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SOFTWARE FOR THE DYNAMIC ANALYSIS OF MACHINES PLACED ON ELASTIC PADS

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Abstract

Software DYNMACH has been designed for the dynamic analysis (vibrations) of machines placed on elastic foundation pads assuming non-linear foundation characteristics. Floating platforms of marine engines are designed for noise and vibration isolation between ship hull and engine. Excessive loadings acting on ship hull can be also eliminated by elastic foundation. Analysis of that kind of marine machines is the main target of presented software. It has been assumed that the machines are modeled as rigid bodies while the foundation is treated as a set of elements of non-linear elastic-damping characteristics. Software DYNMACH is the implementation of the Rigid Finite Element Method. A machine can be modeled as a single rigid finite element composed of an arbitrary number of elementary rigid bodies. The following rigid body types are available in the library of the program: a plate, a brick, and a full cylinder. Elastic elements used for foundation modeling can have non-linear stiffness and damping characteristics. These characteristics can be assumed as functions of the foundation displacement or the expected excitation frequency. A built system can be loaded by an arbitrary number of static forces. These static forces are used for a preliminary calculation of strains and the centre of equilibrium, as well as for determination of the stiffness and damping coefficients of the non-linear elastic damping elements in the equilibrium position. Program DYNMACH can calculate the frequencies and modes of natural vibration of the loaded system, as well as calculates of system responses to dynamic excitations. Assuming only small vibration amplitudes the dynamic behaviour of the system can be considered as linear, however, this linear behaviour is imposed on the non-linear equilibrium conditions obtained from the solution of the coupled static problem. Database of different types of rubber machines couplings has been build-in the DYNMACH software.

Keywords: *marine engine, elastic foundation of power plant, rigid finite element, elastic pads, rubber couplings, non-linear characteristics of rubber machine elements*

1. Introduction

Some of marine propulsion systems are located on the floating platforms and/or on the elastic foundations [1]. Floating platforms of marine engines are performed for noise and vibration isolation between ship hull and the engines. Excessive loadings acting on ship hull, like underwater explosions or ship's grounding, can be also eliminated by elastic foundation and the propulsion system is protected from damages.

Dynamic characteristics of machines placed on elastic foundation should be carefully analysed. Especially resonance curve should be determined. That kind of investigation should be performed during designing process as well as after all serious accidents. Accidents and operation time (for aged systems) may have significant impact on the elastic (rubber) pads characteristics. Commercial Finite Element Method (FEM) [3] software has possibility of that kind of analysis, but there are some disadvantages. Firstly, realistic data of the non-linear, dynamic stiffness characteristics of rubber pads and couplings are difficult to obtain [4]. Secondly, machines' modelling is time consuming, in general FEM software. The stiffness difference between engine body and elastic foundation is so high that the elements of the marine propulsion systems might be treated as rigid body.

Software *DYNMACH* has been designed for the dynamic analysis (vibrations) of machines placed on elastic foundation pads assuming non-linear foundation characteristics. It has been assumed that the machines are modeled as rigid bodies while the foundation is treated as a set of elements of non-linear elastic-damping characteristics. Machines can be modelled as a single rigid finite element composed of an arbitrary number of elementary rigid bodies. Software *DYNMACH* can calculate the frequencies and modes of natural vibration of the loaded system, as well as calculates of system responses to dynamic excitations. Database of different types of rubber machines couplings has been build-in the *DYNMACH* software.

2. General assumptions

A machine can be modelled as a single rigid finite element composed of an arbitrary number of elementary rigid bodies. The following rigid body types are available in the library of the program: a plate, a brick, and a full cylinder of the longitudinal axis parallel to the *x*, *y*, or *z*-axis of the global co-ordinate system.

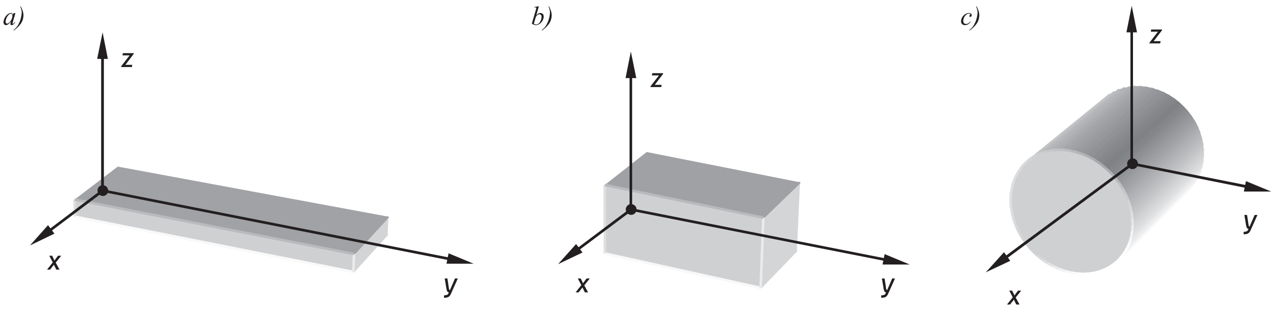

Fig. 1. Elementary rigid bodies available in the program: a) a plate, b) a brick, c) a cylinder

Elastic damping elements used for foundation modelling can have non-linear stiffness and damping characteristics [4]. These characteristics can be assumed as functions of the foundation displacement or the expected excitation frequency and are approximated in the software *DYNMACH* as third order (cubic) polynomials. It is possible to join an arbitrary number of the elastic damping elements to a single rigid finite element in an arbitrary number of points. However, it must be noted that the local coordinate system of each elastic damping element must be parallel to the global coordinate system.

A built system modelled by rigid finite elements can be loaded in the program *DYNMACH* by an arbitrary number of static forces arbitrary oriented in the 3D space. These static forces are used for a preliminary calculation of strains and the centre of equilibrium, as well as for determination of the stiffness and damping coefficients of the non-linear elastic damping elements in the equilibrium position.

Software *DYNMACH* allows calculation of the frequencies and modes of natural vibration of the loaded system, as well as calculation of system responses to dynamic excitations. Assuming only small vibration amplitudes the dynamic behaviour of the system can be considered as linear, however, this linear behaviour is imposed on the non-linear equilibrium conditions obtained from the solution of the coupled static problem. The total number of dynamic forces and their orientation in the 3D space, the same as in the case of the static forces, can be arbitrary.

In the case of the static behaviour calculation the results contain the displacement components in each rigid finite element and the strains in each elastic damping element. In the case of the dynamic behaviour calculation (eigen-value problem) the results contain the natural frequencies and the normalised modes of natural vibration [2]. In the case of the dynamic behaviour calculation (forced vibration) the results contain of the displacement amplitude and phase calculated for the centre of equilibrium of each rigid finite element. It is also possible to obtain the strain amplitude and phase calculated for the attachment points of elastic damping elements [2].

3. Characteristic matrixes

One of the advantages of the Rigid Finite Element Method is the diagonal form of the mass matrix. For a single rigid finite element the mass matrix can be presented in the following form:

$$\mathbf{M} = \text{diag}[m, m, m, J_{gx}, J_{gy}, J_{gz}], \tag{1}$$

where:
m – mass of the element,
J_{gx}, J_{gy}, J_{gz} – main inertia moments.

The total mass of the rigid finite element is computed by the use of the formulas for volume calculation and is expressed as a sum of the constituent masses of the rigid bodies composing the element. On the other hand the main inertia moments of the rigid finite element are the eigen-values of a matrix built out of the inertia and deviation moments of the constituent masses in the local coordinate system as:

$$\mathbf{J}_{gi} = \begin{bmatrix} I_{sx} & -D_{sz} & -D_{sy} \\ -D_{sz} & I_{sy} & -D_{sx} \\ -D_{sy} & -D_{sx} & I_{sz} \end{bmatrix}, \tag{2}$$

where:
I_{sx}, I_{sy}, I_{sz} – structural inertia moments,
D_{sx}, D_{sy}, D_{sz} – structural deviation moments.

The stiffness and damping matrices of an elastic damping element have comparable forms:

$$\mathbf{K} = \begin{bmatrix} \mathbf{S}_{rk}^T \mathbf{\Theta}_{rk}^T \mathbf{k}_k \mathbf{\Theta}_{rk} \mathbf{S}_{rk} & -\mathbf{S}_{rk}^T \mathbf{\Theta}_{rk}^T k_k \mathbf{\Theta}_{pk} \mathbf{S}_{pk} \\ -\mathbf{S}_{pk}^T \mathbf{\Theta}_{prk}^T \mathbf{k}_k \mathbf{\Theta}_{rk} \mathbf{S}_{rk} & \mathbf{S}_{pk}^T \mathbf{\Theta}_{pk}^T \mathbf{k}_k \mathbf{\Theta}_{pk} \mathbf{S}_{pk} \end{bmatrix}, \tag{3}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{S}_{rk}^T \mathbf{\Theta}_{rk}^T \mathbf{c}_k \mathbf{\Theta}_{rk} \mathbf{S}_{rk} & -\mathbf{S}_{rk}^T \mathbf{\Theta}_{rk}^T \mathbf{c}_k \mathbf{\Theta}_{pk} \mathbf{S}_{pk} \\ -\mathbf{S}_{pk}^T \mathbf{\Theta}_{prk}^T \mathbf{c}_k \mathbf{\Theta}_{rk} \mathbf{S}_{rk} & \mathbf{S}_{pk}^T \mathbf{\Theta}_{pk}^T \mathbf{c}_k \mathbf{\Theta}_{pk} \mathbf{S}_{pk} \end{bmatrix}, \tag{4}$$

where:
$\mathbf{k}_k$ – six-element vector of containing the elastic coefficients of the k-th elastic damping element,
$\mathbf{c}_k$ – six-element vector of containing the damping coefficients of the k-th elastic damping element,
$\mathbf{\Theta}$ – directional cosine matrix,
$\mathbf{S}$ – displacement transformation matrix between a rigid finite element of the indexes p and r and the k^{th} elastic damping element.

The r^{th} rigid finite element can be loaded by an arbitrary number of loads. The vector of the loads of the element f_r is calculated by the use of the following equation:

$$\mathbf{f}_r = \sum_{n=1}^{N} \mathbf{S}_{rn}^T \mathbf{\Theta}_{rn}^T \mathbf{p}_n, \tag{5}$$

where:
n – the load number,
$\mathbf{S}_{rn}$ – displacement transformation matrix between the r^{th} rigid finite element and the n^{th} load,
$\mathbf{\Theta}_{rn}$– directional cosine matrix between the r^{th} rigid finite element and the n^{th} load,
$\mathbf{p}_{rn}$ – six-element vector containing the components of the n^{th} load.

The forms of the displacement transformation matrix $\mathbf{S}_{rn}$ and the directional cosine matrix $\mathbf{\Theta}_{rn}$ remains the same as in the case of the computations of the stiffness and damping matrices of an elastic damping element.

4. Solution algorithm

In the first step software *DYNMACH* calculates a non-linear static problem. For loads associated with the weights of all rigid finite elements – these are calculated automatically – as well as for external loads the program calculates strains and reactions for elastic damping elements, which is the basis for computation of new stiffness and damping matrices of the system. This process in realised iteratively until the differences between the values of displacements in two subsequent iterations do not excide the assumed accuracy ε.

In the second step, based on the stiffness and damping matrices calculated in the non-linear static problem, program *DYNMACH* calculates the frequencies and modes of natural vibration of the loaded system.

In the third step program *DYNMACH* calculates dynamic responses of the system to forced vibration by the use of the modal transformation method. For this method the results from the previous dynamic step are used. The responses of the system are calculated in the forms of the displacement amplitude and phase both calculated for the centre of equilibrium of each rigid finite element.

5. Pre-processor

It is recommended to use *DYNMACH* pre-processor in order to create data file. Beside ordinary pre-processor options, there is also a database of pads, dampers and couplings, which can be directly put into the program. In order to have successful data, one has to run *DYNMACH* pre-processor and fulfil all data needed. Main philosophy of *DYNMACH* pre-processor is to enter all data concerning statics and dynamics separately and get results for statics and dynamics separately too. View of the *DYNMACH* pre-processor is presented in Fig. 1. In the pre-processor five types of couplings (elastic pads) are build in. 3-D, three-layer damping pad is shown in Fig. 1. Other elastic coupling elements are presented in Fig. 2–5. Direct data of the user's elastic coupling might be also entered in the program. Static as well as dynamic data of elastic couplings might be taken into account. After entering all needed data, static or dynamic stiffness value of chosen elastic pad will be displayed in the pre-processor. Stiffness is a result of dependence in function of dimension, hardness and temperature.

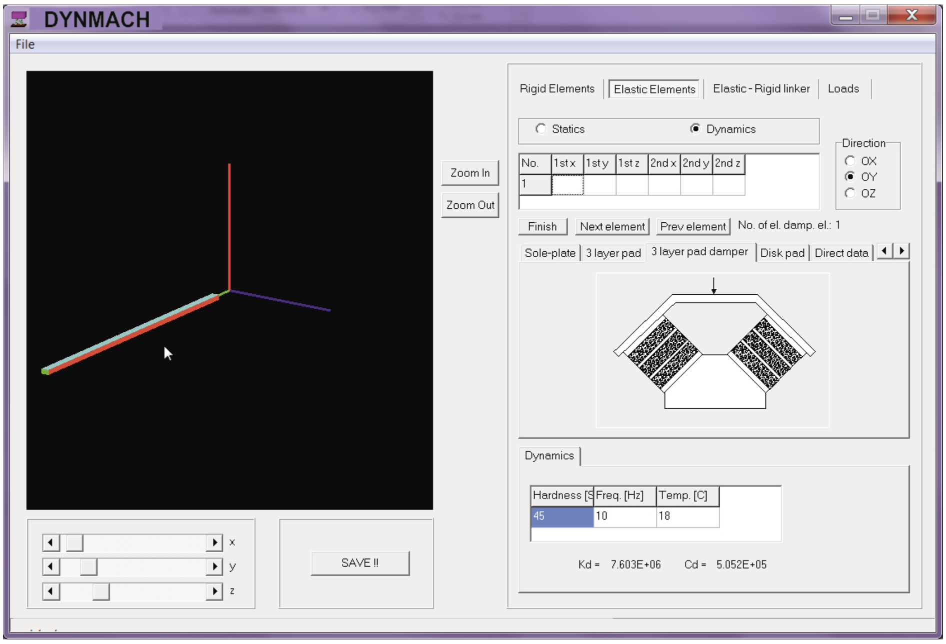

Fig. 1. Pre-processor of the DYNMACH software

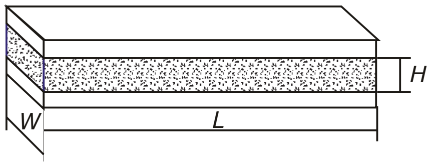

Fig. 2. Sole-pad build in the pre-processor

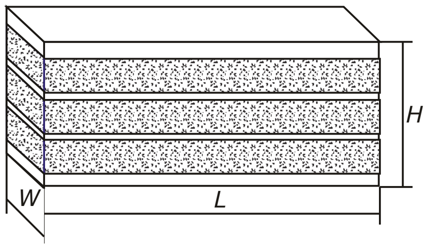

Fig. 3. Three layer pad build in the pre-processor

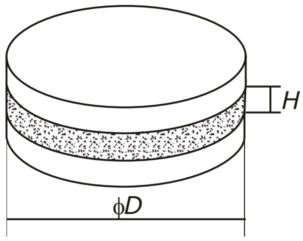

Fig. 4. Disc pad build in the pre-processor

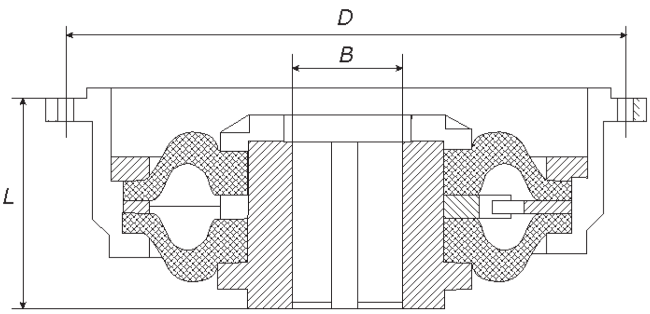

Fig. 5. Flexible coupling build in the pre-processor

6. Post-processor

Post-processor has been developed for results analysing facilitating. Some of the results might be displayed in graphical form. In the post-processor six different panels are made. The first one – *Displacement and rotation* contains the displacement and rotation components (see Fig. 6) calculated for the mass centres of individual rigid finite elements due to static loading in subsequent iterations in the global co-ordinate system. The second panel – *Strains* contains the strains calculated for the mass centres of individual elastic damping elements due to static loading in subsequent iterations in the global co-ordinate system. The third panel – *Natural vibration modes* contains the normalised vectors of natural vibration modes of a discrete model. The fourth panel – *Natural vibration frequencies* contains the values of natural vibration frequencies (see Fig. 7) of a discrete model. The fifth panel – *Vibration amplitudes and phases* contains the vibration amplitudes and phases calculated for the mass centres of individual rigid finite elements due to dynamic loading in subsequent iterations in the global co-ordinate system. The sixth panel – *Strain amplitudes and phases* contains the strain amplitudes and phases calculated for the mass centres of individual elastic damping elements due to dynamic loading in subsequent iterations in the global co-ordinate system.

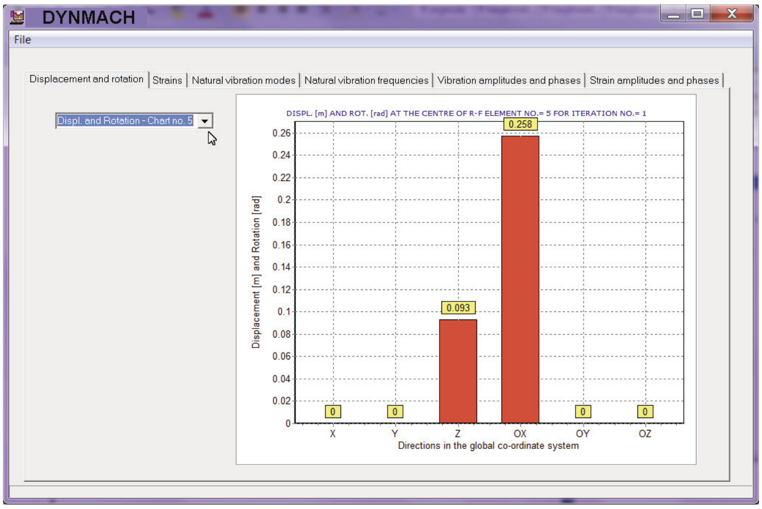

Fig. 6. Displacement and rotation panel of the post-processor

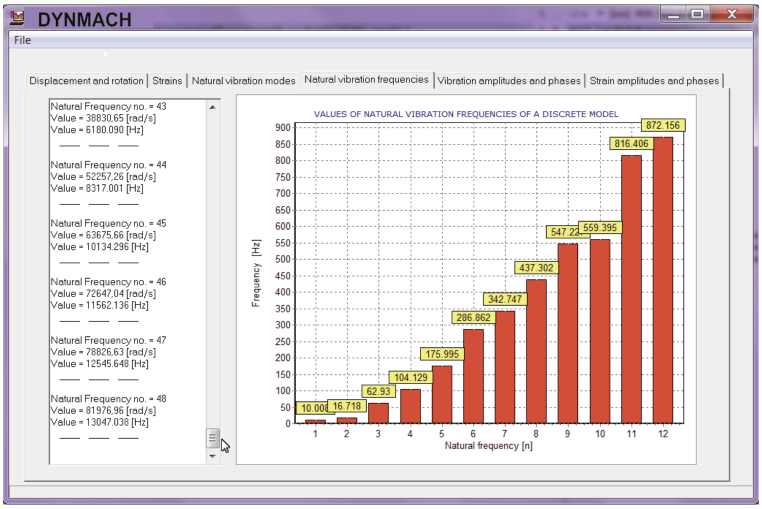

Fig. 7. Natural vibration frequencies panel of the post-processor

7. Software verification

A sample calculations are carried out for a cantilever beam having the following dimensions: the length $L = 1$ m, the width $B = 0.012$ m, and the height $H = 0.02$ m. The beam and its Rigid Finite Element model are presented in Fig. 8. The beam is divided into 8 rigid finite elements connected together by 8 elastic damping elements. The Young's modulus of the beam material is assumed as $E = 210$ GPa, the Kirchoff's modulus $G = 80.6$ GPa, and the density $\rho = 7860$ kg/m^3. The beam is loaded by its own weight and additionally the first rigid finite element is loaded by two dynamic forces having the amplitudes of 200 kN and 100 kN and excitation frequencies 70 rad/s and 10 rad/s, respectively. As the first the static deflection of the beam is calculated, next the frequencies and modes of natural vibration of the beam, and finally the amplitudes and phases of forced vibration due to the acting dynamic load. For this case the elastic and damping properties of the beam are assumed as linear.

A small damping equal for all elastic damping elements is assumed. The results obtained by the use of *DYNMACH* program compared with the results of theoretical calculations and are presented in Tab. 1 and Tab. 2.

The results presented in Tab. 1 and Tab. 2 demonstrate that even for the beam divided only into 8 rigid finite elements and 8 elastic damping elements the results of theoretical calculations and obtained from the program *DYNMACH* are only slightly different.

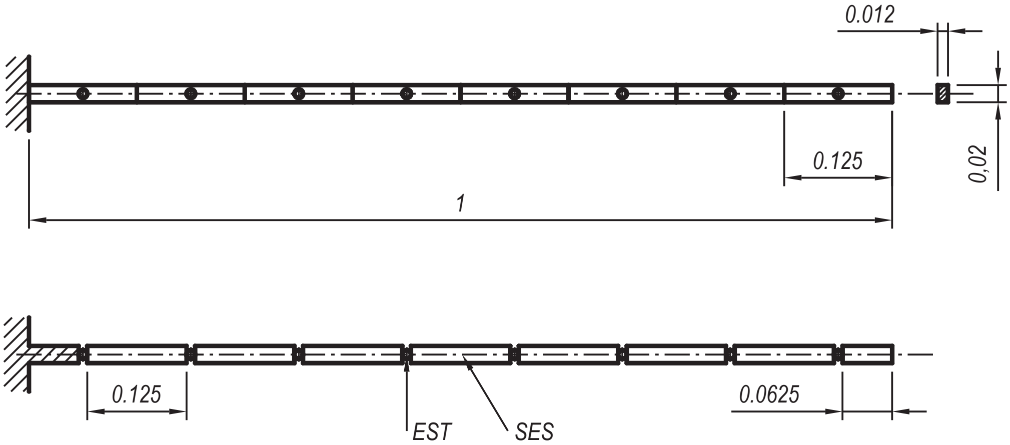

Fig. 8. The dimensions and a discrete model of the beam

Tab. 1. The results of theoretical calculations and obtained from the program DYNMACH in the case of the static loading of the beam

	Theory	Rigid Finite Element Method	Relative error
Rotation angle of the free end of the beam	1.84×10^{-3} rad	1.83×10^{-3}rad	0.38%
Displacement of the free end of the beam	1.38×10^{-3} m	1.31×10^{-3} m	4.51%

Tab. 2. The results of theoretical calculations and obtained from the program DYNMACH in the case of the dynamic loading of the beam

	Theory	Rigid Finite Element Method	Relative error
1st natural frequency for the *x-y* plane mode	62.96 rad/s	62.93 rad/s	0.04%
2nd natural frequency for the *x-y* plane mode	394.54 rad/s	397.75 rad/s	0.81%
3rd natural frequency for the *x-y* plane mode	1104.50 rad/s	1121.60 rad/s	1.54%
1st natural frequency for the *y-z* plane mode	104.92 rad/s	105.04 rad/s	0.11%
2nd natural frequency for the *y-z* plane mode	657.57 rad/s	654.26 rad/s	0.50%
3rd natural frequency for the *y-z* plane mode	1840.84 rad/s	1802.40 rad/s	2.08%

8. Conclusions

Software *DYNMACH* has been verified and might be useful for the vibrations analysis of machines placed on elastic foundation pads assuming their non-linear characteristics. Relatively simple modelling method (machines are modeled as rigid bodies with non-linear foundation pads) can be used in the diagnostics and monitoring systems. Software *DYNMACH* can calculates system responses to even extreme dynamic excitations. Database of different types of rubber machines couplings might be also useful for the user of the software.

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