_1} \right)_{r_1 = \varepsilon_{T1}} d\phi \right] dz_1, \tag{25}$$

The total dimensional load carrying capacities, friction force and total friction coefficient can be determined from the following relationships [4], [10]:

$$C_\Sigma = C_{1\Sigma} \cdot bR\eta_o\omega/\psi^2; \; C^{(0)} = C_1^{(0)} \cdot bR\eta_o\omega/\psi^2; \; C^{(1)} = C_1^{(1)} \cdot bR\eta_o\omega/\psi^2, \tag{26}$$

$$Fr_\Sigma = Fr_{1\Sigma} \cdot bR\eta_o\omega/\psi; \; Fr^{(0)} = Fr_1^{(0)} \cdot bR\eta_o\omega/\psi; \; Fr^{(1)} = Fr_1^{(1)} \cdot bR\eta_o\omega/\psi, \tag{27}$$

$$\left( \frac{\mu}{\psi} \right)_\Sigma = \frac{Fr_\Sigma}{\psi C_\Sigma}; \; \left( \frac{\mu}{\psi} \right)_1^{(0)} = \frac{Fr_1^{(0)}}{C_1^{(0)}}; \; \left( \frac{\mu}{\psi} \right)_1^{(1)} = \frac{Fr_1^{(1)} \cdot C_1^{(0)} - Fr_1^{(0)} \cdot C_1^{(1)}}{C_1^{(0)2}}. \tag{28}$$

## 3. Numerical calculations

The numerical calculation of hydrodynamic pressure distribution, the load carrying capacities, friction force and friction coefficient as well as corrections of: hydrodynamic pressure, the load carrying capacities and the friction force were carried out using both Mathcad 15 software and own calculation procedures. Reynolds' differential equation (13) and (17) were solved by finite difference method using Gumbel's boundary conditions. Based on the determined hydrodynamic pressure distributions there were calculated both dimensionless load carrying capacities (22) and the dimensionless correction of the load carrying capacities (23). The values of these parameters are shown on Fig. 1 for relative eccentricity $\lambda$=0.1÷0.9 and the dimensionless length of the bearing $L_1$=2; 1.5; 1; 1/2; 1/4. Dimensionless friction force and dimensionless correction of friction force were determined based on formulas (24), (25) and they are shown on Fig. 2. Derivatives of the components of velocity vector with a low subscript "s" in the equations (24), (25) mean that part of the derivative of the components of velocity vector (15), (16) and derivatives of the corrections of the components of velocity vector (18), (19), which contain units responsible for the peripheral movement of the journal and magnetic field. Derivatives of the components of velocity vector with a low subscript "p" in the equations (24), (25) mean that part of the derivative of the components of velocity vector (15), (16) and derivatives of the corrections of the components of velocity vector (18), (19), which contain units responsible for velocity caused by pressure gradient. The friction coefficient and the corrections of coefficient of friction were determined based on the formula (29) and there are shown in Fig. 3. Changes in the dynamic viscosity by the external magnetic field for the ferrofluid of 2% concentration of $Fe_3O_4$ magnetic particles are described by the equation $\eta = \eta_0\eta_{1;}$ $\eta_1 = 1 + a\cdot B_1 + b\cdot B_1^2 + c\cdot B_1^3 + d\cdot B_1^4$ where $\eta_0$=0.026 Pas; $B_o$=1T, a=2.44834, b=-7.95749, c=11.72352, d=-6.43172. These data were obtained on the basis of the experimental results presented in the papers [1]. The studied ferrofluid was made based on Pennzoil 15W40 mineral oil. For the purpose of numerical calculations, there were also adopted: the dimensionless magnetic number $R_F$=0.5; the coefficient of magnetic susceptibility $\chi$=0.06, dimensionless flow index n=0.949; small parameter g=0.026.

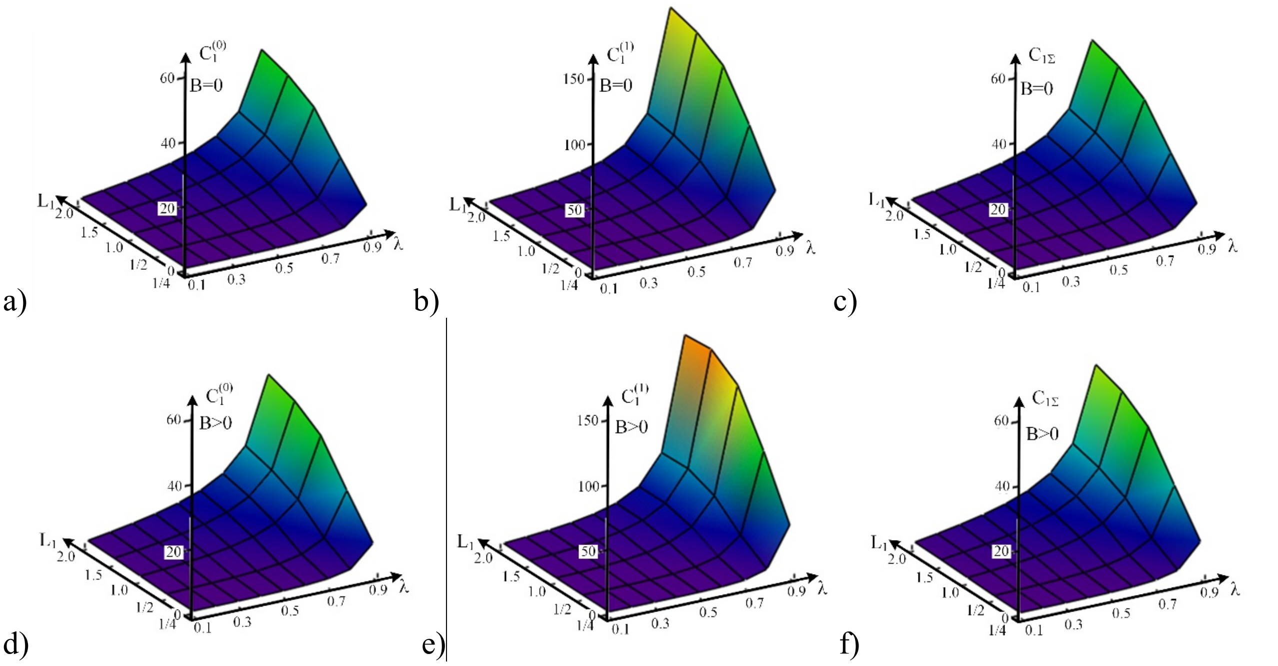


*Fig. 1. Dimensionless load carrying capacity: a), d) for Newtonian oils, b), e) correction caused by the non-Newtonian oil properties, c), f) the final value -* $C_{1\Sigma} = C_1^{(0)} + g\cdot C_1^{(1)}$*; Fig. 3a), b), c) – without magnetic field, Fig. 3d) e), f) – with magnetic field*

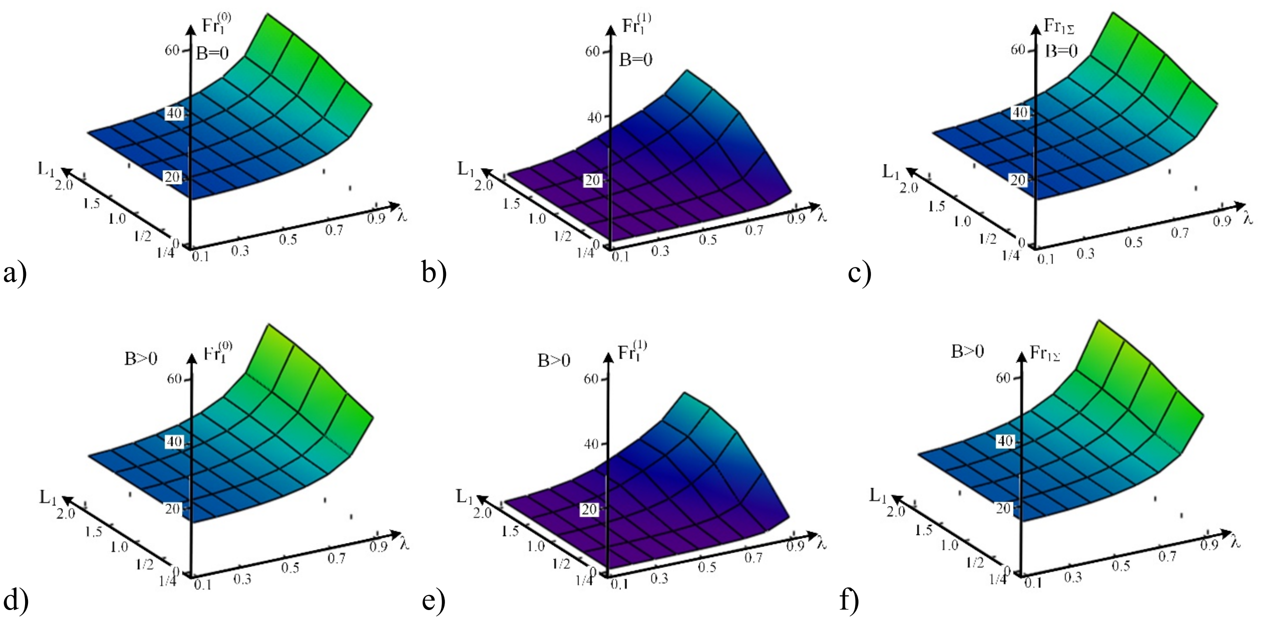


*Fig. 2. Dimensionless friction force: a), d) for Newtonian oils, b), e) correction caused by the non-Newtonian oil properties, c), f) the final value* - $\mathrm{Fr}_{1\Sigma} = \mathrm{Fr}_1^{(0)} + g \cdot \mathrm{Fr}_1^{(1)}$ *; Fig. 3a) ,b), c) – without magnetic field, Fig. 3d) e), f) – with magnetic field*

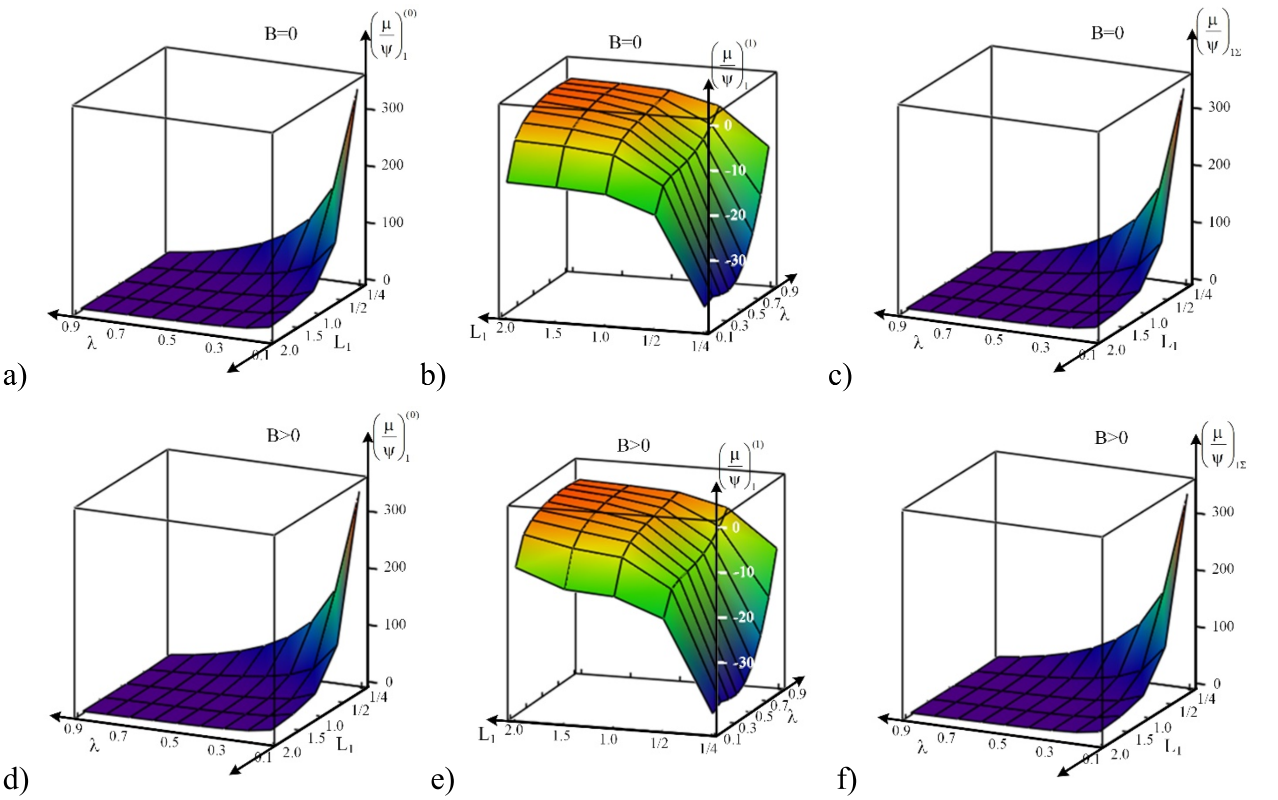


*Fig. 3. Friction coefficient: a), d) for Newtonian oils, b), e) correction caused by the non-Newtonian oil properties, c), f) the final value* - $\left(\frac{\mu}{\psi}\right)_{1\Sigma} = \left(\frac{\mu}{\psi}\right)_1^{(0)} + g \cdot \left(\frac{\mu}{\psi}\right)_1^{(1)}$ *;Fig .3 a) ,b) ,c) - without magnetic field, Fig. 3d) ,e), f) – with magnetic field*

## 4. Observations and conclusions

1. Both load carrying capacities and the friction force change in the range of 13.7% to 17.3% (depending on the relative eccentricities ratio and the dimensionless length of the bearing) for changes of an external magnetic field in the range of 50 mT to 100 mT, which have been simulated. The coefficient of friction does not change significantly.
2. The corrections of load carrying capacities and the friction force change in the range of +22% do -20% (depending on the relative eccentricities ratio and the dimensionless length of the bearing) for changes of an external magnetic field in the range of 50 mT to 100 mT, which have been simulated. The correction of the coefficient of friction varies in the range of + 22% to -2.5%.
3. The total value of the load carrying capacities and the friction forces change in the range of 12.8% to 17.4% for the changing of the external magnetic field in the range of 100 mT to 50 mT. The total coefficient of friction varies in the range of +1% to -1%.
4. Influence of non-Newtonian properties of studied ferrofluid (colloidal mixture of Pennzoil 15W40 oil and 2% $Fe_3O_4$ ) on: the load carrying capacities contains in the range of 0.3% to 10.5%, on the friction force in the range of 0% to 1.5% and on the coefficient of friction in the range of 0.3% to 10.2% (depending on the relative eccentricities ratio and the dimensionless length of the bearing).
5. On the base of numerical calculations follows that for the oils with essential non-Newtonian properties like ferrofluids, it is necessary to use property constitutive models.

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