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THE ANT ALGORITHM FOR SOLVING THE ASSIGNMENT OF VEHICLES TO TASKS IN THE MUNICIPAL SERVICES COMPANIES

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Abstract

In this article, the method of designating the assignment vehicles to tasks in the municipal services companies was described. This method consists of two stages, which were discussed. The first stage of the method of designating the assignment relies on determining the minimum route of the assignment, which consists of all the tasks. The second stage of the assignment is the stage of realization the route by the vehicles. In order to allocate the vehicles to tasks a concept of the elementary route was introduced. The problem of the assignment vehicles to tasks in the municipal services companies was characterized. The mathematical formulation of the assignment vehicles to the tasks was presented. The function of criterion determining the minimum route of the assignment was designated. The limit of driving time of a driver, the limit of the working time of a driver, the limit of realization all tasks, the limit of payload of the vehicle were described. The minimum route of the assignment was designated be means of the ant algorithm. The method was verified in programming language C #. The process of verification was divided into two stages. In the first stage, the best parameters of the ant algorithm were designated. In the second stage, the algorithm was started with these parameters and the result was compared with the random search algorithm. The random search algorithm generates 2000 routes and the best result is compared with the genetic algorithm. The influence of the parameters on quality of the results was examined.

Keywords: *municipal services companies, assignment, optimization, ant algorithm, verification*

1. Introduction

One of the main problems in the municipal services companies is the vehicle routing [1, 2]. The assignment of vehicles to tasks is the second stage of the method designating the route of vehicles in the municipal services companies proposed by authors of this publication. The first stage of designating the tasks is not presented in this paper. It is assumed that all tasks are known. The task is designated when we know the route of the vehicles, which collect the waste, the point of beginning of the route, the points of the route and the point of ending of the route.

The problem of assigning vehicles to tasks in the municipal services companies appears when the vehicle starts realization the first task and leaves the base. We can ask which task will be realized as the first in the route. The problem of the assignment appears again when the vehicle-ended realization the current task, unloaded the waste on the dumping ground and we can ask again, which the next task will be realized in the route. The assumption is that all the tasks must be realized in the minimum route in a working day. We have three points of reference: the base, the task and the dumping ground, which are the elements of a transport system. These points determine the routes of the vehicles. To the task can be reached in two ways, directly from the dumping ground or through the base. The main aim of the assignment is to designate the next task in the route and the minimum route between two tasks. One should remember that on the task the limit of the working time of a driver, the limit of driving time of a driver, the limit of time of all tasks realization are imposed. The method of designating the assignment is described in two stages. In the first stage, the minimum route of the assignment, which consists of all tasks, is designated and in the second stage, this route is realized by the vehicles.

3. The mathematical formulation of the assignment vehicles to the tasks

The first stage of the method of designating the assignment relies on determining the minimum route of the assignment, which consists of all the tasks. In order to present, the mathematical model of the minimum route of the assignment following data has been specified [9]:

- $W^{Zp} = \{1,...,i,...,\overline{W^{zp}}\}$ – the set of points of loading,
- $W^{Zk} = \{1,...,j,...,\overline{W^{zk}}\}$ – the set of points of loading, the points where the vehicle leave the route,
- $W^{Zad} = \{1,...,za,...,\overline{W^{zad}}\}$ – the set of the numbers of tasks,
- $W^{\boldsymbol{B}} = \{1,...,b,...,\overline{W^{b}}\}$ – the set of the bases,
- $W^{W} = \{1,...,k,...,\overline{W^{w}}\}$ – the set of points of unloading of interpretation: the dumping ground,
- $W^{P} = \{1,...,p,...,\overline{W^{p}}\}$ – the set of vehicles' numbers,
- $W^{N} = \{1,...,n,...,\overline{W^{n}}\}$ – the set of drivers' numbers,
- $\mathbf{W} = [\,w(k,i)\,]$ – the matrix of the distance between k -this point of unloading and i – this point of loading,
- $\mathbf{BZ} = [\,bz(b,i)\,]$ – the matrix of the distance between b – this base and i – this point of loading,
- $\boldsymbol{WB} = [\,wb(k,b)\,]$ – the matrix of the distance between k – this point of unloading and b – this base,
- $\boldsymbol{ZW} = [\,zw(j,k)\,]$ – the matrix of the distance between j – this point where the vehicle leave the route and k – this point of unloading,
- $\boldsymbol{T1} = [\,t1(p,n,j,k)\,]$ – the matrix of travel times between j – this point where the vehicle leave the route and k – this point of unloading for p – this vehicle and n – this driver,
- $\boldsymbol{T2} = [\,t2(p,n,b,i)\,]$ – the matrix of travel times between b – this base and i – this point of loading for p – this vehicle and n – this driver,
- $\boldsymbol{T3} = [\,t3(p,n,k,b)\,]$ – the matrix of travel times between k – this point of unloading and b – this base for p – this vehicle and n – this driver,
- $\boldsymbol{T4} = [\,t4(p,n,k,i)\,]$ – the matrix of travel times between k – this point of unloading and i – this point of loading for p – this vehicle and n – this driver,
- $\boldsymbol{T5} = [\,t5(p,n,k)\,]$ – the matrix of times of unloading of a vehicle in k – this point of unloading for p – this vehicle and n – this driver,
- $\boldsymbol{T6} = [\,t6(p,n,za)\,]$ – the matrix of times of loading of a vehicle in za – this task for p – this vehicle and n – this driver,
- $\boldsymbol{T7} = [\,t7(p,n,k)\,]$ – the matrix of waiting time for unloading in k – this point of unloading for p – this vehicle and n – this driver,
- $\phi(p)$ payload p – of this vehicle,
- $\theta(za)$ – the volume of the task,
- T^{rest} – statutory resting time on the route,
- T^{dop1} – the permitted driving time,
- T^{dop} – the permitted working time of driver,
- ΔT – the range of realization of the all tasks.

The main task is to find the following decision variables:

- $\mathbf{XBZ} = [xbz(b,i)]$, $xbz(b,i) \in \{0,1\}$ if $xbz(b,i) = 1$ then the route between b – this base and i – this point of loading is used. Otherwise $xbz(b,i)=0$,

- $\mathbf{X}=[x(k,i)]$, $x(k,i)\in\{0,1\}$ if $x(k,i)=1$ then route between k – this point of unloading and i – this point of loading is used. Otherwise $x(k,i)=0$,
- $\mathbf{XWB}=[xwb(k,b)]$, $xwb(k,b)\in\{0,1\}$ if $xwb(k,b)=1$ then route between k – this point of unloading and b – this base is used. Otherwise $xwb(k,b)=0$,
- $\mathbf{XZW}=[xzw(j,k)]$, $xzw(j,k)\in\{0,1\}$, an auxiliary variable defining the route of the assignment between the point where the vehicle leaves the route and the point unloading, if $xzw(j,k)=1$ then route between j – this point where the vehicle leave the route and k – this point of unloading is used. Otherwise $xzw(j,k)=0$.

To the function of criterion determining the minimum route of the assignment saved by the formulation

$$F(\mathbf{XBZ},\mathbf{X},\mathbf{XWB},\mathbf{XZW})=\sum_{b\in W^B}\sum_{i\in W^{Zp}} xbz(b,i)\cdot bz(b,i)+\sum_{k\in W^W}\sum_{i\in W^{Zp}} x(k,i)\cdot w(k,i)+$$
$$+\sum_{k\in W^W}\sum_{b\in W^B} xwb(k,b)\cdot wb(k,b)+\sum_{j\in W^{Zk}}\sum_{k\in W^W} xzw(j,k)\cdot zw(j,k)\to \min, \tag{1}$$

will take the minimum value.

- Constraints take the form of:
- the limit of driving time of a driver: the sum of travel times to the task both from the base and directly from the point of unloading, the sum of travel times between the point where the vehicle leaves the route and the point of unloading and the sum of travel times between the point of unloading and the base:

$$\forall n\in N, \forall p\in P,$$
$$\sum_{b\in W^B}\sum_{i\in W^{Zp}} xbz(b,i)\cdot t2(p,n,b,i)+\sum_{k\in W^W}\sum_{i\in W^{Zp}} x(k,i)\cdot t4(p,n,k,i)+\sum_{k\in W^W}\sum_{b\in W^B} xwb(k,b)\cdot$$
$$\cdot t3(p,n,k,b)+\sum_{j\in W^{Zk}}\sum_{k\in W^W} xzw(j,k)\cdot t1(p,n,j,k)\le T^{dop1}, \tag{2}$$

- the limit of the working time of a driver: the driving time of a driver, the sum of loading times, the sum of unloading times, the sum of expected times of unloading and resting time on the route, in order to reduce the volume of the formula the variable of the driving time of a driver takes the mark T^{drive}:

$$\forall n\in N, \forall p\in P,$$
$$T^{drive}+\sum_{b\in W^B}\sum_{i\in W^{Zp}}\sum_{za\in W^{Zad}} xbz(b,i)\cdot t6(p,n,za)+\sum_{k\in W^W}\sum_{i\in W^{Zp}}\sum_{za\in W^{Zad}} x(k,i)\cdot t6(p,n,za)$$
$$+\sum_{j\in W^{Zk}}\sum_{k\in W^W} xzw(j,k)\cdot t5(p,n,k)+\sum_{j\in W^{Zk}}\sum_{k\in W^W} xzw(j,k)\cdot t7(p,n,k)+T^{rest}\le T^{dop}, \tag{3}$$

- the limit of realization all tasks: the task must be realized within a given range of time, T^{work} – the working time of a driver:

$$T^{work}\le \Delta T, \tag{4}$$

- the limit of payload of the vehicle: the vehicle can realize the task if payload of this vehicle is greater or equal to the size of this task:

$$\forall za\in W^{Zad}, \forall p\in P,$$
$$\theta(za)\le \varphi(p). \tag{5}$$

The second stage of the assignment is the stage of realization the route by the vehicles. In order to allocate the vehicles to tasks a concept of the elementary route needs to be introduced. The elementary route is a part of the minimum route. The beginning and the ending of the elementary route is always the base. For each route, all aforementioned limits are checked. The next step of the assignment is the step in which we add the elementary routes together designating the set of tasks for each vehicle. In case of exceeding the time of realization all tasks for added routs, other elementary routes are realized by other vehicles.

4. The algorithm of designating the minimum route of the assignment

The minimum route of the assignment was designated be means of the ant algorithm. At the beginning, we must determine the input parameters of the algorithm: a size of the population, the number of iterations, parameters characterizing the algorithm e.g. ρ – a factor pheromone. The ant algorithm is repeated until the stop condition is achieved. The stop condition in this algorithm is determined by a predetermined number of generations (iterations).

In order to implement the ant algorithm we assume that route of the ants consists of the elements such as the task and the base [8]. The dumping ground is not treated as the separate point of route because it is the ending point of the task. Ants can make a choice and they either move to the base and then to the task, or directly to the task. The elements of the route of the ants were called a common term as the point of the assigned route.

The starting point of each ant is in the base. The further route of the ant and thereby the choice of the next point of the assigned route is selected from the specified probability [3-5, 7]. Each point of the route is visited only once. In order to define the ant algorithm the set of the points of the assigned route was designated $W^{Tp} = \{1,...,y,z,...,\overline{W^{tp}}\}$, where y,z – another elements of set W^{Tp}, $y \neq z$. The probability of selection of the route from y to z by the ant takes the form:

$$P^{mr}{}_{yz}(t) = \begin{cases} \dfrac{\left[\tau_{yz}(t)\right]^{\alpha} \cdot \left[\eta_{yz}(t)\right]^{\beta}}{\sum_{l \in \Omega^{mr}} \left[\tau_{yl}(t)\right]^{\alpha} \cdot \left[\eta_{yl}(t)\right]^{\beta}}, & z \in \Omega^{mr}, \\ 0, & z \notin \Omega^{mr}, \end{cases} \tag{6}$$

where:

$\tau_{yz}(t)$ – the intensity of pheromone trail on the section between y – this point of the route and z – this point of the route,

η_{yz} – the heuristic information e.g. $\eta_{yz} = \dfrac{1}{w(y,z)}$, where $w(y,z)$ a distance between y – this point of the route and z – this point of the route,

α, β – parameters determining the effect of pheromones and the heuristic information on the behaviour of ants,

Ω^{mr} – the set of vertices which hasn't been yet visited by the ant, where l – the element of set Ω^{mr}.

The route of an ant ends after realization all the point of route. The ending point is also in the base. Then another ant begins the process of creating new routes. In the final step of the iteration, the pheromone trail is updated. In order to update the pheromone in the route the ant – cycle was used as the most efficient version of the ant algorithms [6]. The pheromone trail is updated after the implementation of all the routes by ants. In the first iteration, the pheromone trail is equally strong in all connections between the points, equal to τ_0. In other iterations, the pheromone trail we can calculate:

$$\tau_{yz}(t+1) = (1-\rho) + \sum_{mr=1}^{Mr} \Delta\tau_{yz}{}^{mr}(t), \tag{7}$$

where:

mr – another ant in the population $mr \in MR$, $MR = \{1,...,mr,...\overline{Mr}\}$ – the set of the all ants,

ρ – a factor pheromone, $(0 < \rho \le 1)$,

$\tau_{yz}(t+1)$ – the strengthening of the pheromone in $t+1$ – this iteration, we assume τ_0 for the first iteration on all connections.

The partially strengthening of the pheromone in t-this iteration takes the form:

$$\Delta\tau_{yz}^{mr}(t) = \begin{cases} \dfrac{1}{L^{mr}(t)}, \\ 0, \end{cases} \tag{8}$$

where $L^{mr}(t)$ – the length of the all route realized in t–this iteration by mr – this ant, if the route (y,z) was realized by mr – this ant then the value $\Delta\tau_{yz}^{mr}(t)$ is equal to $\dfrac{1}{L^{mr}(t)}$, otherwise 0.

The number of the point of the assigned route is equal [8] to $2za+1$ where za – the number of tasks. This number takes into account the situation where each arrival to the task is preceded by the visit of the ant in the base.

5. Verification of the method

The method was verified using programming language C #. Verification of the method takes place in the phase of designating the minimum route of the assignment and relies on comparing the result of the ant algorithm with the result of the random search algorithm. The random search algorithm generates 2000 routes and the best result is compared with the genetic algorithm. The method was verified for 30 tasks. The ant algorithm was started 50 times so the result is the average of all starts. The number of iterations is equal to 200; the population is 100. The parameters of the algorithm take the values: $\alpha = 1{,}3{,}5{,}10{,}20$; $\beta = 0{,}5;1{,}0;5$; $\rho = 0{,}2;0{,}4;0{,}6;0{,}8$.

The 60 combinations of these parameters were checked and the best combination of parameters, where the algorithm gave the best solution, was found. Combination of parameters was shown in Tab. 1.

Tab. 1. Combination of parameters in the ant algorithm

Lp.	α	β	ρ	Lp.	α	β	ρ	Lp.	α	β	ρ
1	1	0.5	0.2	21	1	1	0.2	41	1	5	0.2
2	1	0.5	0.4	22	1	1	0.4	42	1	5	0.4
3	1	0.5	0.6	23	1	1	0.6	43	1	5	0.6
4	1	0.5	0.8	24	1	1	0.8	44	1	5	0.8
5	3	0.5	0.2	25	3	1	0.2	45	3	5	0.2
6	3	0.5	0.4	26	3	1	0.4	46	3	5	0.4
7	3	0.5	0.6	27	3	1	0.6	47	3	5	0.6
8	3	0.5	0.8	28	3	1	0.8	48	3	5	0.8
9	5	0.5	0.2	29	5	1	0.2	49	5	5	0.2
10	5	0.5	0.4	30	5	1	0.4	50	5	5	0.4
11	5	0.5	0.6	31	5	1	0.6	51	5	5	0.6
12	5	0.5	0.8	32	5	1	0.8	52	5	5	0.8
13	10	0.5	0.2	33	10	1	0.2	53	10	5	0.2
14	10	0.5	0.4	34	10	1	0.4	54	10	5	0.4
15	10	0.5	0.6	35	10	1	0.6	55	10	5	0.6
16	10	0.5	0.8	36	10	1	0.8	56	10	5	0.8
17	20	0.5	0.2	37	20	1	0.2	57	20	5	0.2
18	20	0.5	0.4	38	20	1	0.4	58	20	5	0.4
19	20	0.5	0.6	39	20	1	0.6	59	20	5	0.6
20	20	0.5	0.8	40	20	1	0.8	60	20	5	0.8

Verification of the method consists of two stages. In the first stage, the best parameters of algorithm are designated. In the second stage, the algorithm is started with these parameters and the result is compared with the random search algorithm. In the first stage the influence of parameters α, β, ρ on quality of the results are examined. The influence of parameters α, β, ρ on quality of the results were shown in Fig. 1-3.

The best result was designated for the parameters: $\alpha = 1$, $\beta = 1$, $\rho = 0.2$. In the second stage, the algorithm was started with these parameters and the result was compared with the random search algorithm. The result was shown in Fig. 4.

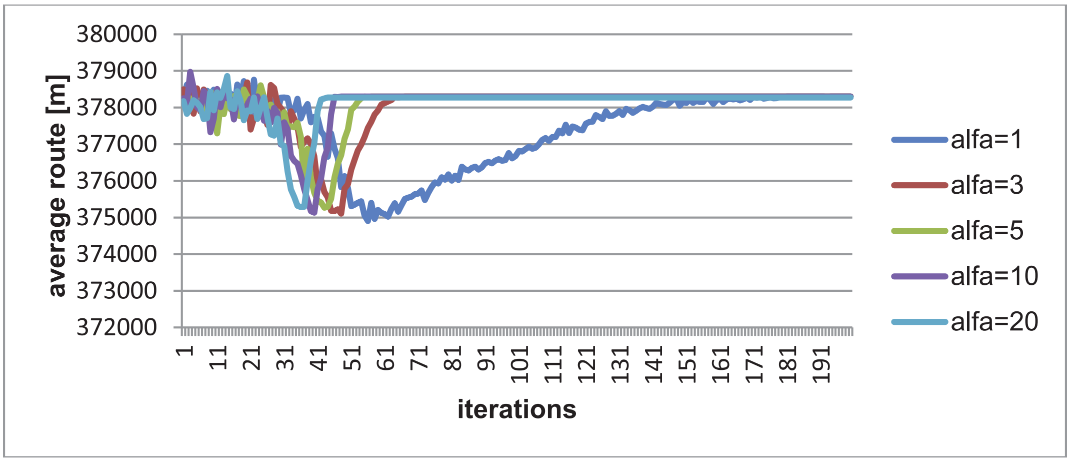

Fig. 1. The influence of the parameter α on quality of the results

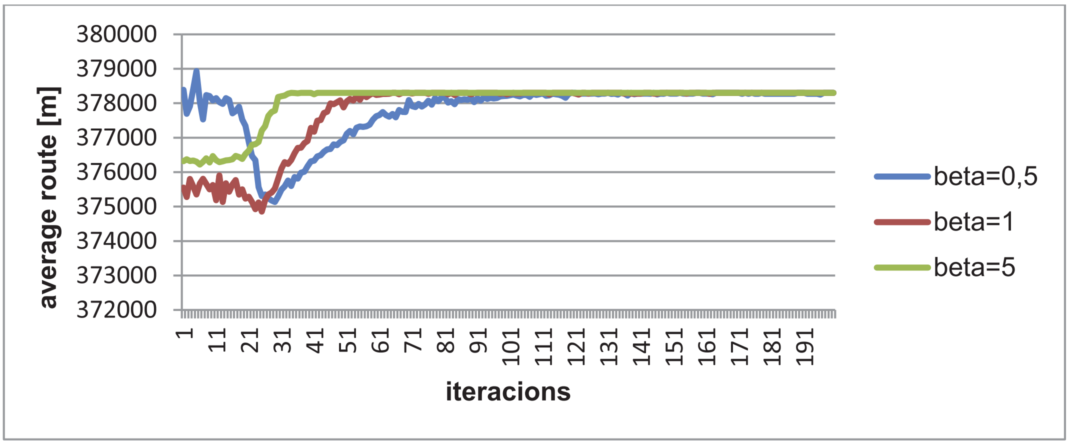

Fig. 2. The influence of the parameter β on quality of the results

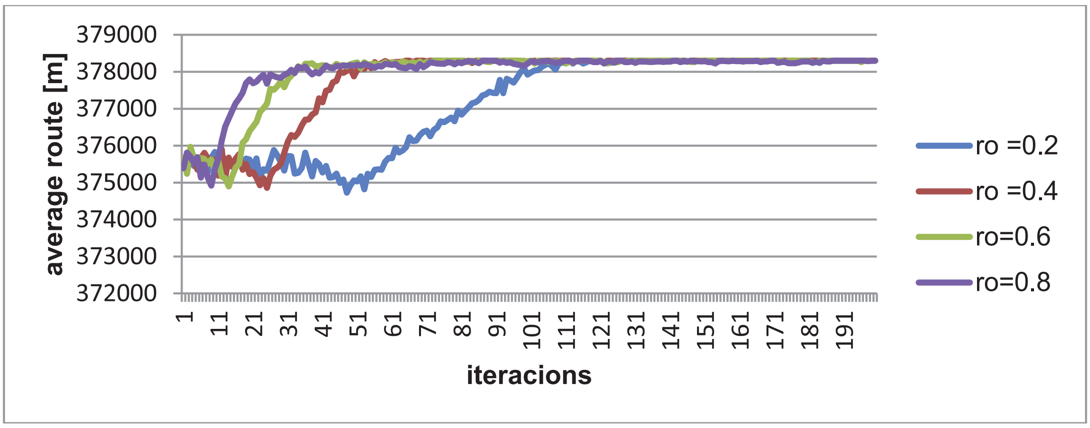

Fig. 3. The influence of the parameter ρ on quality of the results

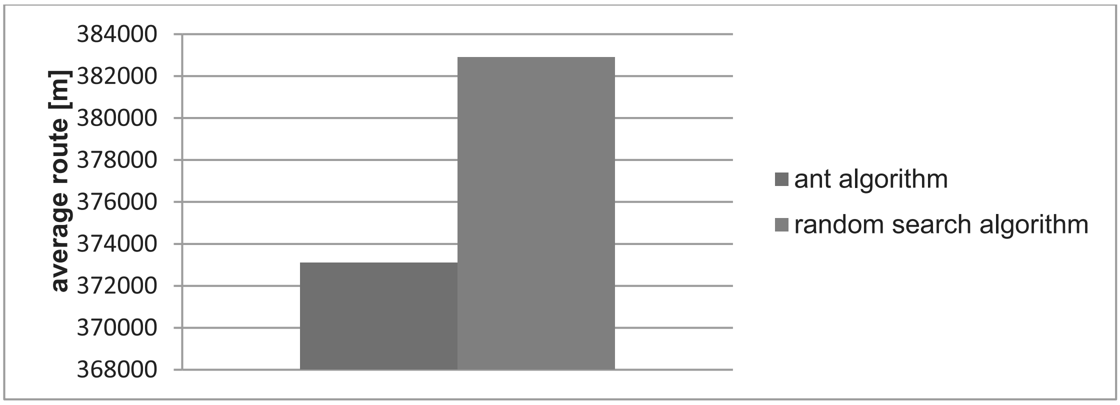

Fig. 4. Comparison of algorithms

6. Summary

Presented method of designating the assignment vehicles to tasks was verified in programming language C #. The most important stage of this method is the stage of designating the minimum route of the assignment, which consists of tasks. The length of the generated route influences the number of the elementary routes; therefore, verification of the method must be done in this stage. After analysing the results of the ant algorithm, we have come to conclusion that with the increase of the parameter ρ algorithm reaches the minimum faster. In each case with the increase of the parameter α , β we can observe the faster convergence of the algorithm to a specified value.

The result is a combination of three parameters and it is very important to perform the stage of designating these parameters.

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