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IMPULSIVE AND PERIODIC CLASS OF SOLUTIONS FOR HYDRODYNAMIC THEORY OF LUBRICATION

Krzysztof Wierzcholski

Technical University of Koszalin
Institute of Mechatronics, Nanotechnology and Vacuum Technique
Śniadeckich Street 2, 75-453 Koszalin, Poland
tel.:+48 94 3478344, fax: +48 94 3426753
e-mail: krzysztof.wierzcholski@wp.pl

Andrzej Miszczak

Gdynia Maritime University
Faculty of Marine Engineering
Morska Street 81-87, 81-225 Gdynia, Poland
tel.:+48 58 6901348, fax: +48 58 6901399
e-mail: miszczak@wm.am.gdynia.pl

Abstract

In this paper, two different classes of lubricant flow conditions are basically indicated, i.e. for periodic solutions when pressure values and other flow parameters change periodically and for non-periodic solutions in the case of lubricating in the conditions of impulses and strokes. The Authors formulate the primary problem in the form of a system of 9 nonlinear non-homogeneous partial differential equations with variable and random coefficients in a curvilinear orthogonal system of coordinates which is supplemented with suitable constitutive dependences and conjugated with magnetic field equations with the Ohm equation. These equations include the following: three conservation equations of the lubricating liquid momentum, the stream continuity equation, the energy conservation equation, three equilibrium equations of the thin elastic superficial layer that are reduced to the differential equation in displacements, the heat transfer equation of the superficial layer that is flown around by the lubricating liquid. The following include those equations that describe constitutive dependences the Rivlin-Ericksen equation of physical dependences for viscoelastic ferrofluid, the equation of the physical dependences of the superficial layer of the surfaces lubricated, the equation of the physical dependences of the magnetic field. The unknown values of the material coefficients shall be determined experimentally. The following 9 unknowns are determined from the system of partial differential equations: three velocity components of the lubricating liquid, the hydrodynamic pressure, the temperature in the lubricating liquid, the three components of the superficial layer displacement and the temperature in the superficial layer. The mathematical solution of the problem presented requires a number of boundary conditions to be imposed. The Authors foresee quasi-analytical solutions of the system described of partial differential equations.

Keywords: *injury solutions, periodic solutions, theory of lubrication, partial differential equations*

1. Some introduction remarks

The problem method of impulsive and periodic solution of lubrication problem had been considered already in Authors papers [2, 4]. In mentioned considerations, the computational model had been not accommodated to the curvilinear coordinates in non-isothermal magnetic flow and had been not coupled with the unified calculation algorithm referring to the lubricant and deformable bearing surfaces. In contrary to the foregoing papers [2, 4] the presented paper utilizes a new unified calculation algorithm for Rivlin-Ericksen viscoelastic oil properties in electro-magnetic field connected with stochastic changes of flow parameters [1-6]. Such algorithm satisfies stability conditions of numerical solutions of partial differential equations and gives real values of fluid velocity components and carrying capacities occurring in journal bearing.

In this paper is presented a semi analytical method of solution of the asymmetrical, laminar, unsteady periodic and impulsive, non-Newtonian lubrication problem flow between two non-rotational and rotational, movable surfaces. The parallel and longitudinal intersections of mentioned surfaces are curvilinear and non-monotone in general. The solutions are made in local curvilinear and orthogonal coordinate system (α_1, α_2, α_3) connected with the one of movable surfaces, where α_2 denote the direction of hap height.

The fluid apparent fluid viscosity η_p is variable in (α_1, α_2, α_3) directions and depends on pressure, temperature and flow shear ratio.

2. General basic equations for elasto-hydro-electro-magnetic conjugated mechanical fields

The results of the applied mathematical achievements are demonstrated in the form of semi analytical and numerical methods of approximate solutions of the set of 16 partial second order, non-linear, inhomogeneous differential equations of elasticity, hydrodynamics and electro-magnetic conjugated fields described in a two and three-dimensional form. The presented method is useful for a finite body or fluid regions of arbitrary shapes [6].

The system, which describes the electro-magneto-thermo-elasticity problem in stresses for two solid surfaces restricting the thin fluid layer, consists of the three partial differential equations (1) in a vector form. To this set of equations, we add the heat conductivity equation in a solid body (2) and we obtain a system in the following form [6]:

$$Div\,\mathbf{S}^* + \mathbf{J}^* \times \mathbf{B}^* + \mu_o\left(\mathbf{N}^*\nabla\right)\mathbf{H}^* = \rho^* \frac{\partial^2 \mathbf{u}}{\partial t^2}, \tag{1}$$

$$div\left(\kappa^* gradT^*\right) = \rho^* c_v^* \frac{\partial T^*}{\partial t}. \tag{2}$$

The fluid flow between two above mentioned solid surfaces in the electromagnetic field will be described by the three equilibrium of momentum equations in a vector form (3), a fluid continuity equation (4) and a conservation of energy equation in a scalar form (5), hence we obtain the following system [3, 6]:

$$Div\;\mathbf{S} + \mu_o(\mathbf{N}\nabla)\mathbf{H} + \frac{1}{2}\mu_o rot(\mathbf{N}\times\mathbf{H}) + \mathbf{J}\times\mathbf{B} = \rho\left(grad\,\frac{\mathbf{vv}}{2} - \mathbf{v}\times rot\;\mathbf{v}\right) + \rho\frac{\partial \mathbf{v}}{\partial t}, \tag{3}$$

$$\frac{\partial \rho}{\partial t} + div(\rho\mathbf{v}) = 0, \tag{4}$$

$$div(\kappa\;grad\;T) + \phi_F = \rho\frac{d}{dt}(c_v T) + \mu_o T\Xi(\mathbf{v}\nabla)\mathbf{H} + \mathbf{J}^2/\sigma. \tag{5}$$

The above-mentioned system of equations is completed by Maxwell and Ohm equations as well for two surfaces as for the thin boundary liquid layer between two surfaces. Thus, we consider equations [3, 6]:

$$\nabla\cdot\mathbf{B}^* = 0, \quad \nabla\cdot\mathbf{B} = 0, \nabla\times\mathbf{H}^* = \mathbf{J}^* + \frac{\partial \mathbf{D}^*}{\partial t}, \quad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{6}$$

$$\nabla\times\mathbf{E}^* = -\frac{\partial \mathbf{B}^*}{\partial t}, \quad \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \mathbf{J}^* = \sigma^*\mathbf{E}^*, \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}), \tag{7}$$

We assume following notations: μ_o – magnetic permeability in vacuum H/m, T – fluid temperature in K, T^* – solid body temperature in K, **B** – magnetic induction vector in T, **N** − magnetization vector A/m, **E** – electric intensity vector V/m, **H** – magnetic intensity vector A/m, Ξ – first derivative of magnetization vector respect to temperature A/mK, σ − electrical conductivity coefficient S/m,

J – electric current density in A/m^2, **D** – electric induction vector As/m, ρ – fluid density kg/m^3, κ – thermal conductivity coefficient W/mK, **v** – fluid velocity vector in m/s, ϕ_F – dissipation of energy in W/m^3, **S** – stress tensor in the fluid in Pa, **u** – displacement vector of the solid body in m, t – time in s, c_v – specific heat in J/kgK. The symbols with an asterisk related to the solid body.

Second order approximation of the general constitutive equation given by Rivlin-Ericksen can be written in the following form [6, 7]:

$$\mathbf{S} = -p\mathbf{I} + \eta\mathbf{A}_1 + \alpha\mathbf{A}_1^2 + \beta\mathbf{A}_2, \tag{8}$$

where: p – pressure in Pa, **I** – the unit tensor, $\mathbf{A}_1$, and $\mathbf{A}_2$ – the first two Rivlin-Ericksen tensors and η, α, β – three material constants, where $\eta=\eta_0\eta_1$ denotes dynamic viscosity in Pas, and α,β are pseudo-viscosity coefficients in Pas^2.

Tensors $\mathbf{A}_1$, and $\mathbf{A}_2$ are given by symmetric matrices defined by [1, 5-7]:

$$\mathbf{A}_1 \equiv \mathbf{L}+\mathbf{L}^T, \quad \mathbf{A}_2 \equiv grad\, \mathbf{a} + (grad\, \mathbf{a})^\mathbf{T} + 2\mathbf{L}^\mathbf{T}\mathbf{L}, \quad \mathbf{a} \equiv \mathbf{L}\,\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t}, \tag{9}$$

where: **L** – tensor of fluid velocity gradient vector in s^{-1}, $\mathbf{L}^\mathbf{T}$ – tensor for transpose of a matrix of gradient vector of a fluid in s^{-1}, **a** – acceleration vector m/s^2. Symbol grad(**a**) denotes tensor of rank two.

3. Semi-analytical series solutions

LEMMA 1

An estimation of the terms with respect to the thin layer boundary simplifications for the equations of the conservation of the momentum, continuity, the energy equation, and constitutive equation given by Rivlin-Ericksen formula in the curvilinear orthogonal co-ordinates (α_1, α_2, α_3) for constant coefficients $\rho,\alpha,\beta\neq 0,\kappa\neq 0$ {i.e. an incompressible, viscoelastic flow}, for periodic and non-periodic solutions {i.e. time t depended, and unsymmetrical medium flow} in a thin layer space (α_1, α_2, α_3, t) between two arbitrary movable surfaces, with a non-monotonic curvature line, lead to the following system of basic equations [6-8]:

$$\rho\frac{\partial v_i}{\partial t} + X_{ci} = -\frac{1}{h_1}\frac{\partial p}{\partial \alpha_i} + \frac{\partial}{\partial \alpha_2}\left(\eta\frac{\partial v_i}{\partial \alpha_2}\right) + \beta\frac{\partial^3 v_i}{\partial t\,\partial \alpha_2^2} + M_i + O_i(\alpha,\beta) \quad i = 1,3. \tag{10}$$

$$(\alpha + 2\beta)\frac{\partial}{\partial \alpha_2}\left[\left(\frac{\partial v_1}{\partial \alpha_2}\right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2}\right)^2\right] = \frac{\partial p}{\partial \alpha_2}, \tag{11}$$

$$\frac{\partial}{\partial t}(\rho h_1 h_3) + \frac{\partial}{\partial \alpha_1}(\rho v_1 h_3) + \frac{\partial}{\partial \alpha_2}(\rho v_2 h_1 h_3) + \frac{\partial}{\partial \alpha_3}(\rho v_3 h_1) = 0, \tag{12}$$

$$\frac{\partial}{\partial \alpha_2}\left(\kappa\frac{\partial T}{\partial \alpha_2}\right) + \eta\left[\left(\frac{\partial v_1}{\partial \alpha_2}\right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2}\right)^2\right] = Z + O_T(\alpha,\beta), \tag{13}$$

where the length, width and gap- height directions, are limited respectively: $0<\alpha_1\leq 2\pi$, $-b_m\leq\alpha_3\leq b_s$, $0\leq\alpha_2\leq\varepsilon$. The system of Eq. (10)–(13) contains the following unknowns: $p(\alpha_1,\alpha_2,\alpha_3,t)$, $T(\alpha_1,\alpha_2,\alpha_3,t)$, $v_i(\alpha_1,\alpha_2,\alpha_3,t)$ i.e. pressure, temperature, three medium velocity components for i=1,2,3 in three curvilinear, orthogonal dimensional directions: α_1, α_2, α_3. Lame coefficients are as follows: $h_1(\alpha_1,\alpha_3)$, $h_3(\alpha_1,\alpha_3)$ for non-rotational surfaces and non-monotone curvatures; $h_1(\alpha_3)$, $h_3(\alpha_3)$ for rotational surfaces and its non-monotone generating lines, and $h_1(\alpha_3)$,h_3=1 for rotational surface with monotone generating line {see [6] section 2.1.5}.

Applications: Symbols M_i, denote terms of electro-magnetic field influences. And we have following one order smaller terms describing: X_{ci} – inertia forces without local derivative of velocity, Z – the convection transport of energy, O_i, O_T – the viscoelastic oil properties {see [10] section 2}.

PROOF OF LEMMA 1

The proof of Lemma 1 was performed in the Authors monograph [8] chapter 2, section 2.1, Lemma 2.1.5.2 and in chapter 3 intersection 3.1.5.

LEMMA 2

A system of partial differential equations (10)-(13) defined in thin space $(\alpha_1, \alpha_2, \alpha_3, t)$ between two rotational movable surfaces for M_i=0 {without any electro- magnetic field influences} and for continuous single valued η (α_1, α_3), {i.e. without any medium viscosity changes in gap height direction α_2}, for viscoelastic properties i.e. $\alpha\beta \neq 0$, has an analytical non periodic solution in the form of the following infinite uniform convergent functional-power series with respect to the successive powers of $\beta / \eta_0 t$ (for t >0) {available see [6, 7]}:

$$v_i(\alpha_1,\alpha_2,\alpha_3,t) = v_{i0}(\chi,\alpha_1,\alpha_3) + \frac{\beta}{\eta_0 t} v_{i1}(\chi,\alpha_1,\alpha_3) + \left(\frac{\beta}{\eta_0 t}\right)^2 v_{i2}(\chi,\alpha_1,\alpha_3) + \dots , \quad i = 1,2,3. \tag{14}$$

$$p(\alpha_1, \alpha_2 = 0, \alpha_3, t) = p_{10}(\alpha_1,\alpha_3,t) + \frac{\beta}{\eta_0 t} p_{11}(\alpha_1,\alpha_3,t) + \left(\frac{\beta}{\eta_0 t}\right)^2 p_{12}(\alpha_1,\alpha_3,t) + \dots, \tag{15}$$

$$T(\alpha_1,\alpha_2,\alpha_3,t) = T_{10}(\chi,\alpha_1,\alpha_3) + \frac{\beta}{\eta_0 t} T_{11}(\chi,\alpha_1,\alpha_3) + \left(\frac{\beta}{\eta_0 t}\right)^2 T_{12}(\chi,\alpha_1,\alpha_3) + \dots, \tag{16}$$

with a new variable

$$\chi \equiv \frac{\alpha_2}{2\sqrt{v_v t}}, \quad v_v \equiv \frac{\eta_0}{\rho}, \ t > 0, \ 0 < \frac{\beta}{\eta_0 t} < 1. \tag{17}$$

Applications: The fluid velocity components of v_{1k}, v_{3k}, v_{2k} and pressure p_{1k} and temperature T_{1k} for k=0 depend on the time and the viscosity of the medium but they are independent of the viscoelastic properties. The flow parameters for k=1,2,... describe corrections of fluid velocity components i.e. the pressure and temperature caused by the time dependent viscoelastic properties {available in [6, 7, 9]}.

PROOF OF LEMMA 2

By substituting expressions (14)-(16) into the system of equations (10), (13) one obtains for the first twelve unknown functions v_{10}, v_{20}, v_{30}, T_{10}, v_{11}, v_{21}, v_{31}, T_{11}, v_{12}, v_{22}, v_{32}, T_{12}, … the ordinary differential equations {[6] see chapter 4 intersection 4.1.3}. Imposing the proper boundary conditions on the velocity components v_{20}, v_{21}, v_{22}, … we obtain the successive modified partial differential Reynolds equations determining unknown pressure functions p_{10}, and its corrections p_{11}, p_{12},… {[6] chapter 4 Eq.(4.1.2), (4.1.22)}. The proof of Lemma 2 was completed in the Authors monograph [6].

LEMMA 3

A system of partial differential equations (10)-(13) defined in thin space $(\alpha_1, \alpha_2, \alpha_3, t)$ between two movable rotational surfaces for M_i=0 {without any magnetic field influences} and for continuous, single valued $\eta(\alpha_1, \alpha_3)$, {i.e. without any medium viscosity changes in gap height direction α_2}, for viscoelastic properties i.e. $\alpha\beta \neq 0$ has an analytical periodic solution in the form of the following infinite uniform convergent functional power series with respect to the successive decaying exponential functions {see [6] cf. intersection (4.3)}:

$$v_i(\alpha_1,\alpha_2,\alpha_3,t)=v_i^{(0)}(\alpha_1,\alpha_2,\alpha_3)+\sum_{k=1}^{\infty}v_i^{(k)}(\alpha_1,\alpha_2,\alpha_3)\exp(\mathbf{i}k\omega_o t),\quad i=1,2,3, \tag{18}$$

$$T(\alpha_1,\alpha_2,\alpha_3,t)=T^{(0)}(\alpha_1,\alpha_2,\alpha_3)+\sum_{k=1}^{\infty}T^{(k)}(\alpha_1,\alpha_2,\alpha_3)\exp(\mathbf{i}k\omega_o t), \tag{19}$$

$$p(\alpha_1,\alpha_2=0,\alpha_3,t)=p^{(0)}(\alpha_1,\alpha_3)+\sum_{k=1}^{\infty}p^{(k)}(\alpha_1,\alpha_3)\exp(\mathbf{i}k\omega_o t). \tag{20}$$

Applications: *Symbol ω_o denotes the frequency of vibrations in s^{-1} and describes periodical perturbations in an unsteady fluid flow in the gap. Symbol $\mathbf{i}\equiv\sqrt{-1}$ is an imaginary unit. The unknown functions with upper index (0) describe the velocity vector components, the temperature and the pressure for stationary non-viscoelastic fluid properties. The unknown functions with upper index (k) for k=1,2,3,... denote corrections of the velocity vector components, the temperature and the pressure caused by the non-stationary viscoelastic properties of the fluid {available in [6] intersections 4.3.2, 4.3.3}.*

PROOF OF LEMMA 3

We put infinite series (18)-(20) into the set of equations (10)-(13) for $M_i=0$, $\beta=0$, and we equate the terms of the same upper indexes in the same powers of exp functions. By equating the terms with upper index (0), we obtain a sequence of the partial differential equations of motion for Newtonian fluid properties and steady conditions. By equating the terms with upper index (1), (2),… we obtain a sequence of partial differential equations, which determines corrections caused by the motion for the viscoelastic fluid properties and unsteady conditions. The proof was completed in the Authors monograph [6] section 4.3.3-7.

4. Solutions of selected stochastic partial differential equations

Now we describe the distribution of unknown function p (pressure) for the problem described by the system of Eqs.(11)-(15) with viscoelastic oil properties i.e.$\alpha\beta\neq0$, but in the case of $\eta(\alpha_1,\alpha_2,\alpha_3)$ {*i.e.* medium viscosity changes in the gap height direction α_2} and for stochastic conditions. We assume that the dimensionless time depended gap height ε_{T1} consists of two parts [6, 9]:

$$\varepsilon_{T1}=\varepsilon_{T1s}(\alpha_1,\alpha_3,t)+\delta_1(\alpha_1,\alpha_3,\xi,t), \tag{21}$$

where ε_{T1s} denotes the total dimensionless nominal smooth part of the geometrical form of the thin fluid layer. This part of the gap height contains the corrections of the gap height caused by time deformations. Symbol δ_1 denotes the dimensionless random part of the gap height changes resulting from vibrations, unsteady loading and the surface roughness measured from the nominal mean level. Symbol ξ describes the random variable, which characterizes the roughness arrangement. Expectation operator E has the following form {[6] cf. intersections 4.2.3 and 4.2.6, [9] intersection 3.2 and [10]}:

$$E(*)=\int_{-\infty}^{+\infty}(*)\times f_g(\delta_1)d\delta_1,\ \sigma_{s1}=\frac{c_1}{\sqrt{13}}=0{,}375,\quad f_g(\delta_1)\equiv\begin{cases}\left(1-\dfrac{\delta_1^2}{c_1^2}\right)^5 & for-c_1\le\delta_1\le+c_1,\\ 0 & for\quad|\delta_1|>c_1,\end{cases} \tag{22}$$

whereas: c_1 – half of the total range of dimensionless random changes for the quantity considered, σ_{s1} – dimensional standard deviations, f_g – dimensionless probability density function. The definite integral in interval $(-\infty,\infty)$ from each probability density function is always equal to unity. In Eq.(22) is presented Pseudo-Gaussian symmetric density function. Such function presents case where probabilities of the gap height decreases are the same as probabilities of gap height increases. Presented theory valid for asymmetric density functions too. Now we formulate {[12] intersection 8.4.2, [6] section 3.2}.

THEOREM 1. If continuous single valued function $\eta(\alpha_1,\alpha_2,\alpha_3)$ {i.e. the medium viscosity} is not constant in the gap height direction, $M_i\neq 0$ {i.e. magnetic field forces are taken into consideration}, $\alpha,\beta\neq 0$ {i.e. viscoelastic properties of the medium are considered}, then the unknown function $p(\alpha_1,\alpha_2=0,\alpha_3,t)$ {i.e. hydrodynamic pressure} in curvilinear co-ordinates $(\alpha_1,\alpha_2,\alpha_3)$ on the journal non rotational movable surface, with non-monotone curvature line in general, satisfies the following time t depended form of modified stochastic Reynolds equation {[12] section 8.4}:

$$\frac{\partial}{\partial\alpha_1}\left[\frac{h_3}{h_1}\left(\frac{\partial E(p)}{\partial\alpha_1}-h_1M_1\right)E\left(\int_0^{\varepsilon_T}A_\eta d\alpha_2\right)\right]+\frac{\partial}{\partial\alpha_3}\left[\frac{h_1}{h_3}\left(\frac{\partial E(p)}{\partial\alpha_3}-h_3M_3\right)E\left(\int_0^{\varepsilon_T}A_\eta d\alpha_2\right)\right]=$$

$$=\omega\frac{\partial}{\partial\alpha_1}h_1h_3\left[E\left(\int_0^{\varepsilon_T}A_s d\alpha_2\right)-E(\varepsilon_T)\right]-h_1h_3\frac{\partial E(\varepsilon_T)}{\partial t}+O_p(\alpha,\beta), \tag{23}$$

where:

$$A_s(\alpha_1,\alpha_2,\alpha_3)\equiv\int_0^{\alpha_2}\frac{1}{\eta}d\alpha_2\left(\int_0^{\varepsilon_T}\frac{1}{\eta}d\alpha_2\right)^{-1},\quad A_\eta(\alpha_1,\alpha_2,\alpha_3)\equiv\int_0^{\alpha_2}\frac{\alpha_2}{\eta}d\alpha_2-A_s(\alpha_1,\alpha_2,\alpha_3)\int_0^{\varepsilon_T}\frac{\alpha_2}{\eta}d\alpha_2, \tag{24}$$

whereas: $0\le\alpha_1\le 2\pi\theta_1$, $0\le\theta_1\le 1$, $b_m\le\alpha_3\le b_s$, $0\le\alpha_2\le\varepsilon_T$, $\varepsilon_T=\varepsilon_T(\alpha_1,\alpha_3,t)$, $\eta(\alpha_1,\alpha_2,\alpha_3)$ $h_1(\alpha_1,\alpha_3)$, $h_3(\alpha_1,\alpha_3)$, O_p-terms caused by the visco-elastic oil properties.

PROOF SKETCH OF THEOREM 1.

The results of the applied mathematical achievements are presented in the form of a derivation and approximate solution of the stochastic, modified Reynolds equation of the second order (23) using semi analytical and numerical methods in curvilinear orthogonal coordinates. Such a modified Reynolds equation was obtained by imposing at first the specific boundary conditions on differential equations (1)-(7) for the liquid flow in the thin boundary layer and in the next step, we impose the proper boundary condition on the fluid velocity component in gap height direction. Taking into account the magnetic field and stochastic changes of the gap height between two surfaces, and changes of the dynamic lubricant viscosity in the gap height direction, then modified Reynolds equation in curvilinear orthogonal coordinates formulated by Eq.(23) {[6, 11]} was obtained. In equation (23) hydrodynamic pressure p is the unknown function. Symbols $h_i(\alpha_1,\alpha_3)$ for i=1,3 denote Lame coefficients for non-rotational surfaces and non-monotone curvatures and $h_i(\alpha_3)$ for rotational surfaces and its non-monotone generating lines {[6], section 2.1}, ω – denotes the angular velocity of the rotational surface, t – time. A full proof was completed in the Authors monograph {[12] (Chapter 8.4)}.

COROLLARY FROM THEOREM 1

If the surface deformations and $M_i\neq 0$ {i.e. the magnetic field are taken into account} and stochastic conditions as well $\eta(\alpha_1,\alpha_3)$ {i.e. viscosity changes in gap height direction α_2 are neglected}, then modified Reynolds equation (23) for rotational deformable surfaces in curvilinear orthogonal coordinates tends to the following form see [6] {intersection 3.1.5}

$$\frac{1}{h_1}\frac{\partial}{\partial\alpha_1}\left[\frac{\varepsilon^3(u_2)}{\eta_o}\left(\frac{\partial p}{\partial\alpha_1}-M_1(t)h_1\right)\right]+\frac{1}{h_3}\frac{\partial}{\partial\alpha_3}\left[\frac{h_1\varepsilon^3(u_2)}{h_3\eta_o}\left(\frac{\partial p}{\partial\alpha_3}-M_3(t)h_3\right)\right]=6\omega h_1\frac{\partial\varepsilon(u_2)}{\partial\alpha_1}. \tag{25}$$

$$M_i \equiv \mu_o(\mathbf{N}\nabla)H_i + 0.5\mu_o \mathrm{rot}(\mathbf{N}\times\mathbf{H})_i \quad \text{for} \quad i=1,3. \tag{26}$$

Applications: p is an unknown pressure function, ε – gap height restricted by the two surfaces, u_2– deformations in the gap height directions. Symbols $h_1(\alpha_3)$, $h_3(\alpha_3)$ denote Lame coefficients depended on the surface geometry, ω – angular velocity of the surface, t – time.

5. Numerical calculations

FEM, or difference and Mathcad tools are useful during the pressure calculations from (25). For spherical coordinates $\alpha_1=\varphi$, $\alpha_2=r$, $\alpha_3=\vartheta$ we have: $h_1=R\sin\vartheta_1$, $h_3=1$. Fig. 1 and Fig. 2 shows the time -variable, distribution of function p on the spherical calculation region Ω: $0\le\varphi\le\pi$, $\pi R/8\le\alpha_3\equiv\vartheta\le\pi R/2$, $\vartheta_1=\vartheta/R$ for the periodic stochastic flow. We assume $p=p_{at}$ on the boundary of region Ω. Semi numerical pressure calculations by virtue of series (20) and Eq.(25) are performed in Matlab 7.2 and Mathcad 12 Professional Program by means of the finite difference method. We assume the radius of the spherical surface R= 0.0265m, the angular velocities in circumferential and meridian directions ω_1= 1.40 s^{-1}, ω_3= –0.45 s^{-1} and the angular perturbations in the above-mentioned directions ω_{1o}= 0.100 s, ω_{3o}= 0.025 s. The angular velocity describing periodical perturbations equals ω_o=500 s^{-1}. Components of eccentricities are as follows $\Delta\varepsilon_x$= 2.5 μm, $\Delta\varepsilon_y$ = 0.5 μm, $\Delta\varepsilon_z$ = 2.0 μm. We assume following values of fluid viscosity and density: η_o= 0.15 Pas, ρ_o= 1000 kg/m^3. The minimum value of the gap height equals ε_{min}=4.8 μm, the maximum value attains ε_{max}= 10.50 μm. Pressure values are calculated in the following instants within the time period: t=0 s, $t=\pi/\omega_o$ s, $t=2\pi/\omega_o$ s. Stochastic dimensionless coefficient caused by the roughness is defined in the following form: $\wp \equiv \sigma_S / E(\varepsilon_T)$ see [6]. Magnetic effects are omitted i.e. M_i=0.

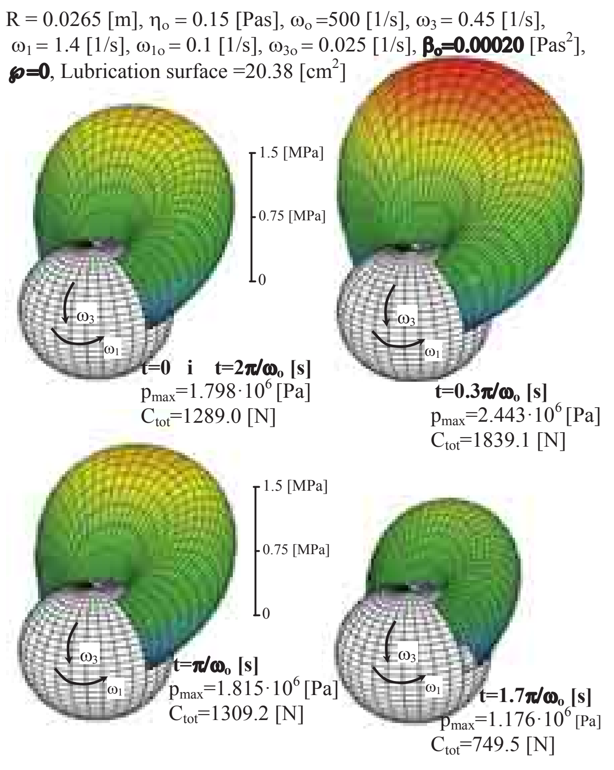

Fig. 1. The distributions of unknown function p presenting the solution of the partial differential and recurrence equation on the spherical surfaces restricting thin gap, and caused by the rotation of the spherical surface in circumferential direction φ and meridian direction ϑ, simultaneously

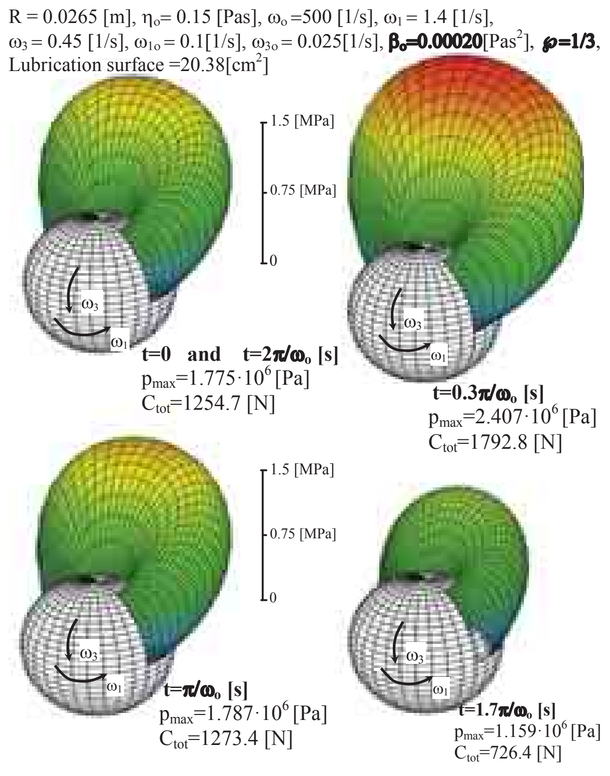

Fig. 2. The distributions of unknown function p (i.e. pressure) in gap, between two spherical surfaces caused by the rotation of the spherical surface in circumferential direction φ and meridian direction ϑ, simultaneously. Magnetic effects are omitted i.e. M_i=0

6. Conclusions

1. The analytical models that describe those electromagnetic, non-Newtonian hydrodynamic effects, which really occur in the areas of slide bearing surfaces, will be prepared with the use of Maxwell's equations, liquid motion equations and the energy equation [1-3, 5]. Such models will allow one to track the real changes of magneto-hydrodynamic effects and particularly the influence of variable or constant magnetic fields or force impulses on operating processes in the course of the lubrication of the surfaces of bearings.
2. For the partial differential hydrodynamic system of equations, two main forms of solutions namely impulsive and periodical are considered and derived
3. For liquids with viscoelastic properties in the thin lubricating layer, the Rivlin-Ericksen's non-linear constitutive dependences are accepted taking into account pseudoviscosity coefficients presented in obtained solutions.

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