ISSN: 1231-4005
e-ISSN: 2354-0133
DOI: 10.5604/12314005.1213583

USE OF RANDOM WALK IN TWO-DIMENSIONAL LATTICE GRAPHS TO DESCRIBE INFLUENCE OF WIND AND SEA CURRENTS ON OIL SLICK MOVEMENT

Sambor Guze, Jolanta Mazurek

Gdynia Maritime University
Department of Mathematics
Morska Street 81-87, 81-225 Gdynia, Poland
tel.: +48 58 55861231, +48 58 5586387
e-mail: s.guze@wn.am.gdynia.pl, j.mazurek@wn.am.gdynia.pl

Leszek Smolarek

Gdynia Maritime University
Department of Transport and Logistics
Morska Street 81-87, 81-225 Gdynia, Poland
tel.: +48 58 5586838
e-mail: l.smolarek@wn.am.gdynia.pl

Abstract

The concept of oil slick movement being influenced by wind and sea currents is elementary for the decision model of the distribution of large oil spill emergency control means at sea. The analysis of water area conditions such as wind and sea currents is elementary for the concept of oil slick movement. The article presents the model of oil slick movement under the influence of wind and sea currents. In building the model, random walk in two-dimensional lattice graphs has been used. The movement of oil slick is analysed in two ways. In the paper, the movement of the oil slick is analysed in two ways without wind and focus on surface sea currents and with wind and currents. Case one assumes no wind and focuses on surface sea currents only using random walk in a two-dimensional square grid graph. Case two assumes that wind is in place, so oil slick is moving due to surface sea currents and wind currents. The description of movement in case two is based on a two-dimensional lattice graph, which is a combination of a triangular grid graph and a hexagonal grid graph. The article also describes the basic assumptions of oil slick model: the definition of water area, oil slick and algorithm for rescue action to contain the oil spill. Oil slick movement concept is elementary for the decision model of oil spill control at sea. The model allows estimating the distance of oil slick from coastal areas.

Keywords: *random walk, oil spill, currents*

1. Introduction

The concept of oil slick movement being influenced by wind and sea currents is elementary for the decision model of the distribution of large oil spill emergency control means at sea. The concept allows for the action-time and resources analysis and assessment of the distribution of large oil spill emergency control means.

Firstly, the article describes the basic concept of oil spill control model. The papers [4] and [6] show random temporal model as an analysis of conditions influencing the time required to prepare and perform oil spill control and rescue.

The analysis of water area conditions such as wind and sea currents is elementary for the concept of oil slick movement. The use of random walk in two-dimensional lattice graphs accounts for wind and sea current influence and thus allows for real-time estimate of the distance of the oil slick to coastal areas as well as to other areas endangered by the oil slick. The parameters given by such estimation will allow creating a database of key decision-making criteria. The database will be used to form a decision model of oil spill control at sea. Such a model will aid oil spill control strategic decision management.

2. Random walks in graphs

The concept of random walks is used to many mathematical and physics models. Generally, the random walk is the process by which randomly moving objects wander away from where they started.

Let us consider the undirected graph $G(V, E)$, where $V = \{v_1, v_2, ...\}$ is the set of the n graph vertices and $E = \{e_1, e_2, ...\}$ is the set of m graph edges. The pair of the vertices v_i, $v_i \in V$ is called adjacent vertices if $(v_i, v_j) \in E$, i, j, = 1, 2, ... An edge and a vertex on common edge are called incident. The number of edges incident to vertex $v_i \in V$ is called the vertex degree and denoted by $\deg(v_i)$. The set of the all-actual neighbours of vertex $v_i \in V$ without itself is called the open neighbourhood and denoted by $N(v_i)$.

Then according to [4] the random walk of length t in given graph G and some starting vertex s, is defined as a randomized process in which, starting from the vertex s, we repeat t times a step that consists of choosing at random one of the neighbours $v' \in V$ of the vertex $v_i \in V$ we are at and moving to it. The well-known fact is that the random walk on undirected graph can be the view of the time-reversible Markov chain [1-3].

Let us consider a random walk on graph G starts at a vertex v_i. When at the t-th step the walk is in at a vertex v_t, the next move is the neighbour of this vertex with probability $1/\deg(v_t)$. Then the sequence of random vertices is a Markov chain. The starting vertex can be fixed or it may be drawn from some initial distribution.

The probability of occupation the vertex v_i after t-th steps is denoted by:

$$P_t(v_i) = P(v_t = v_i),$$

what is equivalent to:

$$P_t(i) = P(v_t = i). \tag{1}$$

Then, the matrix of transition probabilities of this Markov chain over the graph G is described by $M = (p_{v_i v_j})_{v_i, v_j \in V} = (p_{ij})_{i,j \in V}$, where:

$$p_{ij} = \begin{cases} \dfrac{1}{\deg(i)} & \text{if } (i, j) \in E, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Furthermore, the M_G is denoted an adjacency matrix of graph G and D is the diagonal matrix with $D_{ii} = 1/\deg(i)$. Under these assumptions, the matrix M can be described by following formula:

$$M = D \cdot M_G. \tag{3}$$

Then, the rule of the random walk is expressed by:

$$P_{t+1} = M^T P_t, \tag{4}$$

where:

$$P_t = (M^T)^t P_0. \tag{5}$$

Thus, the probability that starting at vertex v_i, we reach v_j in t steps is given by the ij-entry of matrix $M^t = (p_{ij}^t)_{i,j \in V}$.

To complete the Markov chain description, the steady state distribution for Markov chain over G graph may be defined by:

$$\Pi_i = \frac{\deg(i)}{2m}, \tag{6}$$

where m denotes the number of all graph edges.

Finally, for each vertex, the following is true:

$$\sum_{v_j \in N(v_i)} p_{ij} = 1. \tag{7}$$

3. Oil spill control model – submodel of the decision model

Oil spill control model describes the action to encircle the oil spill at sea. The model is shown on a square grid graph [4, 5]. The vertices and edges denote the phenomenon of oil spill as well as the course of action to encircle the oil spill.

3.1. Water area

The water area is shown on a square grid graph. In the model, the water area is divided into equal-size squares. The square size may be adjusted according to the type of oil spilled, degree of danger as well as to the location of oil slick. Each square is represented by a grid vertex. The vertex state represents the state of a given water area square. A vertex state may be empty, oil-affected or barrier-filled. The squares share the grid edges. The initial state of all vertices is “empty”. The appearance of oil slick changes the vertices state to “oil-affected”. Oil affects subsequent vertices until it is contained by a series of barriers. Barriers may be set in place of “empty” vertices. A vertex state already “oil-affected” or “barrier-filled” cannot be changed. The model uses discrete-time measured in cycles.

3.2. Oil spill

Oil spill appears on the water area in cycle 0. The vertex representing the area of the oil spill changes its state from “empty” to “oil-affected” – this vertex is the oil spill source. The oil spill source is designated by the grid centre. The grid centre, as it is a vertex $(0, 0)$, divides the grid into four quarters and sets the system of coordinates. In each subsequent cycle, the oil spill spreads from the “oil-affected” vertices to every adjacent “empty” vertex. In a square grid graph, called the Cartesian grid graph, each vertex is adjacent to four other vertices. It is also possible to use a triangular grid graph with each vertex adjacent to six other vertices or a strong grid graph with each vertex adjacent to eight other vertices. The choice of a grid graph may be adjusted to the degree of danger. The higher the number of adjacent vertices, the larger amount of containing measures must be used. If the oil slick spreads to a larger water area represented by a larger number of vertices, it is possible to assume that an ensemble of vertices is “oil-affected” in cycle 0. In such a case, it is necessary to designate the central vertex as an oil spill centre.

3.3. Algorithm for encircling the oil spill

The containing of the oil spill is performed according to an algorithm built on the basis of “firefighter algorithm”. In order to contain the oil spill, a series of barriers surrounds the oil-affected area. The barriers may be set on the “empty” vertices only. If we assume that the oil spill spreads during N cycles, the oil-affected area will be represented by a square formed by $(N, 0)$, $(0, N)$, $(-N, 0)$, $(0, -N)$ vertices. The action to contain the oil spill starts in cycle $N+1$. The action is divided into “attack” and “defense” modes. The first barrier is set in defense mode. In defense mode, each barrier is set in every other cycle. The aim of defense mode is to use a minimum number of barriers allowing controlling the oil spill so that it does not surround the series of barriers. All remaining barriers are set in attack mode. Attack mode starts at a location where the first defense mode barrier was set. Attack mode is performed simultaneously to defense mode, according to the barrier-setting algorithm and until the oil spill is contained. An oil spill contained by a series of barriers becomes an oil slick, which is unable to spread further.

Figure 1 represents an oil slick on the Cartesian grid graph (oil spill source marked with white). When the action to contain the oil spill starts in cycle $N+1$ and 2 barriers are available in each cycle, the procedure will last for $32N+1$ cycles and the oil slick will occupy $318N^2+14N+1$ vertices. In Fig. 1, the action started in cycle 3 and lasted for 97 cycles. 194 barriers were used. Oil spill spreaded to 1301 vertices.

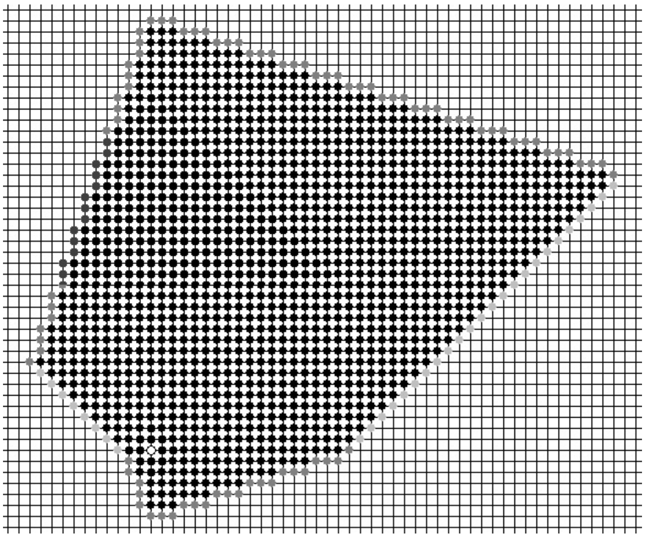

Fig. 1. Oil slick on the Cartesian grid graph

4. Oil slick movement model

Oil slick movement is analysed in two ways. Case one assumes no wind and focuses on surface sea currents only. Case two assumes that wind is in place, so oil slick is moving due to surface sea currents and wind currents. The model allows estimating the size of oil slick so the oil slick movement analysis may be limited to the movement of oil spill source. The description of oil slick movement is based on random walk in a two-dimensional lattice graph. The movement caused by surface sea currents is described on a square grid graph. The movement caused by both surface sea currents and wind currents is described on a grid graph combined from a triangular grid graph and two hexagonal grid graphs.

Generally, the grid graph is defined as a simple undirected graph $G(V,E)$ described in Section 2.

4.1. Oil slick movement caused by a surface sea current

Oil slick movement caused by a surface sea current is a simplified model, which assumes no wind current influence. Thus, the oil slick movement is caused by a surface sea current only. The surface sea current is circular [5].

A two-dimensional lattice graph oriented according to geographical coordinates has been used to describe the movement of the oil slick caused by a surface sea current. The grid size may be adjusted to the degree of danger as well as the water area conditions such as the velocity of the current. The oil spill source is moving across the vertices in a direction set by the grid edges. The oil spill source may move in one of the four directions. In each movement, the oil spill source will move onto one of the four adjacent vertices.

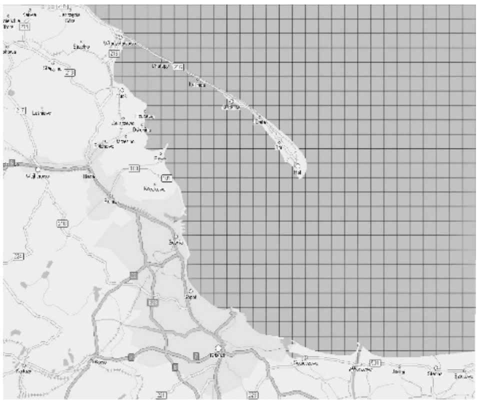

Fig. 2. The lattice graph on the sea

To describe this case, the square grid graph, also called the Cartesian grid graph, which is a Cartesian product of two paths of infinite length, is used. The probability of movement from one vertex v_i to another vertex v_j in a single step is designated by p_{ij} described by equation (2). The movement probability ensemble is an ensemble of graph edges appropriate for a given movement direction. For each vertex, the formula (7) is true.

At equal distribution of surface sea currents in every direction, the probability of movement in one-step is defined with accordance to (2) and is as follows:

$$p_{ij} = \frac{1}{\deg(v_i)}. \tag{8}$$

The probability of movement changes according to the direction of the current dominant on a water area. In Fig. 3, the three exemplary scenarios of movement probability height are shown:
- Scenario one assumes equal distribution of surface sea currents in all four directions.
- Scenario two assumes dominant north current.
- Scenario three assumes dominant north-west current.

The probability measures in other scenarios is analogous, it may be obtained by rotating figures by 90°, 180° or 270° degrees.

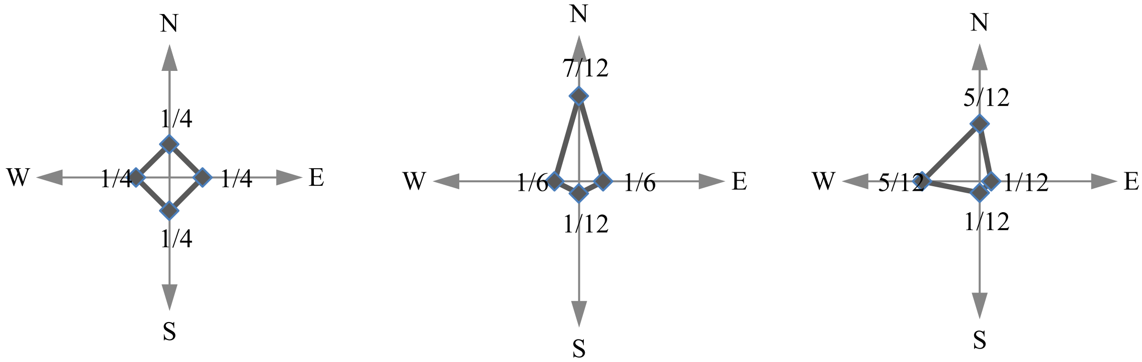

Fig. 3. Examples of movement probability measure

4.2. Oil slick movement caused by a surface sea current and wind current

The model assumes wind current influence. Therefore, the oil slick movement is caused simultaneously by a surface sea current and a wind current.

A two-dimensional lattice graph combined from a triangular grid graph and two hexagonal grid graphs has been used in the model (Fig. 4). The movement is caused according to the edges of the triangular grid graph to account for the wind current influence or according to the edges of the hexagonal grid graphs to account for the surface sea current influence. Random walk on the grid graph occurs across the vertices of the triangular grid graph so each movement means one movement caused by wind current or two movements caused by surface sea current.

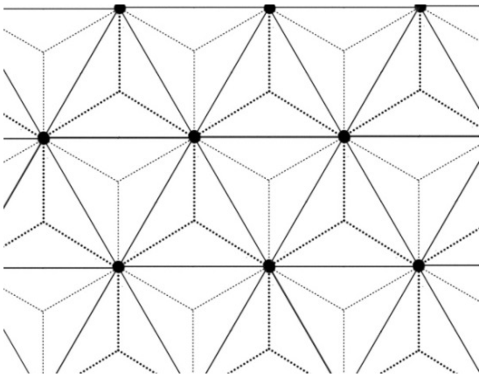

Fig. 4. Model grid

To describe this case, the grid shown in Fig. 4 is used. This is also represented by a simple undirected graph introduced in Section 2.

The Fig. 5 presents the set of oil spill movement vectors influenced by wind current and oil spill movement vectors influenced by surface sea current on a set of coordinates, for a vertex being a vertex from all three-grid graphs.

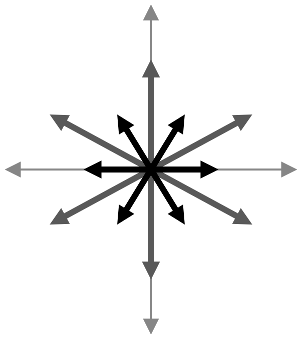

Fig. 5. Set of movement vectors

The probability of movement from one vertex v_i to another vertex v_j in a single step is designated by p_{ij} described by equation (2). The movement probability ensemble is an ensemble of graph edges appropriate for a given movement direction. For each vertex, the formula (7) is true.

Thus, the discussed grid graph has two types of vertices:

- vertices common to all three grid graphs, where the movement probability is $p_{ij} = 1/12$,
- as well as vertices from one of the hexagonal grid graphs, where the movement probability is $p_{ij} = 1/3$.

Movement probability changes according to the direction of surface sea current and wind current dominant on the water area. In Fig. 6 the three exemplary scenarios of movement probability height caused by wind current are presented:

- Scenario one assumes equal distribution of wind currents in all six directions,
- Scenario two assumes dominant southern wind,
- Scenario three assumes dominant eastern wind.

In the examples it is assumed that movement probability influenced by wind current and surface sea current are proportional. Furthermore, a scenario may be analysed where the influence of wind current is greater than that of the surface sea current or vice versa, according to the conditions of the water area.

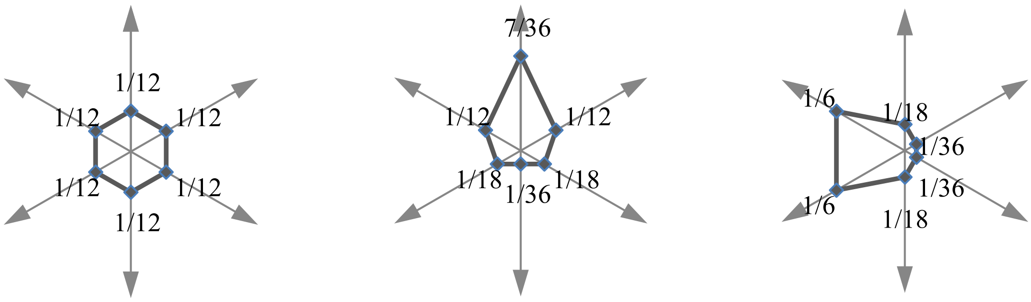

Fig. 6. The spread of movement probability influenced by wind current

For both cases, the random walk on particular grid graphs is described with accordance to formulae (5)-(6). Thus, for given Markov chains over considering grids graphs given by matrix M, the probability of vertex v_i occupation then the probability distribution after t cycles is given by exemplary sequence:

$$(P_1, P_2, P_3, \ldots, P_t)\,. \tag{9}$$

5. Conclusions

The article has presented the model of oil slick movement under the influence of wind and sea currents. The random walk in two-dimensional lattice graphs has been used to build this model. In the paper, the movement of the oil slick is analysed in two ways without wind and focus on surface sea currents and with wind and currents. Second way has been done according to the two-dimensional lattice graph consist of triangular and hexagonal grid graph. The proposed model allows estimating the distance of oil slick from coastal areas.

References

[1] Aldous, D., Fill, J. A., *Reversible Markov Chains and Random Walks on Graphs*, Unfinished monograph, recompiled 2014, available at http://www.stat.berkeley.edu/$\sim$aldous/RWG/book.html, 2002.
[2] Doyle, P. G., Snell, J. L., *Random walks and electric networks*, arXiv.org:math/0001057, [math.PR], 11.01.2000.
[3] Lovász, L., *Random Walks on Graphs: A Survey*, BOLYAI SOCIETY MATHEMATICAL STUDIES, 2, Combinatorics, Paul Erdös is Eighty (Vol. 2) Keszthely (Hungary), pp. 1-46, 1993.
[4] Mazurek, J., Smolarek, L., *Estimation of surrounding the spillage time*, Journal of Polish Safety and Reliability Association, Vol. 3 (2), pp. 245-250, Gdynia 2012.
[5] Mazurek, J., Smolarek, L., *Influence of the action starting time on the total time*, Proceedings of the XIV International Scientific and Technical Conference on Marine Traffic Engineering, pp. 311-318, Szczecin 2011.
[6] Mazurek, J., Smolarek L., *The stochastic model of the oil spill drift*, Zeszyty Naukowe Akademii Morskiej w Gdyni, Problemy bezpieczeństwa w transporcie morskim, No. 82, pp. 98-106, Gdynia 2015.