Journal of KONES Powertrain and Transport, Vol. 21, No. 4 2014
ISSN: 1231-4005
e-ISSN: 2354-0133
ICID: 1130480
DOI: 10.5604/12314005.1130480

FLIGHT PATH MANAGEMENT OF AN AIRCRAFT IN EMERGENCY SITUATION

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Abstract

A distress event i.e. loss of engine power, structural damage etc., creates a major emergency situation in General Aviation (GA) aircraft. It requires location of a safe-to-land runway within reach, and immediately planning and executing an effective flight path towards it. Currently, technological development in avionics allows creating a flight deck decision support tool for trajectory planning of GA aircraft in emergency situation. Principal part of the system constitutes flight path optimization module. The automated path-planning algorithm generates within seconds an optimized trajectory to be followed by the pilot to safe landing. The trajectory planning is formulated as an optimal control problem, with the aircraft kinematics and dynamics expressed by the state equations, and objective functional that may capture the time, length, energy loss, etc. Obstacles and restricted or prohibited zones are represented as constraints on the positional state variables. The purpose of the paper is to present the method of flight path optimization of an aircraft in emergency, after a distress event, which makes impossible continuation of the original flight. The methods allow determining optimal escape flight path from special protection areas (e.g. cities etc.) or danger zones, as well. The simplified realization of the Ritz-Galerkin method was used in this work, which uses an approximate solution to boundary value problems for determining the optimal flight trajectory. The method allows determining the optimal trajectory of the flight satisfying the initial/final conditions and control functions and path constrains for an aircraft.

Keywords: *General Aviation, flight path optimization, flight safety*

1. Introduction

The aim of this work is to present a method of optimizing the trajectory of the approach of the aircraft to the airport or landing field if the necessity to cancel the flight plans or an emergency situation, which disables its continuation, occurs. The method also enables determining the optimal trajectory of the departing from the zone subjected to security (for instance, large populated areas), if the emergency, which disables to continue the performed flight plan takes place.

The problem of the optimal control of the aircraft, especially optimizing of the flight trajectory have been described by many authors, for example [1, 6, 7, 9, 10, 16]. The problem was formed as an alternative question in which the quality functional such as the flight time, consumed fuel etc. was minimized; however, the boundary conditions and limitations imposed on the aircraft trajectory were accomplished. In order to find a solution, the techniques of the dynamic programming established by Bellman [2] and Maximum Pontryagin principle [8] were used.

To develop the described method there was used the simplified of Ritz-Galerkin method of approximate solution of boundary value problems to determine optimal flight trajectory described by Taranienko et al. [14]. The method allows determining the optimal flight trajectory, which fulfils boundary value conditions and limitations imposed on it. The simplified algorithm of optimization uses third degree polynomials to approximate the changes of the selected parameters of the flight.

2. Problem formulation

Any task of flight dynamics consisting in determining the aircraft trajectory based on a point mass model can be formulated as follows: it is necessary to find such a control of the aircraft which can guarantee its transition from the initial point of the trajectory defined by the initial coordinates to the final point of the trajectory defined by final coordinates. Infinitely many trajectories can be drawn between the initial and the final point (Fig. 1) that is why, it can be stated that there are infinitely many control functions, which make a solution to the task formulated above. Each set of the control functions, which meet the above-mentioned condition, is one of the variants of the solution to the task formulated in such a way. As it is known that, the movement of the aircraft centre of gravity in the space is described by a set of 7 nonlinear differential equations, which in a general case does not have an analytical solution; thus, the solution can only be found using the numerical methods, defining the boundary conditions for the state variables. However, it is not known how to determine the control functions. One of the methods is the selection of the control functions so that a certain quality functional, which depends on the state variables and control functions, would reach extreme value (the most often minimal). In this way the task of finding, the flight trajectory of the aircraft comes down to optimization of the quality functional.

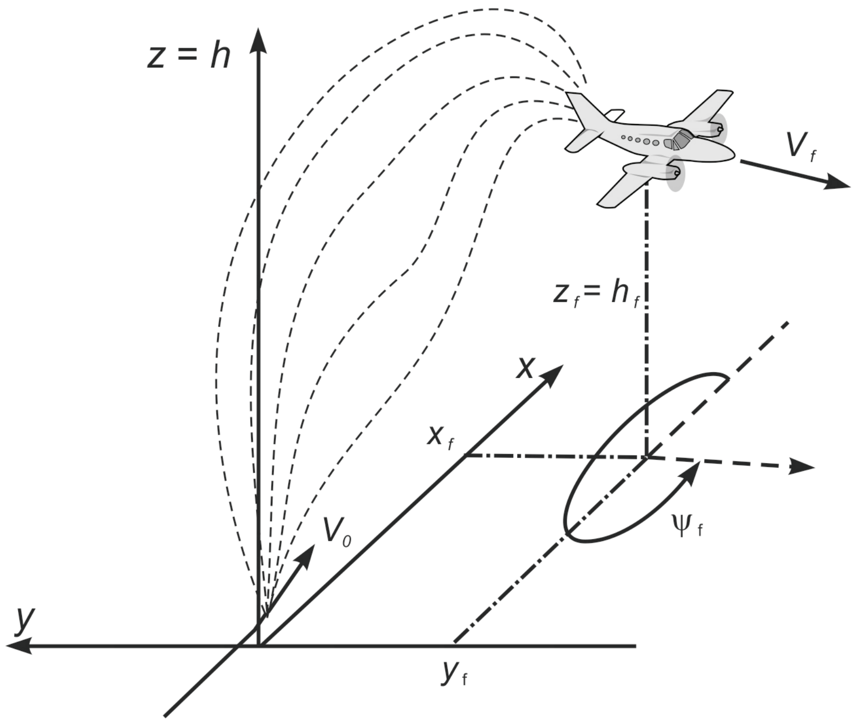

Fig. 1. Trajectory optimization problem

The task of this work is to determine flight parameters of the aircraft performing the flights to the airport or landing field in the aspect of minimal value of the defined functional – depending on the analysed scenario of the events. The problem set up in such a way is a complex task and in general case it requires considering many factors. The value of the adopted criteria factor is influenced by a number of factors, such as height and speed of flight, operating range of engine (engines), time of the flight and weather conditions. When the problem is being solved, the aircraft is treated as a point mass, it means that ideal piloting is assumed; thus, the sum of all the moments applied in each state of the flight becomes zero. The controlling can be performed in two ways: by pulling the control device and changing of the engine power. The first way enables pitching, yawing and rolling. The second way is to change the power of the engine; it allows selecting a proper thrust. Optimization, thus, is provided to determine such a principle of controlling the aircraft so that the curve at which the aircraft moves is extreme. The problem formulated in such a way is a typical alternative task, and the obtained curve is called extreme. There are two main types of these tasks. The first one is when all the values of the determined functions in the initial and final point should be strictly determined. This is the task with the determined boundary conditions. In practical tasks of the flight dynamics only some of these conditions are given. These are the so-called tasks with free ends. The analysed case is an example of such a task. However, all the conditions are given in the initial point only; in the final point, some of them are given. The initial conditions correspond to the initial state of the flight, which follows from the moment when the decision to abort the task performance is made. As far as the final conditions are concerned, some variants can be taken into consideration. The most general is when only one variable is determined (for example, the distance to the place of landing), and the rest of them are resultants. The most often, however, the condition will be formed so that to finish the flight at a given height h_f and speed V_f, which, for example, can follow from the conditions of performing a safe landing. Assuming the final time t_f it is possible to avoid exceeding the maximal time of the flight following, for example, from the number of the cumulated energy in the accumulator (the case of alternator damage).

It is noteworthy that not all the parameters can change arbitrary. Some of them in order to obtain an optimal trajectory could exceed their allowed values. That is why the trajectory obtained as a result of optimization should be imposed by certain limitations according to the required principles and instructions of exploitation. In a general case, these limitations will concern aerodynamics, durability, performance and those depending on the trajectory shape. Aerodynamic and durability limitations are the most important ones, because they guarantee safe exploitation. They should be considered together because in a general case they are determined simultaneously, regarding the durability of the construction, aerodynamic properties of the airframe as well as performance characteristics, which take into account the possibility of an emergency. Besides being an acceptable value, which cannot be exceeded, it is assumed with a certain margin. Under the character of criteria, the following factors are taken into consideration:

- durability of the airframe construction, engines and other elements of the aircraft,
- controllability and stability of the aircraft,
- phenomena connected with low and high flight speeds (stall, flutter, etc.),
- absolute ceiling,
- influence of the turbulent atmosphere,
- change of the performance caused by different emergencies (limitation of the power preceded by the power unit, limitation of the time of the power unit work, complete power loss, etc.).

In the analysed case, the following limitations are assumed: not exceeding the maximal flight speed, allowable load factor and allowable angle of attack.

The limitations depending on the shape of the flight trajectory take into consideration the range of the flight altitude, which is possible to use regarding the movement, minimal altitude over the territory, avoiding forbidden or dangerous zones, etc.

General stating of the task supposes determining the optimal trajectory of movement of a flying vehicle (Fig. 1) described by the system of ordinary differential equations:

$$x_i = f_i\left(x_1,\ldots,x_n,u_1,\ldots,u_m\right), \quad i=1,2,\ldots,n\,, \quad m \le n\,, \tag{1}$$

fulfilling boundary conditions:

$$\begin{aligned} x\left(t_0\right) &= \left\{x_1\left(t_0\right),x_2\left(t_0\right),\ldots,x_n\left(t_0\right)\right\}, \quad x\in\mathbb{R}^n, \\ x\left(t_f\right) &= \left\{x_1\left(t_f\right),x_2\left(t_f\right),\ldots,x_n\left(t_f\right)\right\} \end{aligned} \tag{2}$$

and constraint for state variables and control variables:

$$x_{i\min}(t) \le x_i(t) \le x_{i\max}(t),$$
$$u_{j\min}(t) \le u_j(t) \le u_{j\max}(t), \quad j = 1,2,\ldots,m, \tag{3}$$

where:
x_i – state variables,
u_j – control variables,
t_0, t_f – initial and final times.

The technical characteristics of the flying vehicle are known and can be written as:

$$x_T = (x_{T1},\ldots,x_{Tr}), \quad x_r \in \mathbb{R}^r. \tag{4}$$

The optimal trajectory $x(t)$, $t\in(t_0, t_f)$ has to be found which minimizes the functional:

$$J(x(t)) = \int_{t_0}^{t_f} f_0(X,U)\,dt, \tag{5}$$

and the control corresponding to it is:

$$U(t) = (u_1(t),\ldots,u_m(t)), \quad U \in \mathbb{R}^m. \tag{6}$$

In the case of the current task the system of differential equations (1) commonly employed in the aircraft trajectory analysis is the following six-dimension system derived at the centre of mass of the aircraft [3]:

$$\mathring{V} = g\left(\frac{T\cos\alpha - D}{mg} - \sin\gamma\right),$$
$$\mathring{\gamma} = \frac{1}{mV}\left((T\sin\alpha + L)\cos\varphi - mg\cos\gamma\right),$$
$$\mathring{\psi} = \frac{(T\sin\alpha + L)\sin\varphi}{mV\cos\gamma},$$
$$\mathring{m} = \frac{dm}{dt} = C_S,$$
$$\mathring{x} = V\cos\gamma\cos\psi, \quad \mathring{y} = V\cos\gamma\sin\psi, \quad \mathring{z} = \mathring{h} = V\sin\gamma. \tag{7}$$

V, γ, ψ, α and φ are, respectively the speed, the angle of descent, the yaw angle, the angle of attack and the roll angle. (x, y, $z = h$) is the position of the aircraft. T, D, L, m (technical characteristics – eq. 4) and g are respectively the engine thrust, the drag force, the lift force, the aircraft mass and the gravitational acceleration. It was considering a 2-Degree Of Freedom (DOF) dynamic model that describes the point variable-mass motion of the aircraft over a flat Earth model. A standard atmosphere is defined with ISA (International Standard Atmosphere). Lift coefficient C_L is, in general, a function of the angle of attack α and the Mach number M, i.e., $C_L = C_L(\alpha,M)$. The lift coefficient is used as a variable rather than the angle of attack. The aircraft performance model was worked out on the basis of [4, 5, 11-13, 15].

3. Analysed case – total power loss of the power unit

The situation when the engine stops working during the flight is very dangerous that is why this case will be discussed separately. First of all, the condition of reaching the airport or landing field will be checked in the flight with maximal lift to drag ratio regarding the necessity to perform the manoeuvre in the direction towards this airport. If the airport is situated within the gliding flight range, the optimal trajectory of the approach to this airport will be determined with minimal loss of total energy. It will also be checked if exploitation limitations of the aircraft are not exceeded. The initial value of the total energy is determined on basis of:

$$e_{c0} = h_0 + \frac{V_0^2}{2g}. \tag{17}$$

The flight of the aircraft with a continuous energy loss is possible to the moment when its value does not reach final energy defined by the touchdown conditions (height and speed):

$$e_{cf} = h_f + \frac{V_f^2}{2g}. \tag{18}$$

The variation task in this case is formulated as follows: to find such a controlling of the aircraft which for the given final conditions: x_f, y_f, z_f, γ_f, ψ_f, e_{cf} and initial conditions z_0, V_0, γ_0, ψ_0 will guarantee maximal distance covered by the aircraft to the final point:

$$r = \sqrt{\left(x_0 - x_f\right)^2 + \left(y_0 - y_f\right)^2}\ . \tag{19}$$

Initial values of the coordinates x_0 and y_0 are also determined, however, in this task they subject variation. Three-dimensional motion of the aircraft is described by the set of equations (7) for the thrust equals zero and a constant weight of the aircraft. That is why, the aircraft is controlled by two parameters only: coefficient of load n_z and roll angle φ. Symmetrical coefficients of load are determined on basis of:

$$n_Z = \frac{C_L \rho S V^2}{2mg},\ n_X = \frac{C_D \rho S V^2}{2mg}. \tag{20}$$

The set of equations (7) written regarding τ obtains the following form:

$$\begin{gathered} V' = \frac{g}{\lambda}\left(n_X - \sin\gamma\right),\ \gamma' = \frac{g}{\lambda V}\left(n_Z \cos\varphi - \cos\gamma\right),\ \psi' = \frac{g}{V\lambda\cos\gamma} n_Z \sin\varphi, \\ x' = \frac{V}{\lambda}\cos\gamma\cos\psi,\ y' = \frac{V}{\lambda}\cos\gamma\sin\psi,\ z' = \frac{V}{\lambda}\sin\gamma,\ t' = \frac{1}{\lambda}. \end{gathered} \tag{21}$$

The equation describing the change of total energy of the aircraft completes the set (21):

$$e_c\left(\tau\right) = e_{c0} - e_{cf} - \int_0^{\tau} de_c\ . \tag{22}$$

From the second and the third equations of the set (21) the controlling functions: tgφ and n_Z are determined. From the fourth, fifth and sixth equations values: λ, sinγ and tgψ (equation 12 and 13) are determined. The derivatives γ' and ψ' are determined on basis of:

$$\gamma' = \frac{z''\left(x'^2 + y'^2\right) - z'\left(x'x'' + y'y'' + z'z''\right)}{\left(x'^2 + y'^2 + z'^2\right)^{\frac{3}{2}}\cos\gamma}, \tag{23}$$

$$\psi' = \frac{\cos^2\psi\left(x'y'' - y'x''\right)}{x'^2}. \tag{24}$$

Thus, only the first and the seventh equations will be integrated in the set (21). In the course of calculations in the right directions of the equations (12), (13), (23), (24) the values $V(\tau)$, $x'(\tau)$, $y'(\tau)$, $z'(\tau)$, $x''(\tau)$, $y''(\tau)$, $z''(\tau)$ are substituted. The speed $V(\tau)$ is obtained on basis of integrating of the first equation, and the rest of values are determined on basis of formulas for the support functions $x(\tau)$, $y(\tau)$, $z(\tau)$. So, to solve the task of optimization of spatial flight trajectory of the aircraft with non-operated engines, it is necessary to use three support functions $x(\tau)$, $y(\tau)$, $z(\tau)$ which define the trajectory of motion with the given initial and final conditions.

4. Results

The calculations were made for aircraft with geometry similar to Piper PA-34 Seneca V with two piston engines. General technical characteristics of the aircraft are presented in Tab. 1.

Tab. 1. General characteristics of Piper PA-34 Seneca V

Parameter	Value	Unit
Length	8.72	m
Wingspan	11.86	m
Wing area	19.39	m^2
Gross weight	1999	kg
Maximum speed	378	km/h
Cruise speed (at FL100)	348	km/h
Service ceiling	7 620	m
Rate of climb	7.87	m/s

It was assumed that a distress event i.e. loss of engines power appears during normal flight at 2 000 meters height of flight at speed of 348 km/h. The aircraft flies above Boguchwała zone near airport Jasionka at heading of 270 deg. After loss of engines power the aircraft has to reach runway at Jasionka airport in gliding flight, avoiding Rzeszow city centre. At the final position the aircraft should have at least 300 m of height of flight not far then 2 km from the touch down zone of the runway.

Figure 2 presents the changes of optimal speed of the aircraft during flight and changes of load factor. The flight speed decreases slowly to the assumed final value. The changes of load factor are very small except final manoeuvre. At Fig. 3 the optimal change of trajectory angle and yaw angle is presented. The flight is characterized by small changes of yaw angle. The greater changes of trajectory angle at the beginning of gliding phase are caused by possibility of gliding conditions determination. Fig. 4 presents the situation map with shown prohibited area of Rzeszow-centre.

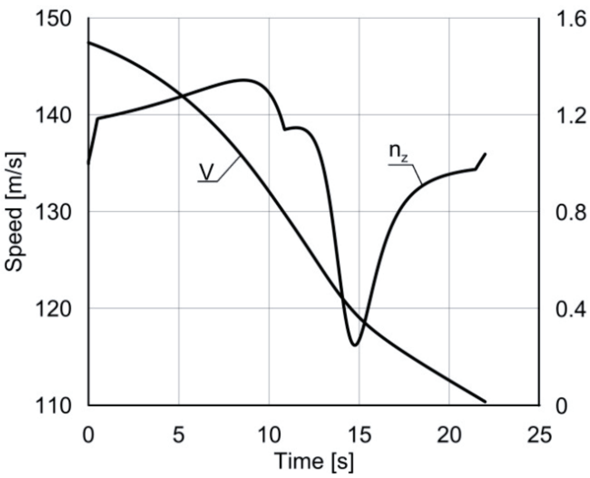

Fig. 2. The optimal change of speed and load factor

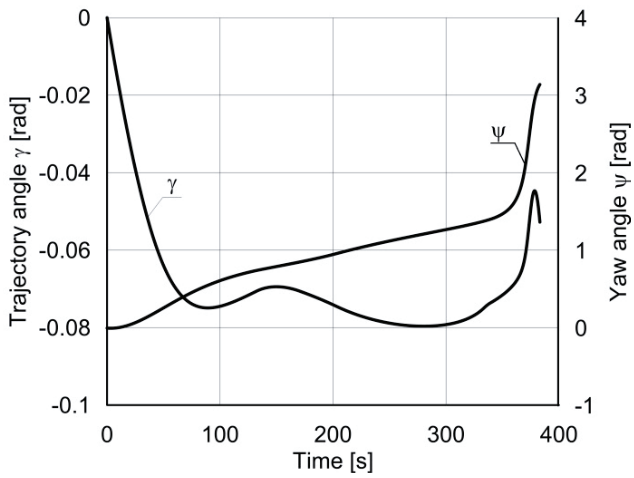

Fig. 3. The optimal change of trajectory and yaw angle

5. Summary

Contemporary General Aviation (GA) aircraft have to be designed considering the established parameters of the flight and performance characteristics, which have a direct connection with flight operating safety. Flight operating safety of the GA should be understood as a level of adjustment of the system of aircraft behaviour, co-existing objects and the environment when performing a flight plan. The flight operating safety and the effectiveness of using the GA aircraft are the two basic requirements concerning the GA.

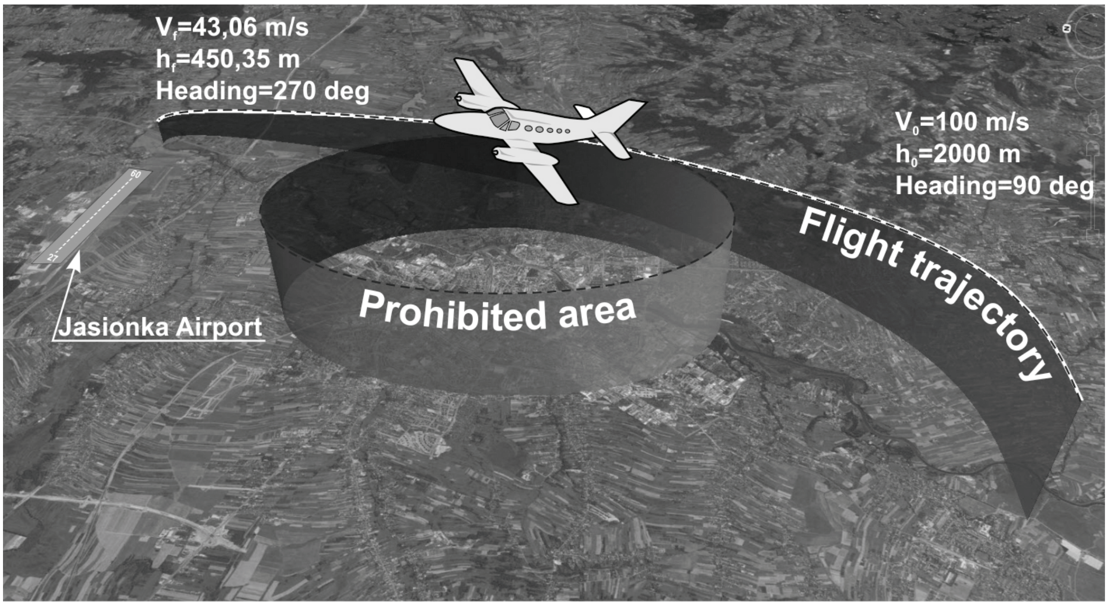

Fig. 4. 3D form of the optimal trajectory with shown high population area as prohibited area

The flight safety when performing the flight is endangered by particular situations, which impose changing of regular operation of the GA aircraft. Dangerous situations or emergencies are frequently connected with the necessity to change the plan, profile and parameters of the flight.

The aim of this work was to present the methods to determine the GA aircraft flight profile after a dangerous situation or emergency occurs. The analysis was limited to the possibility of occurrence of the engine system emergency, and the further flight continued along the trajectory whose shape depends on the kind of the emergency. Three possible scenarios in which the emergency took place were examined:

- it does not influence on the performance of the engine system, however, it limits the time for the aircraft to get to the destination or an alternate airport,
- it causes a partial loss of performance (power) of the engine system, which enables performing the horizontal flight or a flight with a small altitude loss,
- it causes complete power loss, which enables performing a gliding flight only.

The suggested method also enables determination of the optimal flying trajectory from the territory of a special protection zone (for example, large populated areas), in the case of the emergency which disables continuation of the performed task.

The task of this work was to determine the flight parameters of an aircraft performing the flight to an airport or a landing field in the aspect of the minimal value of the assumed functional – depending on the analysed scenario. The method used in this work allows in a simplified way to solve a variation task using the Ritz-Galerkin method consisted in an approximate solution of the boundary value problem to determine the optimal flight path. The choice of the method was dictated by the fact that it was relatively simple and at the first step of optimization it gives a preliminary approximation of the suboptimal path. The aircraft can start the flight according to the initially determined path whose shape is corrected in further steps of optimization. Such an approach minimizes the time to make a decision in an emergency situation. The worked out method can become an element of the on board system supporting the GA aircraft flight control.

Acknowledgement

The research project has been financed from a grant allocated by the National Centre for Research and Development in Poland for the years 2013-2016.

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