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TRANSFORMATION OF METRIC SPACE TOOLS FOR TRANSPORT LOGISTICS

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Abstract

The numerous logistics problems occurring in power train and transport industry lead to wear of vehicles and trains as well as the road or tracks length. The choice of the proper transport requires selecting the kind of the optimum length of the way. For facilities for transport of bulk commodity, very important meaning has the distance between the place of outgoing (x) and delivery point (y). Such problem will be referring for the one, two and three spaces in the case of land transport for road and rail vehicles as well as sea and air transport for ships and airplane.

It is desired that the transport from outgoing point (x) to delivery place (y) ought to be travelled on the shortest way in the presenting geometry sense. In transport, logistics denotes it the least distance between the place of the drive beginning (outgoing) and the place of drive end (delivering place). It is evident that the shortest distance between two points is always if we define this distance in Euclidean geometry sense because Euclidean metric determines the shortest distance between two points. In various transport problems the shortest distance in Euclidean sense, between two various places is not realistic and not possible. Therefore for transport vehicles exists many possibilities of various kinds of access road in Euclidean and non- Euclidean geometry. After Authors, suggestion very interesting is to find the optimum way or optimum track between the trip origin and delivery place.

Such problem demands the more and more information referring the describing the tracks geometry using metric space theory. In this article, especially the non-Euclidean modulus Taxi–Car metrics is considered. Presented metric spaces and their properties are needed and applied in practical transport problems occurring among other in assembly rooms where the way of intelligent shortest truck must be considered. Moreover, in this article the various communication ways will be presented and will be suggested an algorithm construction of the optimum distance problem solutions in non-Euclidean geometry presenting the equivalent and simultaneously most simple communication tracks.

Keywords*: transport logistics, access road, metric spaces, optimization, non-Euclidean geometry, transport engineering Applications*

1. Introduction

The metric space tools are always determine and indicate the choice of distances for many transport problems connected with the road communication network for the moving of vehicles along the concrete track [1-3]. Moreover, the knowledge of the description of the anticipated traffic functioning along the road net is very important during the design of the optimum shape of the various nets of the communication network. The example of configuration of roads and streets is presented in Fig. 1.

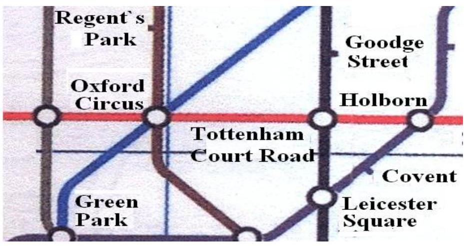


Fig. 1. The arrangement of London subway lines

2. The article tasks

The aim of this article is:

1. Definition and determination of the non- Euclidean metric space and its properties regard to the distances occurring in road, sea and air transport and according to the kind of the vehicle and kind of the transport way.
2. Presentation of the Euclidean and non-Euclidean metric transport in one-, two- and three-dimensional spaces.
3. Illustration and optimization aspects of the transport metric space defining the same distances for the various origin places to the same delivery point.
4. Application of obtained results in transport engineering problems.

3. The definition of metric transport

Metric space is defined by means of an arbitrary set X, where for each pair of elements or points or places $(x,y)\in X$ where x denotes outgoing and y-delivery place and both belong to set X is determined non-negative metric transport function $\rho(x,y)$ with real values is called as metric transport or conventional distance between two places x and y. Metric transport function $\rho(x,y)$ is defined in following form:

$$1^o \; \{ \rho(x,y)=0 \} \Leftrightarrow x\equiv y, \text{ for } x,y\in X, \tag{1}$$

$$2^o \; \rho(x,y)= \rho(y,x), \text{ for } x,y\in X, \tag{2}$$

$$3^o \; \rho(x,y)\leq \rho(x,z) +\rho(z,y) \text{ for } x.y,z\in X . \tag{3}$$

The first axiom (1) shows, that distance in metric sense between two the same places is always zero. Interpretation of second axiom (2) assumes, that distance in metric transport sense between two various places x and y where $x,y\in X$, $x\neq y$ has the same value as the distance between place y and x. Hence, the distance from outgoing place to the delivery place is the same as distance from delivery to outgoing place. Interpretation of third axiom (3) denotes a triangle inequality and shows, that in each metric transport space presenting the distances, the sum of two arbitrary sides of triangle is not smaller than the third side of the triangle. It denotes, that for each various places x,y,z the distance between two arbitrary places for example between x and y, is always smaller or equal to the sum of distances between places (x,z) and (y,z).

4. Classical Euclidean metric transport space

Classical Euclidean metric transport space determines always the least distance between outgoing x and delivery y place (point).

The classical Euclidean metric function, in one-dimensional (1D) space satisfies axioms (1), (2), (3), determines distance between outgoing place $x(x_1)$ and delivery place $y(y_1)$ and it can be defined in following form [4]:

$$\rho_1(x,y) = | x-y |. \tag{4}$$

Above distance illustrates Fig. 2.

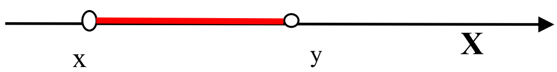


Fig. 2. The Euclidean metric transport distance between outgoing place x and delivery place y, defined in the one dimensional space D1

The classical Euclidean metric function in two dimensional (2D) space satisfies axioms (1), (2), (3), determines distance between outgoing place $x(x_1, x_2)$ and delivery place $y(y_1, y_2)$ and can be defined in following form [4]:

$$\rho_2(x,y) = [(x_1 - y_1)^2 + (x_2 - y_2)^2]^{0.5}. \quad (5)$$

Above distance illustrates Fig. 3.

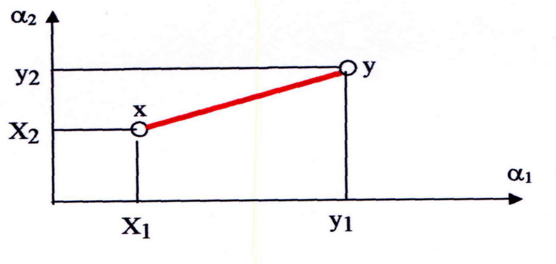


Fig. 3. The Euclidean metric transport distance between outgoing place $x(x_1,x_2)$ *and delivery place* $y(y_1,y_2)$*, defined in the two dimensional space D2*

The classical Euclidean metric function in three dimensional (3D) space satisfies axioms (1),(2),(3), determines distance between outgoing place $x(x_1, x_2, x_3)$ and delivery place $y(y_1, y_2, y_2)$ and can be defined in following form [6]:

$$\rho_3(x,y) = [(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2]^{0.5}. \quad (6)$$

Above distance illustrates Fig. 4.

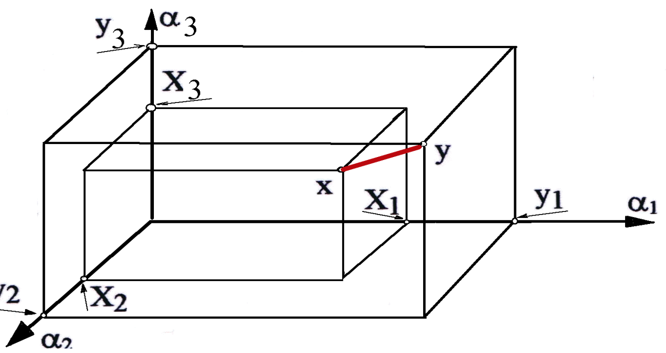


Fig. 4. The Euclidean metric transport distance between outgoing place $x(x_1,x_2, x_3)$ *and delivery place* $y(y_1,y_2,y_3)$*, defined in the three dimensional space D3*

Presented Euclidean metric transport space in Fig. 3, 4 determine the least distance between outgoing x and delivery y place in sea and air transport.

5. Comparisons between Euclidean and non-Euclidean metric transport

Figure 5 and 6 shows comparisons of distances in D2 between places A, B, C as apexes of triangles in classical Euclidean and non-Euclidean metric transport spaces. Fig. 5a shows polygon ADBECFA, as a triangle in non-Euclidean metric transport space where distance between A and B leads only through, distance between B and C leads only through place E.

Figure 6 on the right hand shows polygon a triangle ABC in non-Euclidean metric transport space where distances between apex places A, B, C lead only through the indicated bows.

The D2 metric transport spaces presented in Fig. 5, 6 have applications in road transport.

Figure 7 shows the distances described by the non-Euclidean metric transport. The outgoing and delivery places, P, S, Q are lying on the Earth Ellipsoid and are to lie thousand kilometres away from one other. The shortest distance between places P, Q, R is not rectilinear only curvilinear taking into account the Earth Ellipsoid curvature. The shortest rectilinear way- distance between places P, Q, R we have in Euclidean metric transport space, which is attainable only in air transport between outgoing and delivery places. Fig. 7 illustrates the distances between apexes of spherical-ellipsoidal triangle in non-Euclidean metric transport space [1].

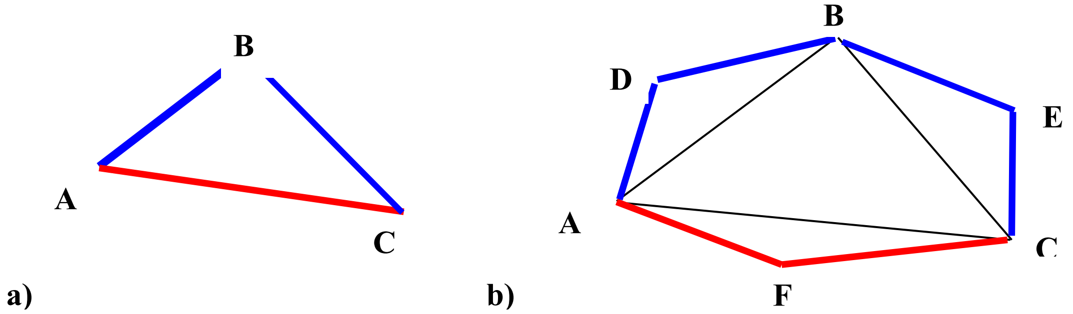


Fig. 5. Triangles in transport metric space D2: a) On the left- the Euclidean triangle with apexes A, B, C in the classical Euclidean metric transport presents way from A direct to B or through the point C; b) On the right- the polygon ADBECF as the triangle in non-Euclidean metric transport space, where way from A to B leads only through the point D

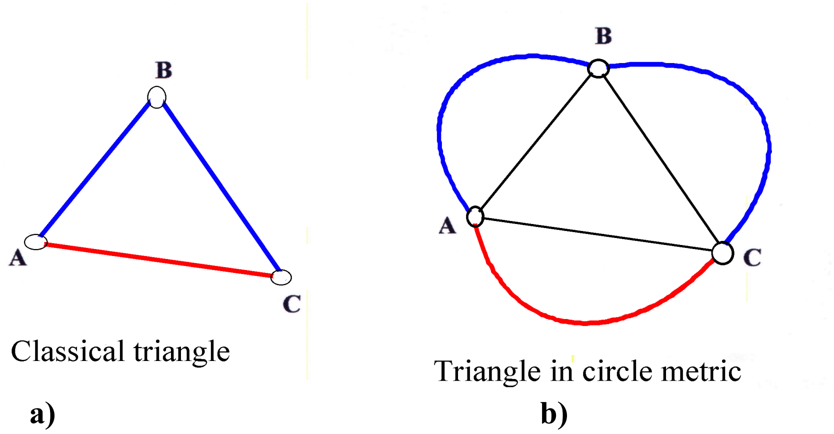


Fig. 6. Triangles in transport metric space D2: a) On the left- the Euclidean triangle with apexes A, B, C in the classical Euclidean metric transport; b) On the right-the triangle ABC in non-Euclidean metric transport space, where distance from A to B leads only through the bow way, similarly distance from B to C leads only through the bow

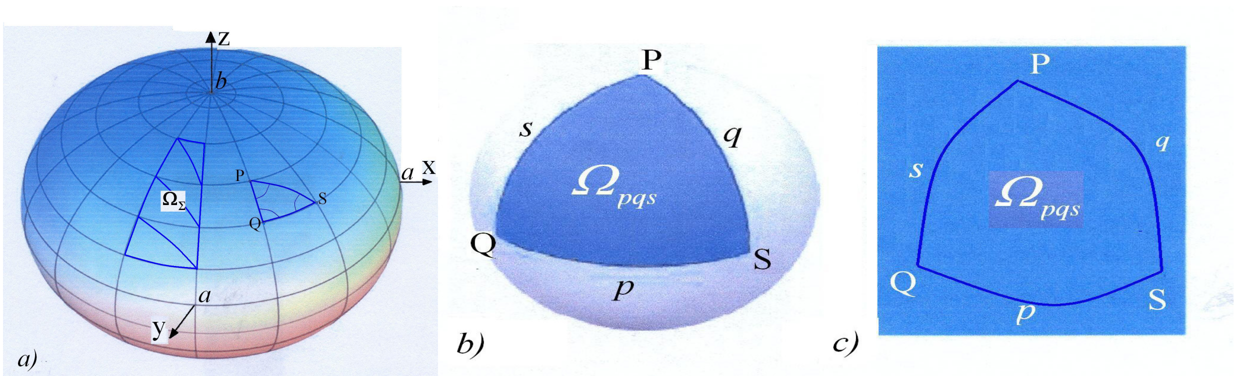


Fig. 7. Elliptical triangle with apex PQR lying on the Earth Ellipsoid as a non-Euclidean metric road transport space: a) triangle on the Earth Ellipsoid, b) approximation of the elliptical triangle, c) a view of elliptical triangle reduced to the plane

Distance differences between roadway on the Earth surface and airway are presented in Fig. 8.

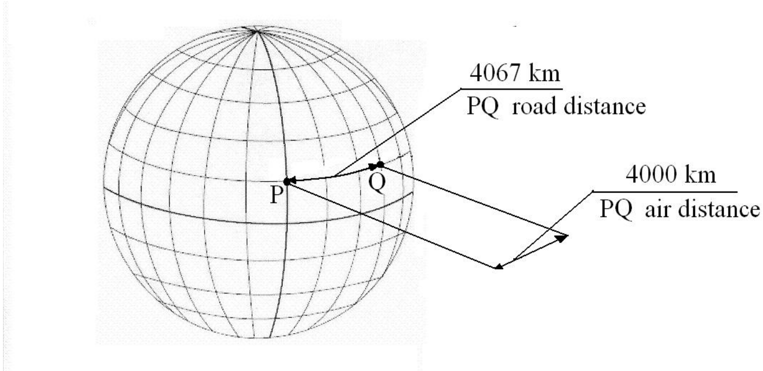


Fig. 8. Differences between metric transport road distance PQ on the Earth surface and metric transport air distance for the same places P and Q

6. Modular metric Taxi-Car

Americans to give the name Taxi-Car for the non-classical non-Euclidean, modular metric transport space, because the roads of big American towns and speed-ways erected in XIX century from outset on, had often the geometry of regular mutually perpendicular i.e. intersected at right angle straight lines, determining and remaining inside buildings or afforested surfaces of the rectangular shapes [9].

Hence, from the outgoing place (x) to the delivering place (y) we cannot to arrive at shortest distance only accordingly with the road or way architecture.

The non-classical, non- Euclidean, modular (Taxi-Car) metric function in three dimensional (2D) space satisfies axioms (1), (2), (3), determines distance between outgoing place $x(x_1, x_2)$ and delivery place $y(y_1, y_2)$ and can be defined in following form [4]:

$$\rho_4(x,y) = |x_1-y_1| + |x_2-y_2|. \tag{7}$$

Above distance illustrates Fig. 9.

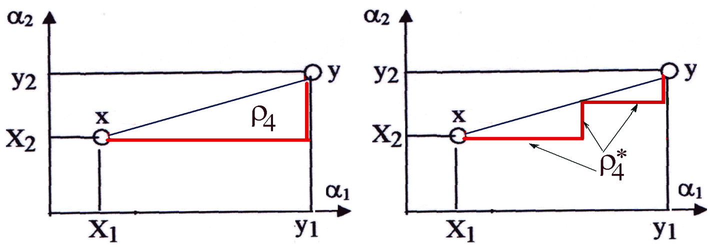


Fig. 9. The non-Euclidean modular (Taxi Car) metric transport distance between outgoing place $x(x_1,x_2)$ and delivery place $y(y_1,y_2)$, defined in the two dimensional space D2: $X=\alpha_1\times\alpha_2$

The non-classical non-Euclidean, modular (Taxi-Car) metric function in three dimensional (3D) space satisfies axioms (1), (2), (3), determines distance between outgoing place $x(x_1,x_2,x_3)$ and delivery place $y(y_1,y_2,y_2)$ and can be defined in following form [6]:

$$\rho_5(x,y) = |x_1-y_1| + |x_2-y_2| + |x_3-y_3|. \tag{8}$$

Above distance illustrates Fig. 10.

Presented on the Fig. 10 the distance in modular (Taxi-Car) metric transport, has the application for the optimum distance determination for the vertical and horizontal route of intelligent, self-propelled load-transport trolleys occurring and working in Federal Mogul bearing factory assembly rooms in Wiesbaden (Germany) and in Otto Bock European Factory of

Endoprosthesis in Dudenstadt (Germany) [10, 11].

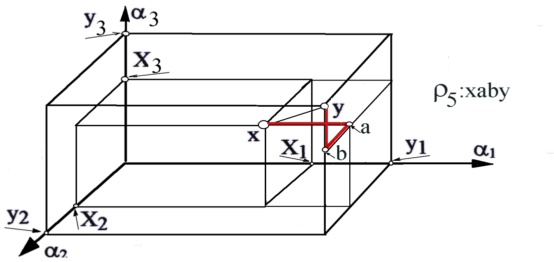


Fig. 10. The non-Euclidean modular (Taxi Car) metric transport distance between outgoing place $x(x_1,x_2,x_3)$ and delivery place $y(y_1,y_2,y_3)$, defined in the two dimensional space D3: $X=\alpha_1\times\alpha_2\times\alpha_3$

7. The first result

At first, we consider various ways from fixed outgoing place $x(x_1,x_2)$ to the fixed delivered place $y(y_1,y_2)$ for modular (Taxi Car) metric transport 2D space. This example is illustrated on the Fig. 11.

Example 1.

Indicate various ways between two fixed places in $x(x_1,x_2)$ and $y(y_1,y_2)$ in 2D modular metric transport.

Solution of Example 1

In modular metric transport D2 space, Fig. 11 shows four various ways between outgoing place $x(x_1,x_2)$ and destination place $y(y_1,y_2)$ namely: xAy, xMBCDy, xLEFy, xGHJKy.

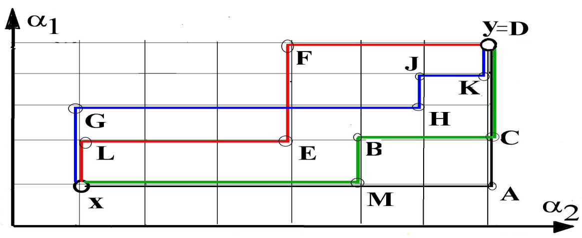


Fig. 11. Various ways with the same length in modular metric transport space D2 between two fixed places $x(x_1,x_2)$ and $y(y_1,y_2)$.

Abovementioned ways have the same distances.

8. The second result

Now for modular (Taxi Car) metric transport 2D space, we consider set of various outgoing points $x^{(n)}(x_1^{(n)},x_2^{(n)})$ for n=1, 2, ... with the same distance from the fixed delivered point (place) $y(y_1,y_2)$. This example is illustrated on the Fig. 12.

Example 2.

Indicate various places $x^{(n)}(x_1^{(n)},x_2^{(n)})$ for n=1, 2, ... with the same distance from fixed delivered place $y(y_1,y_2)$ in 2D modular metric transport.

Solution of Example 2

The places (points) $x^{(n)}(x_1^{(n)},x_2^{(n)})$ for n=1,2,.... with the same distance from fixed delivered place $y(y_1,y_2)$ create the circle presented in Fig. 12 for modular metric transport D2 space.

The radius of the circle in modular space (square in Euclidean space) presented in Fig. 12 equals WC. Distance between each outgoing points lying on the circle circuit from the circle centre i.e. delivering point is equal to the radius. For example distance from outgoing point A to the delivering point C=(y) equals AB+BC =WC. Distance from outgoing point H to the delivering point C=(y) equals HG+GC =WC.

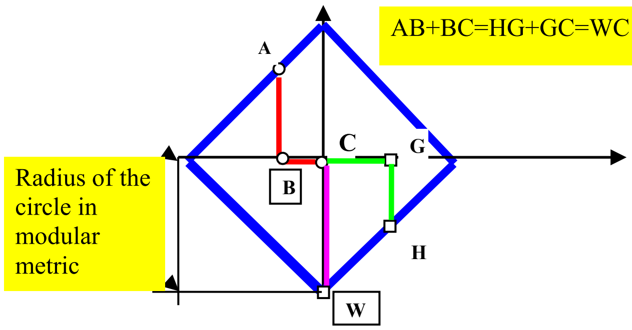


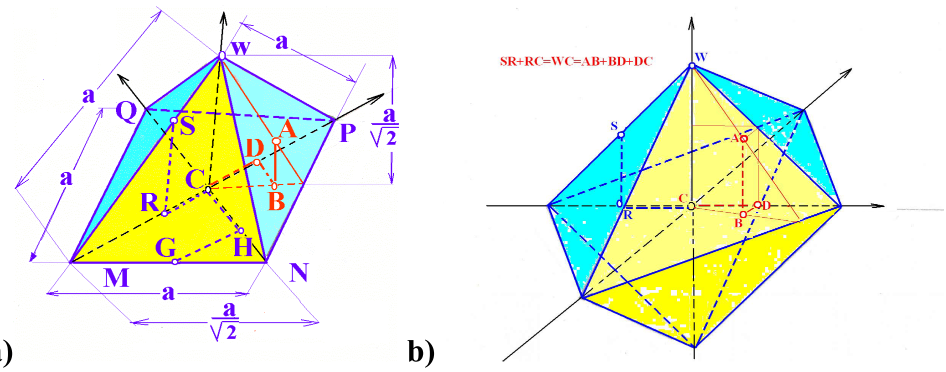
Fig. 12. Delivered point C= $y(y_1,y_2)$ as the centre of the circle in modular metric transport space 2D

Applications

The shortest distances determination for the horizontal route 2D of intelligent, self-propelled load-transport trolleys has applications in Federal Mogul bearing factory assembly rooms. Storage yard of products in point y (centre of the circle) is supplied from the various places lying on the circle circuit with the same distance from y.

9. The third result

Now for modular (Taxi Car) metric transport 3D space, we consider set of various outgoing points $x^{(n)}(x_1^{(n)},x_2^{(n)},x_3^{(n)})$ for n=1, 2, ... with the same distance from the fixed delivered point (place) $y(y_1,y_2,y_3)$. This example is illustrated on the Fig. 13.



a) b)

Fig. 13. Delivered point C= $y(y_1,y_2,y_2)$ as the centre in modular metric transport space 3D: a) outgoing points x: A, W, S, G, Q,... lying on the hemisphere surface in modular metric space (Pyramid surface in Euclidean metric) have the same distance from the delivering point y; b) sphere in modular metric space (two Pyramids in Euclidean metric)

Example 3.

Indicate various places $x^{(n)}(x_1^{(n)},x_2^{(n)},x_3^{(n)})$ for n=1,2,... with the same distance from fixed delivered place $y(y_1,y_2,y_3)$ in 3D modular metric transport.

Solution of Example 3

The places (points) $x^{(n)}(x_1^{(n)},x_2^{(n)},x_3^{(n)})$ for n=1,2,... with the same distance from fixed delivered place $y(y_1,y_2,y_3)$ create the hemisphere presented in Fig. 13 for modular metric transport D3 space(Pyramid in Euclidean metric space).

The radius of the hemisphere in modular space (pyramid in Euclidean space) presented in Fig. 13 equals WC=$a/\sqrt{2}$.. Distance between each outgoing points lying on the hemisphere from the hemisphere centre i.e. delivering point is equal to the radius of hemisphere. For example distance from outgoing point A to the delivering point C=(y) equals AB+BD+DC =WC. Distance from outgoing point S to the delivering point C=(y) equals SR+RC =WC.

Applications

The shortest distances determination for the vertical and horizontal route 3D of intelligent, self-propelled load-transport trolleys has applications in Federal Mogul bearing factory assembly rooms. Storage yard of products in point y(centre of the hemisphere) is supplied along the same distance from y by the transport trolleys from the various places lying on the hemisphere surface.

Remark

The passageways in Egyptian Pyramids Fig. 14 from the external pyramid surface to the pyramid centre have the same length and modular metric architecture.

Fig. 14. Egyptian Pyramids

10. Conclusions

1. On the ground of illustrations presented in Fig. 10, 11 follows, that arbitrary various ways between outgoing and delivering places defined in Taxi-Car modulus metric transport 2D or 3D space have the same length.
2. On the ground of illustrations presented in Fig. 12, 13 follows, that in Taxi-Car modulus metric transport 2D or 3D space we have infinite many various ways with the same length from the various arbitrary outgoing places to the fixed delivering place.
3. Presented results enable to find the optimum localization of places for storage yard (y) of products, which are supplied from the various places (outgoing points), to be the same distance away from storage yard places and lying on the factory assembly rooms.

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