ISSN: 1231-4005
e-ISSN: 2354-0133
DOI: 10.5604/12314005.1213600

NUMERICAL MODEL FOR CALCULATION OF FLUCTUATIONS IN THE MAIN-BEARING FRAME OF RAILCAR WITH CHANGING STIFFNESS AND PHYSICAL PARAMETERS

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Abstract

This article covers the use of analytical technique of solutions for flexural and longitudinal fluctuations of the bearing framework of a railcar body frame in the form of an elastic core of variable section with a variable weight, flexural and longitudinal rigidity. The calculation is performed for the modernization of the body frame of emergency and repair rail service car, taking into account the variability of section, mass, longitudinal and bending stiffness along the length to prolong the service life of their useful operation.

Problems of increasing the reliability and strength of the frames, load-bearing body structure and components for rail vehicles during their design, operation and modernization are extensively studied. An analytical-numerical method based on the dynamic strength of the bearing body frame of emergency and repair rail service car, assuming a beam-type pattern of its fluctuations with elastic fixing of the ends under harmonic load as it moves along the track with periodic joint roughness.

Keywords: *railcar, mainframe of the body, analytical-numerical method, reliability, strength*

1. Introduction

In modern foreign patent and scientific literature the problems of increasing the reliability and strength of the frames, load-bearing body structure and components for rail vehicles during their design, operation and modernization are extensively studied [1-2]. We offer an analytical-numerical method based on the dynamic strength of the bearing body frame of emergency and repair rail service car, assuming a beam-type pattern of its fluctuations with elastic fixing of the ends under harmonic load as it moves along the track with periodic joint roughness.

This article provides a calculation algorithm for the simulation of stress-strain state of load-bearing body frame of emergency and repair rail service car; it gives the results of numerical studies on stress-strain state of bearing body frame structure of emergency and repair rail service car taking into account the variability of section, mass, longitudinal and bending stiffness along its length; it outlines the validity for the choice of diagnostic parameters for the evaluation of dynamic strength, reliability and predictable service life of bearing body frame structure of emergency and repair rail service cars.

Equivalent bearing body frame of emergency and repair rail service car was simulated by an elastic rod with variable cross section, with variable mass, bending and longitudinal stiffness. The difference between the proposed model and the existing ones [1-2] is an account of the variability of cross section, mass, and the longitudinal and bending stiffness along the length of equivalent beam, which corresponds to the actual conditions of operation. In existing methods of calculation, a beam of uniform strength is considered for the simplification, or an approximate calculation is carried out on the model with lumped parameters, excluding elasticity. These approximate models in dynamics may create an error up to 150-200% of the real strains and stresses. Therefore, in practice, pilot studies are always performed and dynamic correction coefficients are introduced into the calculations of strength and stability.

2. Numerical model of oscillations

For the model proposed here, the parameters of the equivalent load-bearing body frame of the locomotive are taken in the form of variable functions:

- the mass per unit length of the body frame of emergency and repair rail service car (kg/m)

$$m_K(X) = m_O * (a_o + a_1 X + a_2 X^2), \quad (1)$$

- the area of cross section:

$$F(X) = F_O *(d_o + d_1 X + d_2 X^2), \quad (2)$$

the length of the main bearing body frame of emergency and repair rail service car is 12.96 meters and the X coordinate varies in the range $0 \le X \le 12.96$ m:

- the reduced moment of inertia of frame section on the axis $X_C - I_X$ (cm^4):

$$I_X(X) = I_O * (b_O + b_1 X + b_2 X^2), \quad (3)$$

- the reduced bending stiffness:

$$S_I(X) = E * I_X(X), \quad (4)$$

where $I_X(X)$ is calculated by the formula (3).

An assumption is made that the body frame of rail service car is represented in the form of an elastic rod (beam) with constant modulus of material elasticity ***E* = *const*** and the density ***ρ* = *const;*** it has some static initial radius of deflection ***R***. The equations of bending and longitudinal oscillations for this model are taken by analogy with [3-4].

To analyse the stress-strain state of equivalent frame of bearing structure of emergency and repair rail service car, the differential equations of bending and longitudinal oscillations of straight rods of variable section are used (considering torsional oscillations relatively small compared to other components) by analogy with [3-4].

$$m_K(X)\frac{\partial^2 U(X,t)}{\partial t^2} - E\frac{\partial F(X)}{\partial X}\cdot\frac{\partial U(X,t)}{\partial X} - EF(X)\frac{\partial^2 U(X,t)}{\partial X^2} =$$
$$= N_Д(X,t) + E\frac{\partial I_X(X)}{\partial X}\cdot\frac{1}{R^2} + 2EI_X(X)\frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3}, \quad (5)$$

$$m_K(X)\frac{\partial^2 W(X,t)}{\partial t^2} + EI_X(X)\frac{\partial^4 W(X,t)}{\partial X^4} + E\frac{\partial^2 I_X(X)}{\partial X^2}\cdot\frac{\partial^2 W(X,t)}{\partial X^2} =$$
$$= P_Д(X,t) + \frac{E}{R}\left[\frac{\partial^2 I_X(X)}{\partial X^2} + 2I_X(X)\cdot\frac{\partial^3 U(X,t)}{\partial X^3}\right], \quad (6)$$

After substituting the Eqs. (1)-(4) and their derivatives in the system of differential Eqs. (5)-(6) we obtain the nonlinear equations of the form:

$$\left[m_0\cdot\left(a_0 + a_1X + a_2X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial t^2} - E\left[F_0\cdot\left(d_1 + 2d_2X\right)\right]\cdot\frac{\partial U(X,t)}{\partial X}$$
$$- E\left[F_0\cdot\left(d_0 + d_1X + d_2X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial X^2} = N_Д(X,t) +$$
$$+ E\cdot\left[I_0\cdot\left(b_1 + 2b_2X\right)\right]\cdot\frac{1}{R^2} + 2E\cdot\left[I_0\cdot\left(b_0 + b_1X + b_2X^2\right)\right]\cdot\frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3}, \quad (7)$$

$$\left[m_0 \cdot \left(a_0 + a_1 X + a_2 X^2\right)\right] \cdot \frac{\partial^2 W(X,t)}{\partial t^2} + E \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right] \cdot \frac{\partial^4 W(X,t)}{\partial X^4} +$$

$$+ E \cdot 2b_2 \cdot I_0 \frac{\partial^2 W(X,t)}{\partial X^2} = P_Д(X,t) + \frac{E}{R} \cdot \left[2b_2 I_0 + 2 \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right] \cdot \frac{\partial^3 U(X,t)}{\partial X^3}\right]. \quad (8)$$

Dividing term by tern each of the Eqs. of the system (7)-(8) by m_K (X), the entire frame of the body is divided into 120 points (X coordinate varies in the range of $0 \le X \le 12.96$ m), for each of the given K-section the coefficients in the Eqs. of the system (7)-(8) are constant and they could be introduced by iteration method (piecewise linear approximation) into computer solution in the procedure similar to the ones in [3-4].

After the introduction of notations, we obtain the Eqs. of the form:

$$\frac{\partial^2 U(X,t)}{\partial t^2} - A_{K1}(X) \cdot \frac{\partial U(X,t)}{\partial X} - B_{K1}(X) \frac{\partial^2 U(X.t)}{\partial X^2} = C_{K1}(X) \cdot \sin n\omega t +$$
$$+ D_{K1}(X) + E_{K1}(X) \cdot \frac{\partial^3 W(X,t)}{\partial X^3}, \quad (9)$$

$$\frac{\partial^2 W(X,t)}{\partial t^2} + A_{K2}(X) \cdot \frac{\partial^4 W(X,t)}{\partial X^4} + B_{K2}(X) \cdot \frac{\partial^2 W(X,t)}{\partial X^2} =$$
$$= C_{K2}(X) \cdot \cos n\omega t + D_{K2}(X) + E_{K2}(X) \cdot \frac{\partial^3 U(X,t)}{\partial X^3}, \quad (10)$$

where the following notation are introduced:

– for longitudinal oscillations of the body frame of rail service car – Eq. (9):

$$A_{K1}(X) = \frac{EF_0(d_1 + 2d_2 X)}{m_K(X)},\ B_{K1}(X) = \frac{EF_0(d_0 + d_1 X + d_2 X^2)}{m_K(X)},$$
$$C_{K1}(X) = \frac{N_{ÄK}(X)}{m_K(X)},\ N_{ÄK}(X) = N_{Ä \cdot n} \sin \frac{(2n-1) \cdot \pi \cdot X}{2\ell_0}.$$

Here the horizontal external dynamic load is taken in the form:

$$N_{ÄK}(X,t) = N_{Ä \cdot n} \sin \frac{(2n-1) \cdot \pi \cdot X}{2\ell_0} \cdot \sin n\omega t, \quad (11)$$

where n = 1, 2, 3…5 – is a number of harmonics, $N_{Дn}$ – is taken according to experimental data obtained, depending on different modes of loading:

$$D_{K1}(X) = \frac{E \cdot (I_0 \cdot (b_1 + 2b_2 X))}{m_K(X)} \cdot \frac{1}{R^2},\ E_{K1}(X) = \frac{2E \cdot (I_0 \cdot (b_0 + b_1 X + b_2 X^2))}{m_K(X)} \cdot \frac{1}{R},$$

– for bending (transverse) oscillations of the body frame of rail service car – Eq. (10):

$$A_{K2}(X) = \frac{E \cdot (I_0 \cdot (b_0 + b_1 X + b_2 X^2))}{m_K(X)},\ B_{K2}(X) = \frac{2E \cdot (I_0 \cdot b_2)}{m_K(X)},$$
$$C_{K2}(X) = \frac{P_{ÄK}(X)}{m_K(X)},\ P_{ÄK}(X) = P_{Ä \cdot n} \sin \frac{\pi \cdot n \cdot X}{\ell_0}.$$

Here the vertical external dynamic load is taken in the form:

$$P_{ÄK}(X,t) = P_{Ä \cdot n} \sin \frac{n \cdot \pi \cdot X}{\ell_0} \cdot \cos n\omega t, \quad (12)$$

where n = 1, 2, 3…5 – is a number of harmonics, $P_{Дn}$ – is taken according to experimental data obtained, depending on different modes of loading.

$$D_{K2}(X) = \frac{E \cdot (2I_0 \cdot b_2)}{R \cdot m_K(X)} \; ; \; E_{K2}(X) = \frac{2E \cdot \left(I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right)}{m_K(X)} \cdot \frac{1}{R} \cdot$$

The solution of the system (7)-(8) is performed with the linearization by Simpson's method, then Fourier method is applied to the differential equations with constant coefficients with further application of operational Laplace transform in time; numerical studies are carried out by the methods of piecewise linear approximation and boundary elements method, similar to the procedures given in [3÷4] in ***Mathcad 14*** programming environment. Initial conditions are taken as zero ones, and the boundary conditions – in the form of elastic fixing of the ends.

Thus, it is possible to find a general solution of differential Eqs. of bending and longitudinal oscillations of the body frame of emergency and repair rail service car (9) and (10) in the form:

$$W(X,t) = \sum_{k=1}^{\infty} W(X) * \{ \frac{C_{K2}}{W(X)} \cdot \frac{\cos n\omega t - \cos \lambda_{2n} t}{\lambda_{2n}^2 - (n\omega)^2} + W_0 \cdot \cos \lambda_{2n} t +$$

$$+ [\frac{D_{K2}}{W(X)} + V_{\hat{I}\ddot{I}} \;] * \frac{1}{\lambda_{2n}} \cdot \sin \lambda_{2n} t \} , \quad (13)$$

$$U(t) = \frac{C_{K1}}{U(X)} \cdot \frac{n\omega \cdot \sin \lambda_{1n} t - \lambda_{1n} \sin n\omega t}{n\omega \cdot \lambda_{1n} \cdot \left(\lambda_{1n}^2 - (n\omega)^2\right)} + U_0 \cdot \cos \lambda_{1n} t +$$

$$+ [\frac{D_{K1}}{U(X)} + V_{\hat{I}} \;] * \frac{1}{\lambda_{1n}} \cdot \sin \lambda_{1n} t + \frac{\hat{O}(X)}{U(X)} * \{ \frac{C_{K2}}{W(X)} * W_1(t) +$$

$$+ W_0 \cdot \frac{\cos \lambda_{2n} t - \cos \lambda_{1n} t}{\lambda_{1n}^2 - \lambda_{2n}^2} + \frac{D_{K2}}{W(X)} * \frac{\sin \lambda_{2n} t - \sin \lambda_{1n} t}{\lambda_{1n}^2 - \lambda_{2n}^2} +$$

$$+ V_{\hat{I}\ddot{I}} \; \cdot \frac{\sin \lambda_{2n} t - \sin \lambda_{1n} t}{\lambda_{1n}^2 - \lambda_{2n}^2} \} , \quad (14)$$

where:

$$W_1(t) = \frac{\cos \lambda_{1n} t}{\left(\lambda_{1n}^2 - (n\omega)^2\right) \cdot \left(\lambda_{2n}^2 - \lambda_{1n}^2\right)} - \frac{\cos n\omega t}{\left(\lambda_{1n}^2 - (n\omega)^2\right) \cdot \left(\lambda_{2n}^2 - (n\omega)^2\right)} -$$

$$- \frac{\cos \lambda_{2n} t}{\left(\lambda_{2n}^2 - (n\omega)^2\right) \cdot \left(\lambda_{2n}^2 - \lambda_{1n}^2\right)} \cdot \quad (15)$$

Thus, as a result of using the method of iterations and piecewise linear approximation we have managed to obtain an analytical and numerical solution for the analysis of joint bending and longitudinal oscillations of the bearing body frame of emergency and repair rail service car in the form of a model of an elastic rod of variable cross section, mass, bending and longitudinal stiffness as it moves along the track with periodic joint roughness.

3. Conclusion

On the basis of numerical studies and comparative analysis with experimental data, we have stated the following quality patterns:

1. Bending stresses appearing in the centre of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, do not exceed the ultimate strength of the material, and in average range from 15 to 40 MPa depending on loading modes (the rate of motion).

2. Longitudinal stresses appearing in the centre of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, are about 20-25% of the bending stresses (from 3 to 10.4 MPa). They reach their maximum values at breakaway and braking modes.

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