Journal of KONES Powertrain and Transport, Vol. 21, No. 2 2014

ISSN: 1231-4005
e-ISSN: 2354-0133
ICID: 1133907
DOI: 10.5604/12314005.1133907

SOME GEOMETRICAL DATA FOR THE ELLIPTICAL BEARINGS IN ARTIFICIAL ROBOTS

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Abstract

In this paper are considered analogies between elliptical-slide bearing occurring in artificial robots and elliptical natural human joints as well artificial human femoral prosthesis. The comparisons between elliptical human femoral prosthesis and mechanical spherical bearings indicate the advantages implicated from the use of elliptical slide bearings. To prove above statement we describe some remarks referring the elliptical femoral prosthesis and natural human joints. Numerous contemporary studies and experimental measurements regarding the human hip joints indicate, that femoral head is, in fact naturally elliptical. Therefore, the femoral elliptical complete prosthesis as well half prosthesis must be better adapted to the natural hip joint with elliptical bonehead and neck of the femur. The elliptical femoral prosthesis was designed for solving the problem of the relationship between the femoral prosthesis head and the anatomical acetabulum and to minimize the various incidence namely wear protrusion and pain effects. Performed clinical tests indicate that elliptical prosthesis head have the most advanced technological shapes because are based on the anatomy and biomechanics features of the natural human hip joint. The same aspects are occurring during functioning the elliptical slide bearing in artificial robots. The mechanics of elliptical-slide bearings can be taken as the appropriate shape in the respect to the operating treatment. Taking into account the hydrodynamic theory of lubrication of cooperating surfaces we must finally find pressure distributions, friction forces, friction coefficients and wear. To prepare this calculations we ought at first determine and calculate the fields of the regions of lubrication on the internal surface lying on the elliptical surfaces and on the external elliptical surface of the sleeve. Surface lubrication regions consist of the sums of spherical triangle, which are lying on the abovementioned surface. In this paper are determined the formulas for the field calculations of elliptical triangle with the three vertexes (points) for coordinates which are foreseen measured.

Keywords: *artificial robots, elliptical slide bearings, comparisons with natural human joints*

1. Substantiation of elliptical bearing application in robots

The humanoid robots with artificial spherical slide bearings presented in Fig. 1 have not harmonic motions and the power transformation is not regular.

The bone head of natural human hip joint is not spherical but elliptical. This fact is confirmed by the numerous anatomical and biomechanical studies on human hip joint, by many scientific literature reports as well long-term results obtained by other Authors [1-8]. We can observe the asymmetrical distribution of the layer of cartilage lying on the bone head surface. The cartilage layer is thickest at the upper pole and thinnest at the equator. From this fact follows, that the head to be elliptical with a wider diameter, a, along the axis of the femoral neck [6-7]. Such different distribution of cartilage is presented in Fig. 2a. Symbol a denotes smaller diameter and b – wider diameter. The square of eccentricity for human hip joint is defined by the following formula [9-12]:

$$\varepsilon^2 = \frac{b^2 - a^2}{b^2}. \tag{1}$$

For b=26.5 mm and a=25.0 mm, eccentricity obtained from formula (1) has the value: ε=0.3316678. If b=26.5 mm and a=26.0 mm then ε=0.1933386. It is worth to noticed that the earth eccentricity ε=0.08182 is smaller than eccentricity of elliptical bonehead. In Fig. 2b, 2c, 2d, 2e, 2f are presented various elliptical acetabulum prosthesis [1] and their localization in human body. Fig. 3a, 3b, 3c, 3d are presenting the spherical and elliptical slide bearings and their localization in robot.

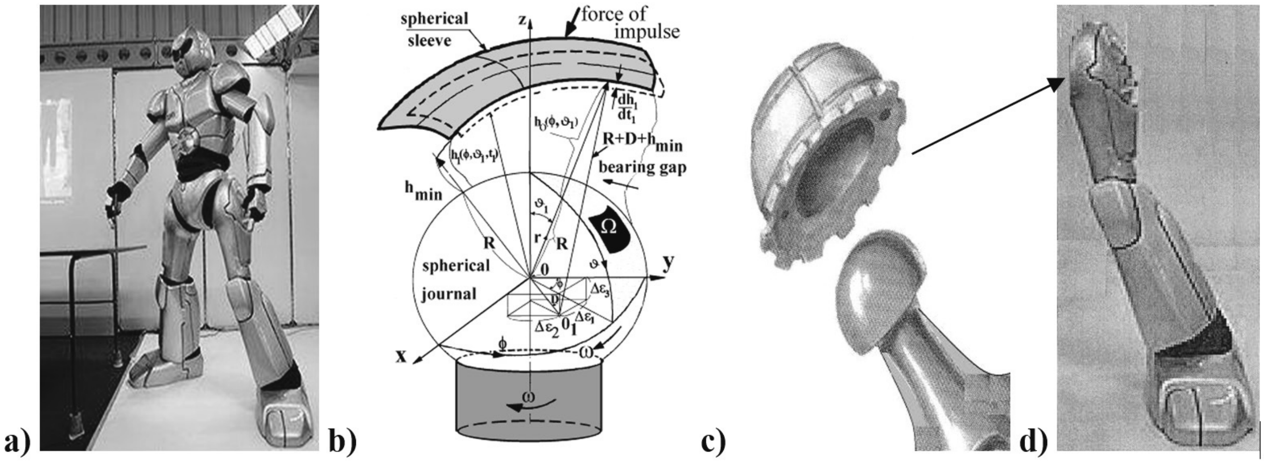


Fig. 1. Robot with spherical bearings: a) Illustration of the robot, b) spherical gap height and eccentricities, c) connection with the artificial sleeve in robot, d) the foot and leg of the humanoid robot

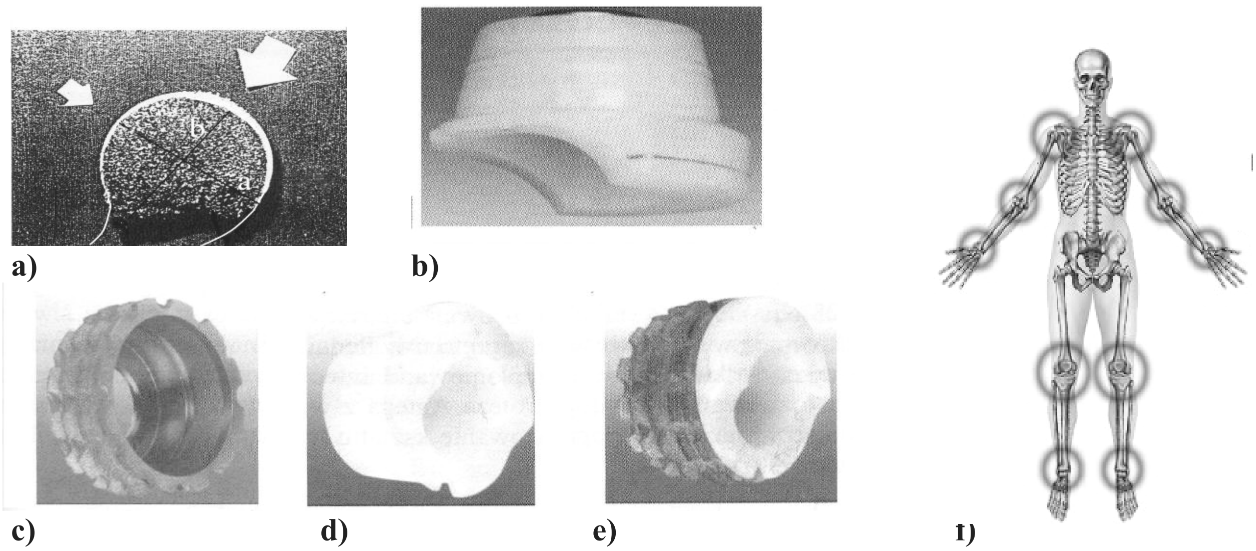


Fig. 2. Ellipsoidal shapes: a) different distribution of cartilage on the elliptical femoral head, b) one element polyethylene, ellipsoidal acetabulum; c, d, e) bipolar acetabulum with the ellipsoidal pad, f) possibilities of elliptical prosthesis localization

The lubrication region lying on the elliptical bearing surface may be calculated approximately and exactly. The approximately value of the half surface of the rotational ellipsoid with diameters *a, b*, $a<b$ can be calculated with the error about 1.061% for p=1.6075 from the following formula:

$$\Omega \cong 2\pi \left[\frac{a^{2p} + 2(ab)^p}{3} \right]^{\frac{1}{p}}. \tag{2}$$

Exactly value of the half-ellipsoid surface can be expressed from the following formula [11]:

$$\Omega_E = \iint\limits_{K(x,y)} \sqrt{1 + \left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial x} \right)^2} \, dxdy \,, \tag{3a}$$

$$z = f(x, y) = \frac{b}{a}\sqrt{a^2 - x^2 - y^2}, \quad K(x, y) : x^2 + y^2 = a^2. \tag{3b}$$

After calculations, we obtain following series:

$$\Omega_E = 2\pi b^2\left[1 - \sum_{n=1}^{\infty}\frac{\varepsilon^{2n}}{(2n-1)(2n+1)}.\right] = 2\pi b^2\left(1 - \frac{1}{3}\varepsilon^2 - \frac{1}{15}\varepsilon^4 - ...\right). \tag{3c}$$

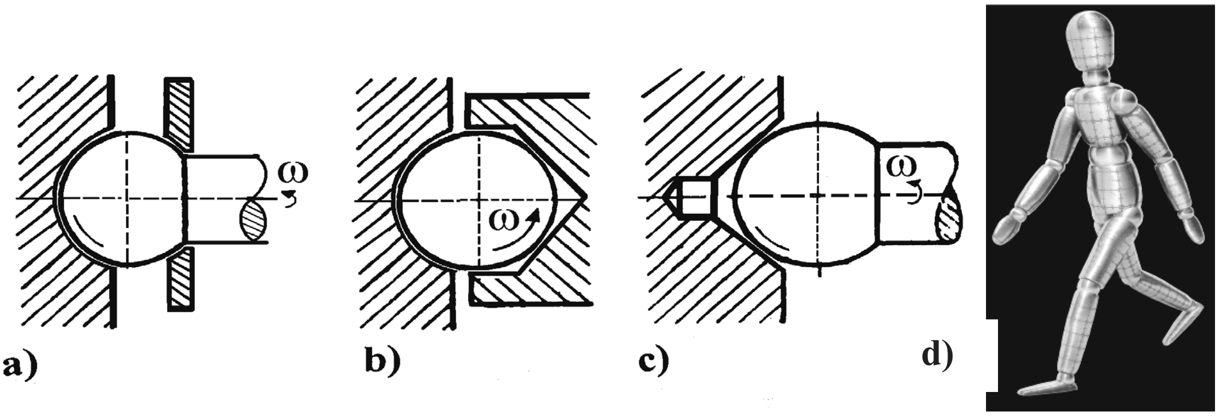
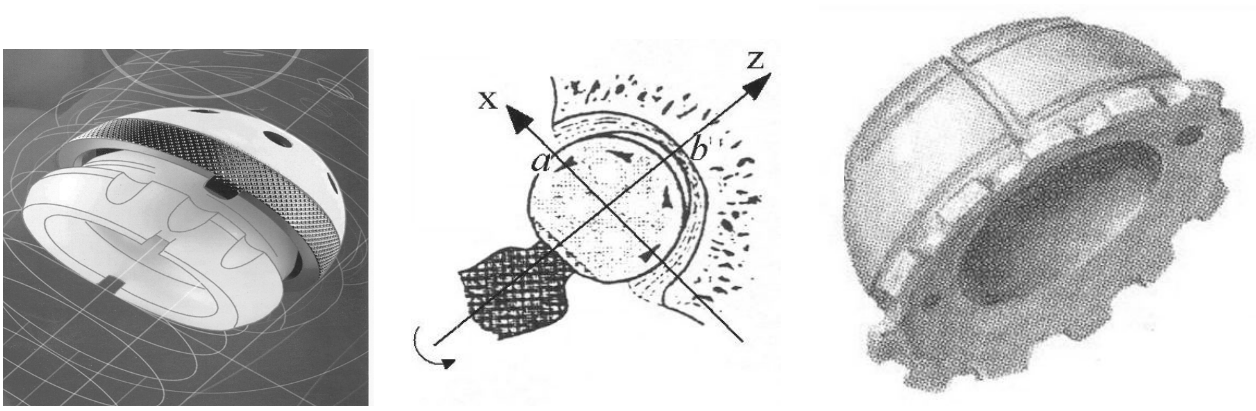


Fig. 3. Spherical and elliptical bearings: a) spherical journal together with a shaft and spherical sleeve, b) individual elliptical ball with conical sleeve, c) elliptical journal together with a shaft and conical sleeve, d) localization possibilities of the elliptical slide bearing between movable limbs occurring in humanoid robots

Figure 4 shows the complete elliptical prosthesis of human hip joint and elliptical bonehead.



a) b) c)

Fig. 4. Elliptical friction nod: a) complete elliptical prosthesis of human hip joint, b) elliptical bone head, c) the elliptical sleeve in the humanoid robot bearing

2. The points on the elliptical surface of slide bearings

This intersection describes the geometry of elliptical journal and sleeve. Therefore at first we define the coordinates of the point P (B,L) lying on the elliptical surface. Wide coordinate B of point P lying on the rotational elliptical surface denotes the angle between perpendicular line to the elliptical surface in point P and plain B=0 i.e. x0y (see Fig. 5a, b).

Length coordinate L of point P lying on the rotational elliptical surface denotes the angle between the projection r_1 of the basis vector **r** on the plain x0y and the plain L=0 (see Fig. 5a, b).

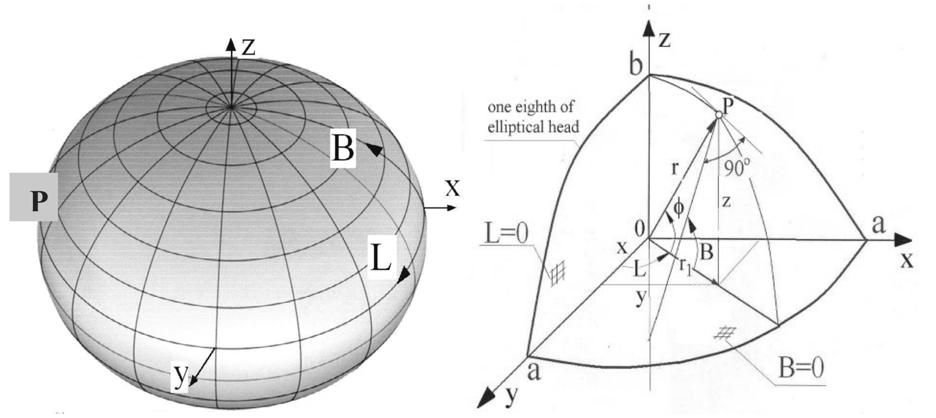


Fig. 5. Coordinates for the arbitrary point laying on the elliptical head surface: a) wide coordinate B and length coordinate L, b) connexion between Cartesian coordinates (x,y,z) of arbitrary point P laying on the elliptical surface and its wide B and length L coordinates

The relationship between coordinates (x,y,z) and (B,L) for arbitrary point laying on the elliptical surface are as follows (compare Fig. 4b):

$$x = \frac{a\cos B\cos L}{\sqrt{1-\varepsilon^2\sin^2 B}},\quad y = \frac{a\cos B\sin L}{\sqrt{1-\varepsilon^2\sin^2 B}},\quad z = \frac{b\sqrt{1-\varepsilon^2}\sin B}{\sqrt{1-\varepsilon^2\sin^2 B}},\quad r_1 = \frac{a\cos B}{\sqrt{1-\varepsilon^2\sin^2 B}}, \tag{4}$$

where $-\pi/2 \le B \le +\pi/2$, $-\pi \le L \le +\pi$.

3. Lubrication region as the sum of elliptical triangle

Now we define the field of elliptical triangle lying on the ellipsoid journal. Lubrication region consists of the sums of selected triangle surfaces. For example, Fig. 6a shows the one PQS of numerous elliptical triangles lying on the elliptical surface. Symbols angP, angQ, angS denote angles in vertexes P,Q,S respectively. Fig. 6b illustrates the real view of the triangle on the elliptical surface and its developed view. The field of the elliptical triangle has the following form:

$$\Omega_{pqs} = \left(\frac{2a+b}{3}\right)^2 \left(angP + angQ + angS - \pi\right). \tag{5}$$

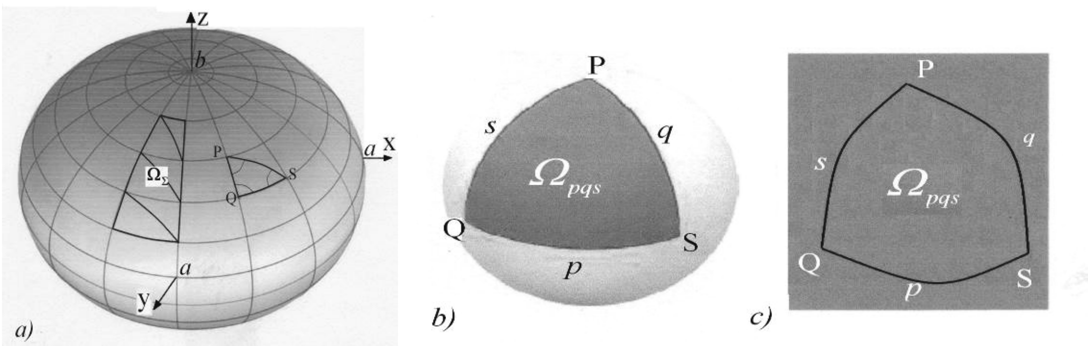
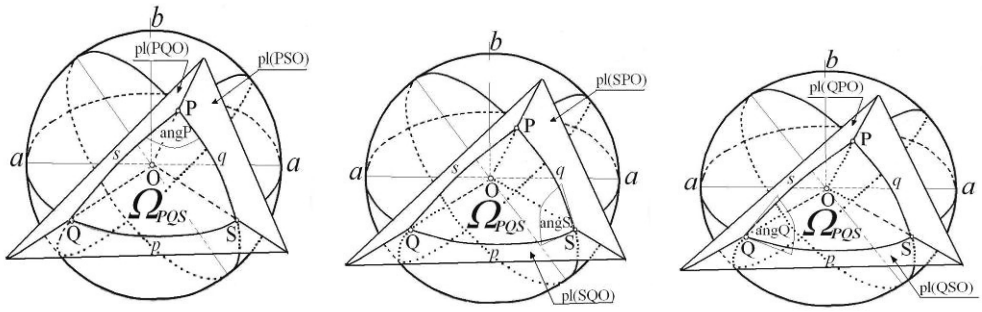


Fig. 6. Elliptical triangles: a) Lubrication region Ω_Σ lying on the elliptical slide bearing surface as a sum of elliptical triangles with the vertexes P,Q,S; b) Enlargements of elliptical triangle PQS on the elliptical surface; c) elliptical triangle on the plane

Unknown angP is the dihedral angle between plane pl(PQO) and pl(PSO).
Unknown angQ is the dihedral angle between plane pl(QPO) and pl(QSO).
Unknown angS is the dihedral angle between plane pl(SQO) and pl(SPO) see Fig. 7.



a) **b)** **c)**

Fig. 7. Elliptical triangle surface Ω_{pqs} and planes pl(PQO), pl(PSO), pl(QSO) to be created by the three points P,Q,O, and P,S,O and Q,S,O respectively, where O is the centre of ellipsoid

The three vertexes of the triangle have the following coordinates: $P(B_p,L_p)=P(x_p,y_p,z_p)$, $Q(B_q,L_q)=Q(x_q,y_q,z_q)$, $S(B_s,L_s)=S(x_s,y_s,z_s)$. By virtue of the formulae (4), the relationships between coordinales x,y,z and B,L for points P,Q,S are as follows:

$$x_p = \frac{a\cos B_p \cos L_p}{\sqrt{1-\varepsilon^2 \sin^2 B_p}}, \quad y_p = \frac{a\cos B_p \sin L_p}{\sqrt{1-\varepsilon^2 \sin^2 B_p}}, \quad z_p = \frac{b\sqrt{1-\varepsilon^2}\sin B_p}{\sqrt{1-\varepsilon^2 \sin^2 B_p}}, \tag{6}$$

$$x_q = \frac{a\cos B_q \cos L_q}{\sqrt{1-\varepsilon^2 \sin^2 B_q}}, \quad y_q = \frac{a\cos B_q \sin L_q}{\sqrt{1-\varepsilon^2 \sin^2 B_q}}, \quad z_q = \frac{b\sqrt{1-\varepsilon^2}\sin B_q}{\sqrt{1-\varepsilon^2 \sin^2 B_q}}, \tag{7}$$

$$x_s = \frac{a\cos B_s \cos L_s}{\sqrt{1-\varepsilon^2 \sin^2 B_s}}, \quad y_s = \frac{a\cos B_p \sin L_s}{\sqrt{1-\varepsilon^2 \sin^2 B_s}}, \quad z_s = \frac{b\sqrt{1-\varepsilon^2}\sin B_s}{\sqrt{1-\varepsilon^2 \sin^2 B_s}}. \tag{8}$$

If eccentricity ε tends to zero, then above formulae will valid for spherical triangles. The angles at the vertexes P,Q,S of the elliptical triangle are as follows:

$$angP = arccos\left(\frac{\left| A_{pq}A_{ps} + B_{pq}B_{ps} + C_{pq}C_{ps} \right|}{\sqrt{A_{pq}^2 + B_{pq}^2 + C_{pq}^2}\sqrt{A_{ps}^2 + B_{ps}^2 + C_{ps}^2}} \right), \tag{9}$$

$$angQ = arccos\left(\frac{\left| A_{qp}A_{qs} + B_{qp}B_{qs} + C_{qp}C_{qs} \right|}{\sqrt{A_{qp}^2 + B_{qp}^2 + C_{qp}^2}\sqrt{A_{qs}^2 + B_{qs}^2 + C_{qs}^2}} \right), \tag{10}$$

$$angS = arccos\left(\frac{\left| A_{sp}A_{sq} + B_{sp}B_{sq} + C_{sp}C_{sq} + \right|}{\sqrt{A_{sp}^2 + B_{sp}^2 + C_{sp}^2}\sqrt{A_{sq}^2 + B_{sq}^2 + C_{sq}^2}} \right), \tag{11}$$

where:

$$A_{pq} \equiv A_{qp} = \begin{vmatrix} y_p & z_p \\ y_q & z_q \end{vmatrix}, \quad B_{pq} \equiv B_{qp} = \begin{vmatrix} z_p & x_p \\ z_q & x_q \end{vmatrix}, \quad C_{pq} \equiv C_{qp} = \begin{vmatrix} x_p & y_p \\ x_q & y_q \end{vmatrix}, \tag{12}$$

$$A_{sq} \equiv A_{qs} = \begin{vmatrix} y_s & z_s \\ y_q & z_q \end{vmatrix}, \quad B_{sq} \equiv B_{qs} = \begin{vmatrix} z_s & x_s \\ z_q & x_q \end{vmatrix}, \quad C_{sq} \equiv C_{qs} = \begin{vmatrix} x_s & y_s \\ x_q & y_q \end{vmatrix}, \tag{13}$$

$$A_{ps} \equiv A_{sp} = \begin{vmatrix} y_p & z_p \\ y_s & z_s \end{vmatrix}, \quad B_{ps} \equiv B_{sp} = \begin{vmatrix} z_p & x_p \\ z_s & x_s \end{vmatrix}, \quad C_{ps} \equiv C_{sp} = \begin{vmatrix} x_p & y_p \\ x_s & y_s \end{vmatrix}. \tag{14}$$

4. Pressure measurements and capacity calculation

The measured pressure values p_1, p_2, p_3, ... in vertexes of particular triangles included in lubrication region Ω_Σ are presented in Fig. 8a, b, c.

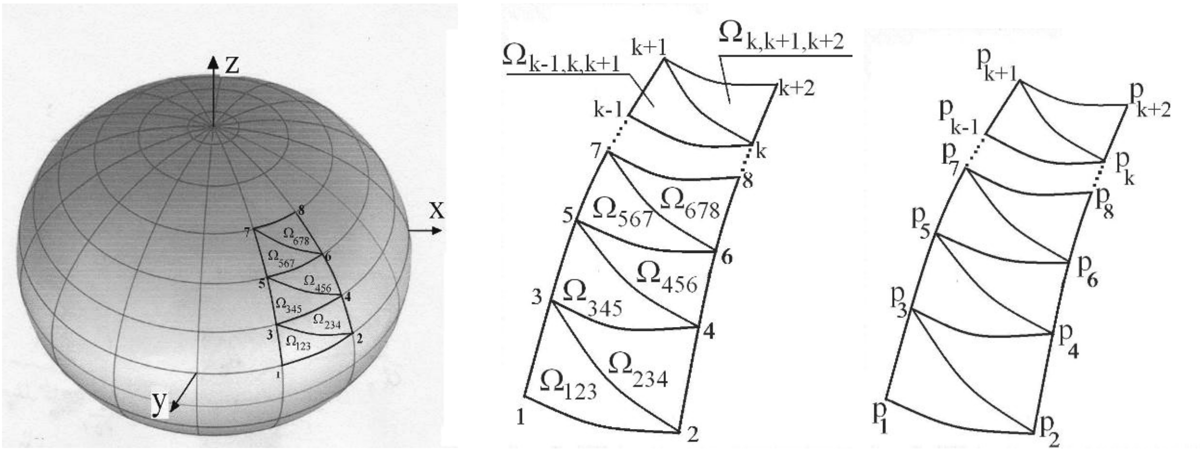


Fig. 8. Lubrication region Ω_Σ :a) localization on the elliptical surface, b) region Ω_Σ as a sum of elliptical triangles, c) measured pressure values in the vertexes of particular elliptical triangles

The load carrying capacity value has the following form:

$$C = (p_\Sigma) \cdot \Omega_\Sigma, \tag{15}$$

where the average value of the pressure in lubrication region is formulated as the arithmetic mean of pressure values in particular vertexes of elliptical triangles and has following form:

$$p_\Sigma = \frac{1}{n+2} \sum_{k=1}^{n+2} p_k, \quad n = 2,4,6,8,... \ . \tag{16}$$

The lubrication region is defines as the sum of particular elliptical triangles in following form:

$$\Omega_\Sigma \equiv \sum_{k=1}^{n} \Omega_{k,k+1,k+2} = \Omega_{123} + \Omega_{234} + \Omega_{345} + \Omega_{456} + \Omega_{567} + \Omega_{567} + ... + \Omega_{k-1,k,k+1} + \Omega_{k,k+1,k+2} + ...$$
$$for \quad n = 2,4,6,8,.... \tag{17}$$

5. Load carrying capacity in analytical form

Taking into account surface integral, we can show the load carrying capacity in following form:

$$C = \iint_{\Omega_\Sigma} p(x,y,z) d\Omega_\Sigma. \tag{18}$$

Pressure function p(x,y,z) is presented in Fig. 9 and determined in elliptical surface Ω_Σ.

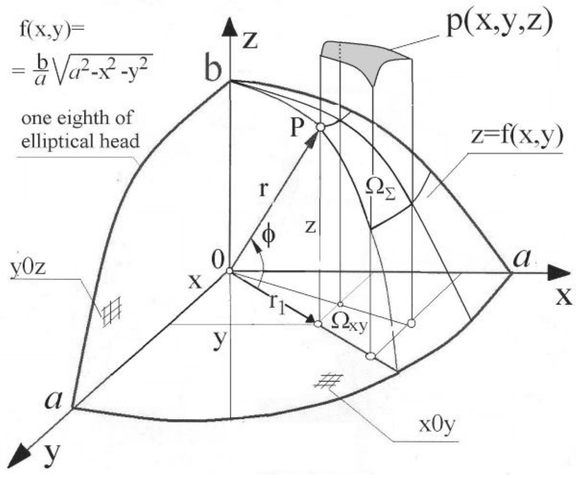


Fig. 9. Integral surface implementation

Transformation of the formula (18) from surface into double integral tends to the following formula presenting the load carrying capacity:

$$C = \iint_{\Omega_{xy}} \left\{ p[\,x, y, z = f(x, y)\,] \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2} \right\} dxdy. \tag{19}$$

Double integral is determined on the region $\Omega_{xy}(x,y)$ lying on the plane x0y and illustrated in Fig. 8. Function z=f(x,y) is defined in Eq.(3b) and presented in Fig. 9. It is the elliptical surface.

6. Conclusions

In this paper is presented the method of lubrication region calculation on the elliptical surfaces lying on the slide elliptical journal bearing applied in robots. The lubrication region consists of elliptical triangles. The values of surfaces of elliptical triangles are derived.

Taking into account the total lubrication surface and mean arithmetic measured hydrodynamic pressure, the formula for capacity calculation is presented.

The functionalities of elliptical human hip, humeral, elbow and other joints and elliptical hip, humeral elbow human prosthesis as well elliptical slide bearings occurring in humanoid robots are in this paper compared.

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