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ANALYSIS OF THE MILITARY VEHICLES OPERATING PROCESS BY MEANS OF MARKOV PROCESSES

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Abstract

The article presents the method of analysing the operating process of Star military vehicles in their readiness aspect. The subject of research was the Military Economic Unit, recently created in the Polish Army. Markov processes were applied. A simple 5-state model was built, and the research was made in a discrete, as well as continuous time. The limit probabilities, both for the chain and the time, were estimated. Thanks to Chapman-Kołomogorov-Smoluchowski equations, the long-term projections could be estimated. To ensure of operating processes, correctly object's permissible transitions from the previous state to the next one were chosen. It was defined based on technical documentation and owned operational knowledge on the considered operating process. The mathematical description of a set of permissible transitions includes a matrix of permissible $S_i \rightarrow S_j$ transitions from the previous state S_i (lines) to the next one. The five-state system has possible and forbidden transitions are presented. Graph of permissible transitions for the five-state operation model usage, standby, maintenance, repair, standstill in repair; correlogram for the average duration of the state; evolution of the probability of the Star cars' staying in the state of usage, standby, maintenance, repair, standstill in repair are presented in the article.

Keywords: *operation, Markov processes, military vehicles, Military Economic Units*

1. Introduction

Military Economic Units are part of the executive logistics of the Polish Armed Forces. Their task is to provide financial and economic services for the military units stationing in the area of their responsibility. The reason for their formation was the necessity to create a uniform security system, which is common for various types of the armed forces, and which provides the implementation of logistics and financial processes by specialised military units, as well as organisation of economic allocations with coordination at the central level, coherent in terms of budgeting and logistics tasks [4]. The tasks implemented by Military Economic Units largely include transport processes, which results from the necessity to carry out supply tasks. The unit, which was selected for studies, released registration and accounting documentation of forty Star vehicles, which covers two calendar years: 2012 and 2013. On the basis of the obtained information, an analysis of this process was made, and the Markov models were formulated in discrete and continuous time.

2. Formulation and analysis of registration states of vehicles

The operation system research requires the determination of all relevant factors that define it. Secondary factors – which unnecessarily complicate the model without a significant improvement in its quality, should be ignored, secondary states should be omitted, and as a result, similar ones should be grouped. A detailed definition of states allows thoroughly studying the system; however, it also imposes high demands on empirical data used to construct the model. However, in case of the overall study of the operation system, the minimum number of its distinguishable states, which allows calculating the system's basic operating indicators, is sufficient. In case of the studied operation system of vehicles, there are the states of usage – S_1, being at a standby – S_2, maintenance – S_3, repairs – S_4 and standstill in repair – S_5.

In order to ensure high quality of a model of operating processes, it is necessary to correctly choose the object's permissible transitions from the previous state to the next one [2]. It was defined on the basis of technical documentation and owned operational knowledge on the considered operating process. The mathematical description of a set of permissible transitions includes a matrix of permissible $S_i \rightarrow S_j$ transitions from the previous state S_i (lines) to the next one S_j (column) during the process. The five-state system has possible and forbidden transitions as presented in Tab. 1 and in Fig. 1.

Tab. 1. Matrix of permissible transitions for the five-state operation model [source: own development]

$\downarrow S_i \rightarrow S_j$	S_1	S_2	S_3	S_4	S_5
S_1	0	1	1	1	1
S_2	1	0	1	1	1
S_3	1	1	0	1	1
S_4	1	1	1	0	1
S_5	0	0	1	1	0

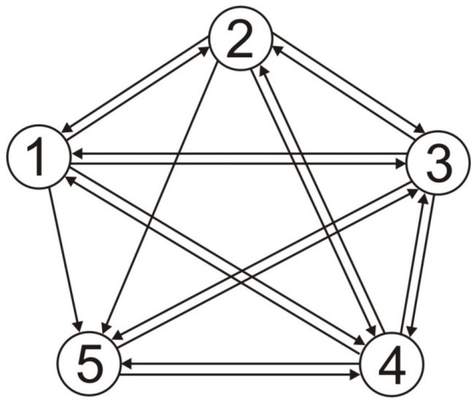

Fig. 1. Graph of permissible transitions for the five-state operation model: S_1 – usage, S_2 – standby, S_3 – maintenance, S_4 – repair, S_5 – standstill in repair [source: own development]

The condition for using the Markov processes is fulfilment of the assumption about the lack of history of a stochastic process. The seasonality of the studied process excludes the strict fulfilment of this assumption, but it does not exclude the use of the Markov models for sufficiently long periods of time, when deterministic seasonal fluctuations are averaged. The minimum averaging time is normally determined on the basis of the AFC and PACF autocorrelation functions [3].

The example graph of ACF and PACF functions for the S_2 state was presented below.

The Gretl charts, as shown in Fig. 2, reflect the weekly seasonality (ACF) and ignorable treatment of the process history older than 30 days (ACF < 0.3). Therefore, the Markov models cannot refer to the periods shorter than averaging/smoothing the estimates of parameters and projections.

In case of shorter periods of averaging the parameters and projections, p_{ij} and p_i standardisation conditions are not met. The probabilities and characteristic times, which are calculated on the basis of the Markov models of the studied operating processes, should be averaged for the periods of at least one month.

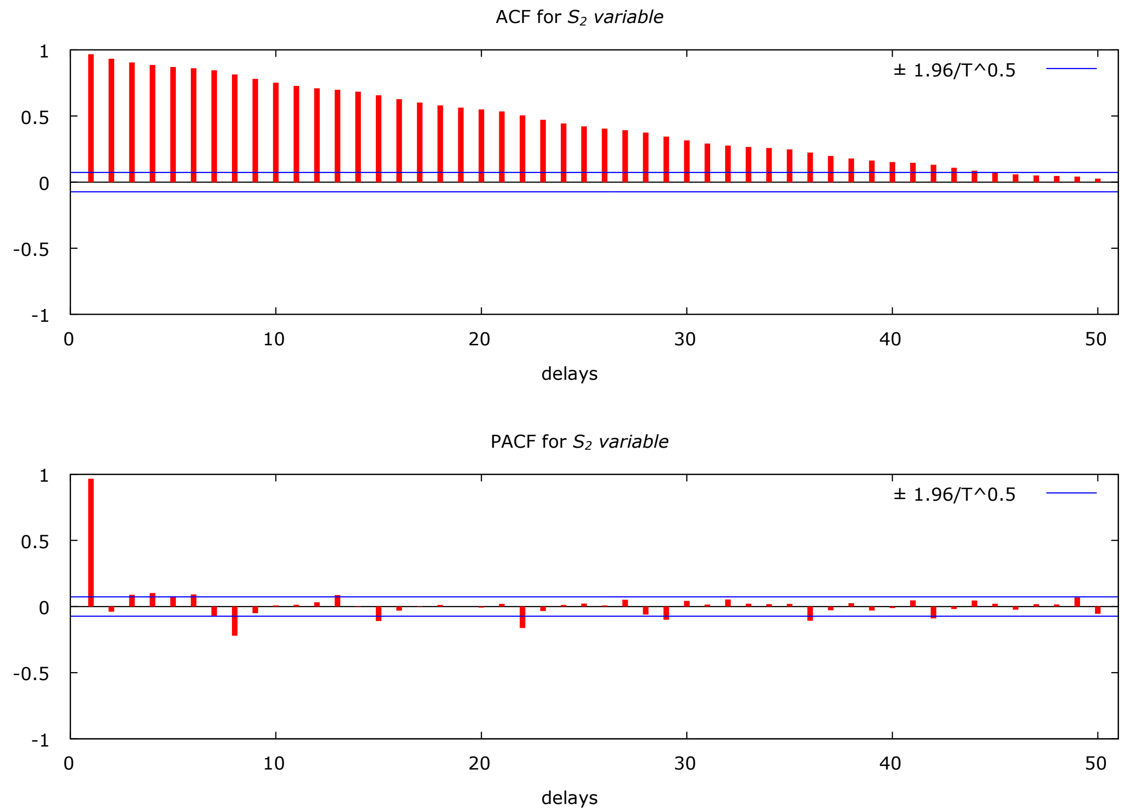

Fig. 2. Correlogram for the average duration of the S_2 state. Delays up to 50 [source: own development]

3. Markov model in discrete time

The first stage of the construction of the five-state Markov model is the estimation of transition probabilities, as the values of $p_{ij}^{\wedge}$ estimators of the p_{ij} elements, and the P matrix of transition probabilities. The values of these estimators of the sample are frequencies of the w_{ij} transitions from the S_i state to the S_j state:

$$p_{ij}^{\wedge} = w_{ij} = n_{ij} / {}_{tr}n_i \,, \tag{1}$$

where:

n_{ij} – number of transitions from the S_i state to the S_j state,
${}_{tr}n_i$ – number of all transitions (output) from the S_i state,
w_{ij} – frequency of the w_{ij} transitions from the S_i state to the S_j state,
n_i – number of observations of the S_i states in the sample.

Estimated p_{ij} elements of the P matrix were given in Tab. 2.

Tab. 2. Values of p_{ij} elements of the P matrix of the five-state operation model [source: own development]

p_{ij}	S_1	S_2	S_3	S_4	S_5
S_1	0	0.298367	0.254286	0.155918	0.29143
S_2	0.264756	0	0.266467	0.163388	0.30539
S_3	0.253066	0.298851	0	0.156173	0.29191
S_4	0.230368	0.272055	0.231857	0	0.26572
S_5	0	0	0.6199	0.3801	0

In order to assess simulation usefulness of the $p_{ij}^{\wedge}$ estimators, maximum errors of the Δ estimation were calculated for central confidence intervals, and the δ relative estimation errors. The results of the calculations were shown in the below tables (Tab. 3 and 4), for the confidence interval of 95%.

Tab. 3. Relative percentage errors of estimation of the P matrix elements of the five-state operation model [source: own development]

δ_{ij} %	S_1	S_2	S_3	S_4	S_5
S_1	0	6.59881	6.802945	7.23774	6.631359
S_2	6.21603	0	6.208793	6.630703	6.041825
S_3	6.786613	6.575305	0	7.213378	6.607813
S_4	8.797633	8.556083	8.78912	0	8.593184
S_5	0	0	4.522261	5.775212	0

Tab. 4. Absolute percentage errors of estimation of the P matrix elements of the operation model [source: own development]

Δ_{ij} %	S_1	S_2	S_3	S_4	S_5
S_1	0	22.1164	26.75316	46.42006	22.75466
S_2	23.47832	0	23.30041	40.58268	19.78402
S_3	26.81754	22.00164	0	46.18828	22.63685
S_4	38.1894	31.45033	37.90749	0	32.33877
S_5	0	0	7.29514	15.19395	0

The estimation errors of the P matrix elements of the five-state model are not so large. Most of the errors have a limit value of 20%, and prognostic quality of this model is fairly satisfactory.

Then, the ergodic probabilities were calculated. In case of the Markov chain, the linear matrix equations are solved:

$$(P^T * \prod = \prod) \wedge (\Sigma_j \, p_j = 1) \Leftrightarrow (\prod^T *(P - I) = 0) \wedge (\Sigma_j \, p_j = 1)\,, \qquad (2)$$

where $\prod = [p_j]_{nsx1}$, I – unit matrix.

The linear matrix equations for p_j with Λ and P matrices are homogeneous. According to the theory of linear equation systems, they always have zero singular solutions, and can have ∞ non-zero solutions dependent on one or more parameters [4]. Therefore, they are solvable only with the standardization condition of limit probabilities. After the conversion to the canonical form, a system of $ns + 1$ linear equations with the reduced by 1 ns number of the p_j unknowns, which should be examined according to the Kronecker-Cappella theorem, and if necessary, it is important to assume the number of selected p_j, appropriate to the research results, as parameters.

The analytical calculation of ergodic probabilities for a studied model is hindered by the P stochastic matrix determinant around zero (det $P = 0.00406431$ is similar to the numerical procedure error). This is the effect of similar values of some p_{ij}, which specify deterministic sections of the phase trajectory. Therefore, the P-I matrix is singular for numerical procedures, and it is not possible to calculate the p_j limits of the equation (2).

The rank of the P-I 5-degree matrix is 4, and the appropriate system of 6 equations with five unknowns can have exactly one solution. It was verified that the system satisfies the asymptotic solution found by the QSB and Mathematica programmes (Tab. 5), but without additional studies of this system, it cannot be stated that it is the only one.

Tab. 5. p_j ergodic probabilities and w_j observation frequencies for S_1-S_5 states of the chain [source: own development]

p_j, w_j	S_1	S_2	S_3	S_4	S_5
p_j Star	0.155082	0.17477	0.266537	0.179533	0.224078
w_j Star	0.198101	0.239358	0.203667	0.125082	0.233792

The highest limit probability has the S_3 maintenance state. It results from the necessity of implementation of daily services before departure and on return, and frequent refilling of operating

Tab. 6. Deviations [%] of w_j frequencies from p_j ergodic probabilities of S_1-S_5 states for the chain [source: own development]

Dev.[%]	S_1	S_2	S_3	S_4	S_5
Star	27.74	36.96	–23.59	–30.33	4.33

fluids. The difference in frequency and ergodic probability is a measure of deviation of the objects' sample during data collection from the asymptotic equilibrium of the studied process. The percentage deviation of the w_j frequencies from the p_j ergodic probabilities for the S_1-S_5 states is presented in Tab. 6.

The highest deviation from the hypothetical equilibrium was noticed in the second state, which constitutes being at a standby. The reason may include frequent standstills of vehicles in the garage, while waiting for the task implementation, and low intensity of the operating process. The number of observations of this state is higher by approx. 20% more time, and in the entire considered period, the vehicles were waiting in the readiness state in relation to the implementation of tasks. The deviations of operating frequencies from limit probabilities in a set of states are satisfactorily low, which reflects the system proper operation.

4. Markov model in continuous time

In order to examine the Markov model in continuous time, the values of the λ_{ij} elements of the Λ matrix of transition intensity were estimated (Tab. 7). The intensity of transitions $\lambda_{ij} \geq 0$ for $i \neq j$ is defined as a right-hand derivative of transition probabilities with respect to time.

$$\lambda_{ij}(t_0) = \left. \frac{\mathrm{d} p_{ij}}{\mathrm{d} t} \right|_{t=t_0+} . \tag{3}$$

The intensities $\lambda_{ii} \leq 0$ for $i = j$ are defined as a complement of the sum of the transition intensities from the S_i state for $i \neq j$ to 0:

$$\lambda_{ii} + \Sigma_j \lambda_{ij} = 0 , \tag{4}$$

hence:

$$\lambda_{ii} = -\Sigma_j \lambda_{ij} . \tag{5}$$

Modules $|\lambda_{ii}| = -\lambda_{ii}$ are called the intensities of transitions from the S_i state. They are not the intensities of return from the S_i state to the S_i state – as suggested in the notation. In case of the Markov homogeneous processes, the intensity of transitions is constant and equal to the inverse of the ${}_{av}t_{ij}$ average times of the object's staying in the S_i state before the S_j state:

$$\lambda_{ij}\hat{} = 1 / {}_{av}t_{ij} , \tag{6}$$

$${}_{av}t_{ij} = (\Sigma_j t_{ij}) / {}_{tr}n_i , \tag{7}$$

where $t_{ij} = (t_{k+1} - t_k)$ only for ${}_lS_k = S_j$ – time of the object's staying in the S_i state before the S_j state, which is equal to the value of the discrete and continuous variable for observation of the k number. ${}_{av}t_{ij} = (\Sigma_j t_{ij}) / {}_{tr}n_i$ – average time of staying in the S_i state before the S_j state.

Tab. 7. Values of the λij elements of the Λ matrix of the five-state operation model [source: own development]

λ_{ij}	S_1	S_2	S_3	S_4	S_5
S_1	–85.5929	16.74754	19.6508	32.0483	17.14629
S_2	0.238864	–1.07034	0.237331	0.38706	0.207083
S_3	13.95653	11.81818	–60.4897	22.61543	12.09957
S_4	12.32043	10.43276	12.24133	–45.6757	10.68116
S_5	0	0	0.226997	0.370207	–0.5972

The intensities of transitions are expressed in the number of transitions per hour for a single object. They should be interpreted for an hour of the initial state duration, not an hour of the process.

4.1. Determination of ergodic probabilities of the Markov process

The first stage in determining the limit probabilities for the Markov process is the formulation on the basis of the matrix of transition intensities of a system of linear equations. In case of continuous time, the equations in relation to the p_j limit probabilities are solved:

$$(\prod^T * \Lambda = 0) \wedge (\Sigma_j\, p_j = 1\ ). \tag{8}$$

$\prod^T = [p_j]^T = [p_1;; p_{ns}]$ is a transpose (row) vector of the p_j limit probabilities of the S_j states of the number of $j \in \{1;; ns\}$.

For the studied process, it is possible to obtain the following matrix equation:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} & \lambda_{34} & \lambda_{35} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -\lambda_{44} & \lambda_{45} \\ 0 & 0 & \lambda_{53} & \lambda_{54} & -\lambda_{55} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

Therefore, determination of the p_j limit probabilities in continuous physical time requires the solution of the following system of equations:

$$\begin{cases} -\lambda_{11}\, p_1 + \lambda_{12}\, p_2 + \lambda_{13}\, p_3 + \lambda_{14}\, p_4 + \lambda_{15}\, p_5 = 0, \\ \lambda_{21}\, p_1 - \lambda_{22}\, p_2 + \lambda_{23}\, p_3 + \lambda_{24}\, p_4 + \lambda_{25}\, p_5 = 0\,, \\ \lambda_{31}\, p_1 + \lambda_{32}\, p_2 - \lambda_{33}\, p_3 + \lambda_{34}\, p_4 + \lambda_{35}\, p_5 = 0\,, \\ \lambda_{41}\, p_1 + \lambda_{42}\, p_2 + \lambda_{43}\, p_3 - \lambda_{44}\, p_4 + \lambda_{45}\, p_5 = 0, \\ \lambda_{53}\, p_3 + \lambda_{54}\, p_4 - \lambda_{55}\, p_5 = 0\ . \end{cases} \tag{10}$$

It is a homogeneous system, which has an infinite number of solutions, among which there may be the solutions to meet the standardisation condition:

$$\sum_{j=1}^{13} p_j = 1\,. \tag{11}$$

The solution of the above system (10) with a restriction – standardisation condition – was found with the use of the WinQSB and Mathematica programmes, which differ in terms of numerical methods (Tab. 8).

Tab. 8. p_j limit probabilities of the operating system's staying in the S_1-S_5 states in continuous physical time [source: own development]

p_j	S_1	S_2	S_3	S_4	S_5
p_j	0.004359	0.305616	0.008173	0.015099	0.666754
p_j [%]	0.435889	30.5616	0.817279	1.5099	66.6754

According to this model, the majority of vehicles are in the limit state of the standstill in repair. It is consistent with the observed long times of waiting for spare parts, specialists and equipment, which are the cause of low performance of the maintenance and repair subsystem. About 30.5% of the car population is at a standby, and only 0.436% of cars are used.

Tab. 9. Limit probabilities of the system observation in the S_1-S_5 states for the chain and continuous time, as well as the measured observation frequencies of the S_1-S_5 states in time [source: own development]

Operation system state of the Star cars	p_j [%] of the chain	p_j [%] in time	w_j [%] in time
S_1 – Usage	15.5082	0.4359	0.7351
S_2 – Standby	17.4770	30.5616	68.5704
S_3 – Maintenance	26.6537	0.8173	1.0083
S_4 – Repairs	17.9533	1.5099	0.7650
S_5 – Standstill in repair	22.4078	66.6754	28.9213

Limit probabilities of the 5-state system in the field of time and for the chain are substantially different (Tab. 9). The causes result from different interpretations of the relation of frequency and process intensity (space of changes in the chain's states is not physical time). In case of the chain, the frequently implemented S_3 state has the highest probability, but it also has the average duration and low probability of observation in physical time. However, in the physical time, the S_2 and S_5 states with large average times of their duration, and observation frequencies in time, have the highest probabilities. The long-term projection of the usage indicator (p_1 = 0.436%), which shows the operational failure of a studied system, is very pessimistic.

Tab. 10. Deviations of the observation frequencies of the S1-S5 states from limit probabilities in time [source: own development]

Operation system state	Deviation [%] of w_j from p_j	p_j [%] in time	w_j [%] in time
S_1 – Usage	68.6439	0.4359	0.7351
S_2 – Standby	124.3678	30.5616	68.5704
S_3 – Maintenance	23.3728	0.8173	1.0083
S_4 – Repairs	–49.3344	1.5099	0.7650
S_5 – Standstill in repair	–56.6237	66.6754	28.9213

The deviation study of the observation frequencies of the S_1-S_5 states from limit probabilities in time (Tab. 10) showed that satisfactorily small deviations from limit probabilities are demonstrated by five states: S_1, S_3, S_4 and S_5. In addition, in case of the S_2 state, the deviation of frequencies from the limit probability is not very high.

Therefore, it concerns sufficient prognostic reliability of a studied model.

4.2. Study of the dynamics of the 5-state system based on the system of Chapman-Kołomogorov--Smoluchowski equations

The systems of the Chapman-Kołomogorov-Smoluchowski equations are studied and solved in order to determine the characteristic times of the object's search for the stationary state after a specified set of initial states, e.g. times of determination of ergodic pj with an error of 1%.

Systems of the Chapman-Kołomogorov-Smoluchowski equations have a matrix form:

$$(\prod_t{}' = \mathrm{d}\prod/\mathrm{d}t = \Lambda * \prod) \wedge (\Sigma_j\, p_j = 1)\,. \tag{12}$$

In case of the studied Markov process, they have the following form:

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} & \lambda_{15} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & \lambda_{25} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} & \lambda_{34} & \lambda_{35} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -\lambda_{44} & \lambda_{45} \\ 0 & 0 & \lambda_{53} & \lambda_{54} & -\lambda_{55} \end{bmatrix} = \begin{bmatrix} p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \\ p_5'(t) \end{bmatrix}. \tag{13}$$

The equivalent system of differential equations has the following form:

$$\begin{cases} p_1'(t) = -\lambda_{12}p_2(t) - \lambda_{13}p_3(t) - \lambda_{14}p_4(t) - \lambda_{15}p_5(t) + \lambda_{21}p_2(t) + \lambda_{31}p_3(t) + \lambda_{41}p_4(t), \\ p_2'(t) = -\lambda_{21}p_1(t) - \lambda_{23}p_3(t) - \lambda_{24}p_4(t) - \lambda_{25}p_5(t) + \lambda_{12}p_1(t) + \lambda_{32}p_3(t) + \lambda_{42}p_4(t), \\ p_3'(t) = -\lambda_{31}p_1(t) - \lambda_{32}p_2(t) - \lambda_{34}p_4(t) - \lambda_{35}p_5(t) + \lambda_{13}p_1(t) + \lambda_{23}p_2(t) + \lambda_{43}p_4(t) + \lambda_{53}p_5(t), \\ p_4'(t) = -\lambda_{41}p_1(t) - \lambda_{42}p_2(t) - \lambda_{43}p_3(t) - \lambda_{45}p_5(t) + \lambda_{14}p_1(t) + \lambda_{24}p_2(t) + \lambda_{34}p_3(t) + \lambda_{54}p_5(t), \\ p_5'(t) = -\lambda_{53}p_3(t) - \lambda_{54}p_4(t) + \lambda_{15}p_1(t) + \lambda_{25}p_2(t) + \lambda_{35}p_3(t) + \lambda_{45}p_4(t). \end{cases} \tag{14}$$

The attempt to solve the system of the Chapman-Kołomogorov-Smoluchowski equations with an operational method at restrictions for $0 \le p \le 1$ probabilities with the use of the Mathematica programme failed, because this programme, with the use of with the NSolve function, finds the Laplace transforms, the originals of which have the values outside the range of $\langle 0; 1 \rangle$.

According to the system of differential equations of 1 order, it is known that the existence and form of its solutions depend on eigenvalues of the matrices of intensity Λ [1]. Therefore, the time dependencies of solutions can be assessed on the basis of calculation of time constants and times of determination of the exponential solutions' components, which were provided in Tab. 11.

Tab. 11. Eigenvalues of the Λ intensity matrix and time constants of 2 system with the S_1-S_5 states [source: own development]

No.	Eigenvalue [1/h]	Time constant [h]	Time constant [min]	Time constant [s]	Settling time 99% [s]
1	–96.6416	–0.010348	–0.620851	–37.251039	171.1685237
2	–71.0114	–0.014082	–0.844935	–50.696085	232.9485125
3	–24.8357	–0.040265	–2.415877	–144.952629	666.0573288
4	–0.9372	–1.067043	–64.022604	–3841.356255	17651.03199
5	1.966310E–06	5.0857E+05	3.0514E+07	1.8308E+09	not applicable

The estimated Λ 5-state intensity matrix has five different r_i actual eigenvalues, including four negative values and one positive value. The measurement unit of eigenvalues is an intensity unit of transitions – 1/h for the Λ estimated matrix. According to the theory of a system of linear differential equations of 1 order, the studied system of the Chapman-Kołomogorov-Smoluchowski equations has ∞ actual solutions constituting linear combinations of exponential functions with the exponents equal to the quotient of time and time constants $\tau_i = 1/r_i$. Negative time constants $\tau_i = 1/r_i$ represent declining components, and in case of positive time constants – the increasing ones, of the linear combination with the A_i actual ratios equal to initial values of these components. The declining components decrease, in relation to the module, to 1% of the initial value in time of $t_{99\%} = 4.595*|\tau_i|$. In accordance with data on the system dynamics (Tab. 11), 4 declining components of the system of the Chapman-Kołomogorov-Smoluchowski equations are determined with an error of 1% in time from 171 s to 17651 s = 4.9 h, and the increasing component with the time constant 1.831 E9 h = 1.831 giga hours = 209 thousand years will represent a residual linear trend of one component of solutions during the life of a studied operation system (several decades). According to the settling time values (Tab. 11), it can be found that the observation probabilities of the studied system's states will quickly change in the time ranges from approx. 3 minutes to about 5 hours from the initial values, but the increasing component of solutions can complicate the course of their search for limit values. The correct analytical solution of a system of the Chapman-Kołomogorov-Smoluchowski equations with a restriction of the standardisation condition determined the *Mathematica Markov Continuous* module. It was assumed that in the initial time $t = 0$, the $X(t)$ process is located in the S_1 state. The obtained probabilities of observation of the S_1-S_5 states are complex functions (these are not the solutions by a classical method). In accordance with the settling time values of the *5*-state system is close to the equilibrium after *5* hours from forcing of the S_1 initial state (usage) of the entire population. Dependency graphs of the time probabilities of observation of the S_1-S_5 states were provided in Fig. 3-7.

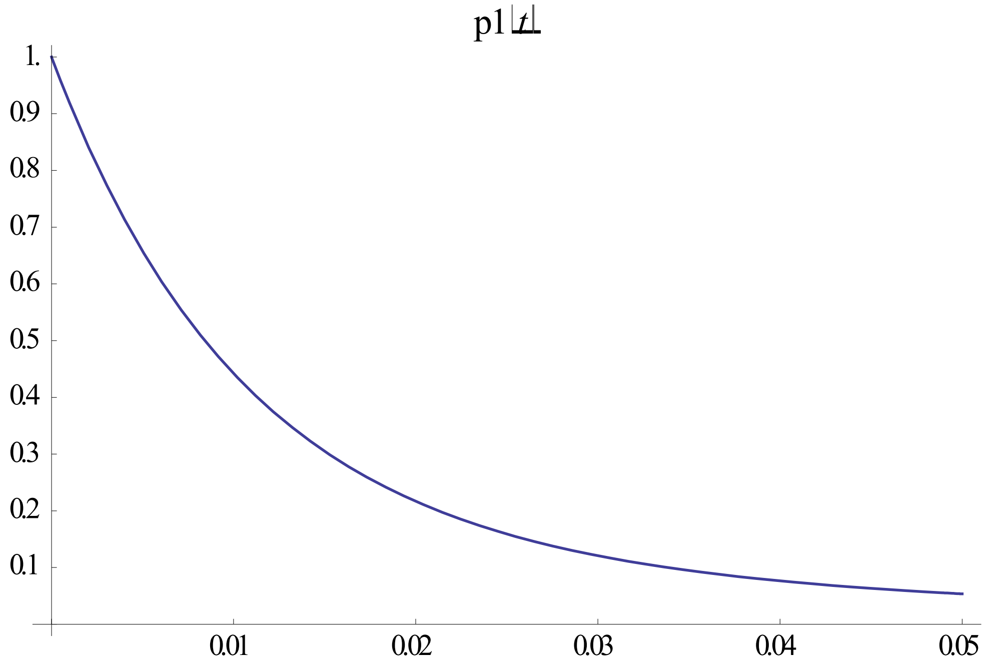

Fig. 3. Evolution of the probability of the Star cars' staying in the S_1 state of usage in the time of 0.05 h = 180 seconds from forcing the S_1 state for t = 0 [source: own development]

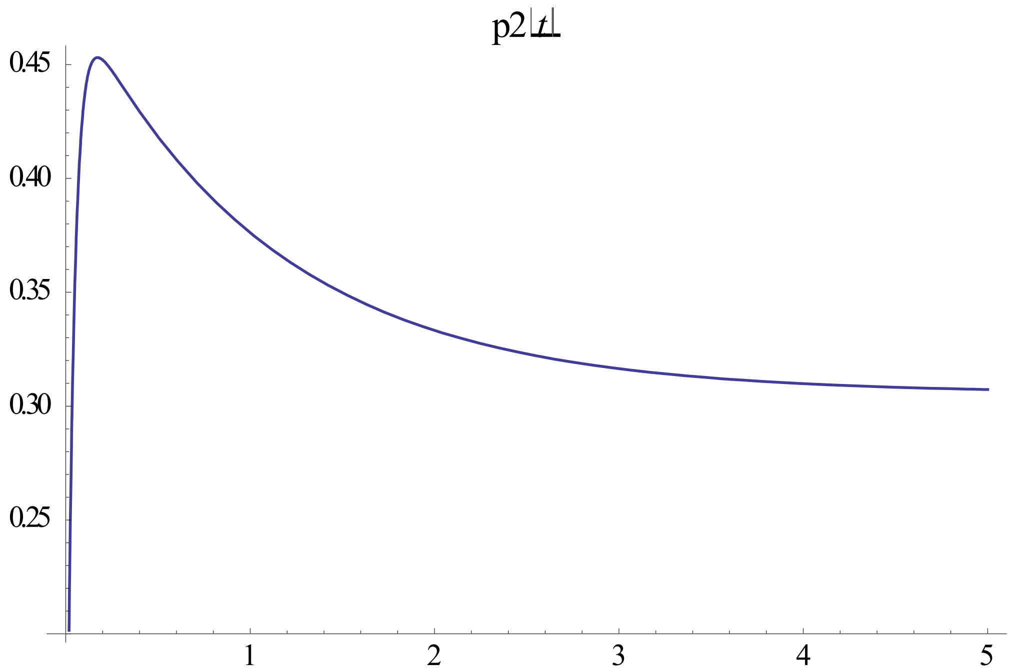

Fig. 4. Evolution of the probability of the Star cars' staying in the S_2 state of being at a standby in the time of 5 hours from forcing the S_1 state for t = 0 [source: own development]

The probabilities of the S_5 states (standstill in repair) and the S_2 state (standby) about the highest limit values are determined at the slowest rate – Fig. 4 and 7. The probability of the S_1 state (usage) within a few minutes – Fig. 3, and probabilities of the S_3 (maintenance) and S_4 (repairs) states are determined between ten and twenty minutes after forcing the S_1 state – Fig. 5 and 6.

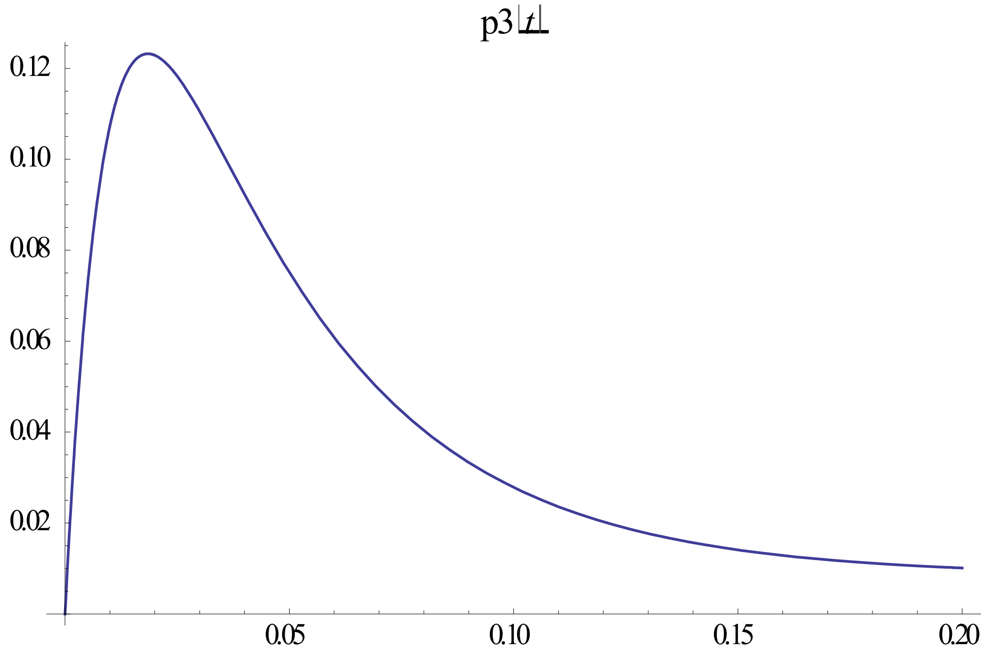

Fig. 5. Evolution of the probability of the Star cars' staying in the S_3 state of maintenance in the time of 0.2 h = 12 minutes from forcing the S_1 state for t = 0 [source: own development]

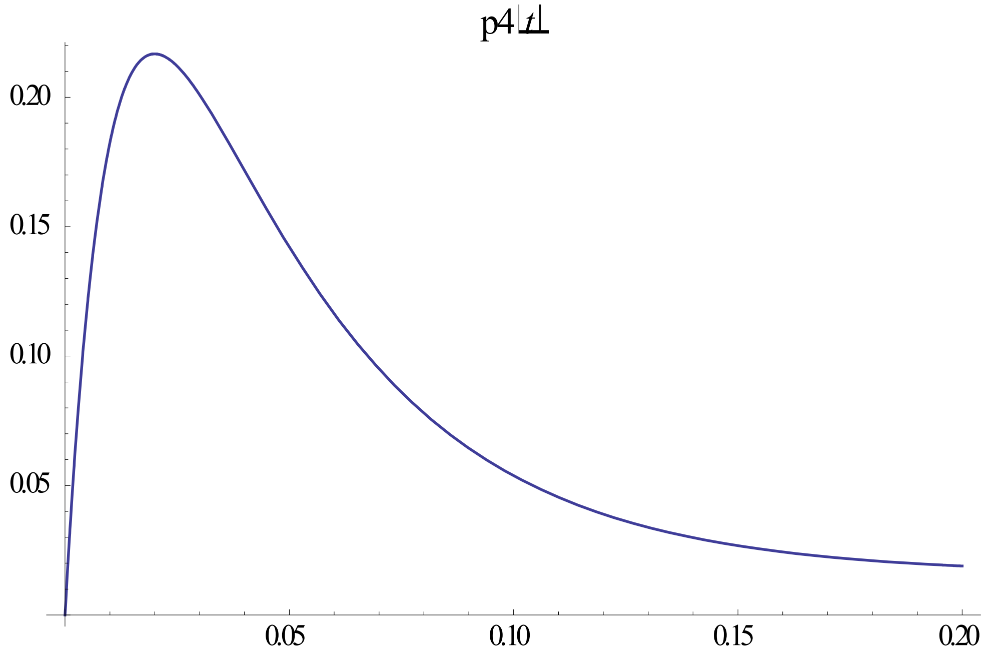

Fig. 6. Evolution of the probability of the Star cars' staying in the S_4 state of repair in the time of 0.2 h = 12 minutes from forcing the s_1 state for t = 0 [source: own development]

5. Conclusion

The presented Markov models do not demonstrate high prognostic reliability and they can be applied only for general research in the qualitative and cognitive analyses. The responsible decisions in terms of the operation system optimisation cannot be taken on their basis, because they were formulated in accordance with the records of poor quality. Due to the small number of vehicles and low intensity of operation, the estimation errors of the parameters of models and projections were not small, but despite of this fact, the obtained results quite well described the functioning of military transport systems, at the same time, showing some of its deficiencies in the form of e.g.

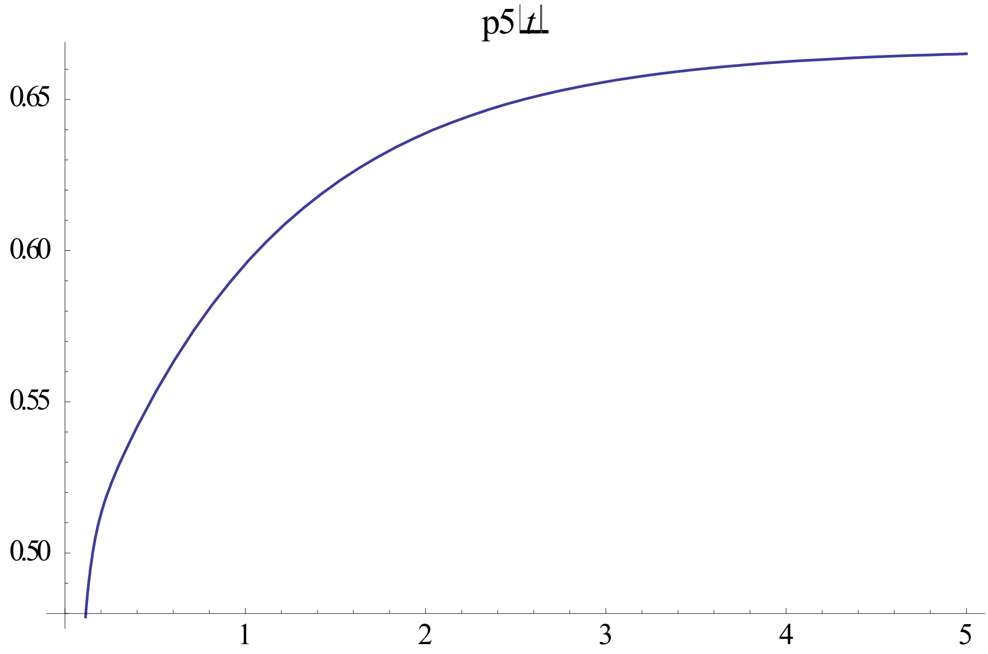

Fig. 7. Evolution of the probability of the Star cars' staying in the S_5 state of the standstill in repair in the time of 5 hours from forcing the S_1 state for $t = 0$ [source: own development]

long periods of the standstill in repair. The estimation of the applicably reliable Markov models would require collecting data on phase trajectories of more vehicles that are best equipped with on-board time recorders of the task implementation, which guarantees the reliability of input data.

However, the aim of this paper was to indicate a test method of the operating process with the use of the Markov models. The proposed method is universal and allows examining any uniform and modernised operation systems (with undifferentiated objects of the same type). They can constitute not only operation systems of vehicles but also of other technical objects, e.g. aircraft, if they are of one type and version in the stable external environment [5, 6]. The models of the Markov processes can be used only in case of operating processes without the effect of their history memory.

References

[1] Białynicki-Birula, A., *Algebra*, PWN, Warsaw 2009.

[2] Brandimarte, P., *Handbook in Monte Carlo Simulation: Applications in Financial Engineering, Risk Management, and Economics*, Wiley 2014.

[3] *StatSoft Electronic Statistics Textbook*, StatSoft company website.

[4] *Wojskowe Oddziały Gospodarcze, Terenowe Organy Systemu Logistycznego SZ RP, Zbiór Podstawowych Zasad Funkcjonowania* (Poradnik), Sztab Generalny WP Zarząd Planowania Logistyki – P4 [*Military Economic Units, Logistics System Local Authorities of the Polish Armed Forces, Collection of Core Functioning Principles* (Guide), Polish General Staff Logistics Planning Department – P4], Warsaw 2011.

[5] Zieja, M., Ważny, M., *A model for service life control of selected device systems*, Polish Maritime Research, Vol. 21, No. 2, 2014.

[6] Zieja, M., Ważny, M., Stępień, S., *Distribution determination of time of exceeding permissible condition as used to determine lifetimes of selected aeronautical devices/systems*, Eksploatacja i Niezawodność – Maintenance and Reliability, Vol. 18, No. 1, 2016.