ISSN: 1231-4005
e-ISSN: 2354-0133
DOI: 10.5604/12314005.1138272

HOW TO FORMULATE RATES TO ASSESS OPERATIONAL EFFECTIVENESS OF MILITARY AIRCRAFT – AN INTRODUCTION

Henryk Tomaszek, Ryszard Kaleta, Mariusz Zieja

Air Force Institute of Technology
Księcia Bolesława Street 6, 01-494 Warszawa, Poland
tel.: +48 22 6851956, fax: +48 22 6851012
e-mail: poczta@itwl.pl, ryszard.kaleta@itwl.pl, mariusz.zieja@itwl.pl

Abstract

The paper has been intended to deliver a brief introduction to the forecast on the potential military advantages for some selected aircraft using data from the TURAWA and SAMANTA computer-based systems for tracking aircraft operation and maintenance. The term "operational effectiveness" of a military aircraft may be read as a sum of effects gained from particular sorties by this aircraft. In the simplest case, the operational effectiveness is nothing more but the number of destroyed hostile targets. The number of destroyed hostile targets is a random variable. With account taken of hostile counteractions and unreliability of engineered systems, the number of possible sorties by one aircraft is also a random variable. Therefore, the assessment of predicted operational effects gained by the aircraft may result from, either: a single operational (combat) flight (sortie) by one aircraft or flights (sorties) within some assumed time interval (time of an air mission), or the aircraft's total lifetime (service life). Findings of the study may find their application in an attempt to use the data collected in the TURAWA and SAMANTA systems to construct rates of aircraft effectiveness in the forms of: expected value of effects gained from a single sortie, expected value of effects gained from some assumed time interval (flying time), expected value of effects gained throughout the aircraft's service life. All the relationships gained may then be applied to forecast the effectiveness of operating some selected types of aircraft using data collected throughout the service, and verified under field conditions while performing exercises.

Keywords: *dependability, availability, effectiveness, survivability, service life*

1. Introduction

There are various measures to assess the effectiveness [1, 2, 7]. In the case discussed below, what has been assumed a measure of the effectiveness is the expected value of the number of destroyed hostile targets either:

– in a single sortie - $E_1[z]$, or
– as related to a given time interval, e.g. to the time of an air mission $E_t[z]$, or
– throughout the aircraft's total lifetime (service life) - $E_\infty[z]$.

The question of assessing the operational effectiveness of a military aircraft is relatively complicated, but at the same time, of great importance and quite often given careful consideration [1, 2, 7]. The complexity of this issue results from the need for the account to be taken of many and various interrelated threads and aspects; these interrelationships are sometimes very complicated, too. Partial solutions require, in turn, some assumptions to be made. These are as follows:

1. The expected value of the number of destroyed hostile targets is assumed a measure of operational effectiveness of a military aircraft.
2. The aircraft assumed a fighter and flying fighter mission(s) has been adapted to fight hostile targets with self-guided missiles.
3. The following rates will be taken into account in the developed method for the effectiveness assessment:

a) suitability of an aircraft for operation and hostile targets destruction;
b) operational readiness of:
   – the aircraft,
   – the pilot;
c) in-flight reliability of:
   – the aircraft,
   – the pilot;
d) survivability of an aircraft under combat conditions;
e) effectiveness of air warfare agents carried by the aircraft.

4. A simplified way of applying warfare agents to fight hostile targets has been assumed.
5. It has also been assumed that aerial targets are homogeneous as far as their vulnerability is concerned.

2. Determination of the density function of effects of combat application of military aircraft

Of great interest and significance, with respect to the assessment in question, may prove the answer to the question: How do the above-mentioned rates, i.e. the suitability of the aircraft, the readiness and reliability thereof as well as those of the pilot, the life-time of the aircraft, and finally, the effectiveness of the air warfare agents influence the extent of the pre-planned results of the operation.

To describe the increment in effect with time, we are going to use a difference equation, where the following probabilities play a vital role:

P – probability of the need to perform (demand for) a sortie,

P_1 – probability that a given aircraft is suitable to perform this sortie owing to its parameters and weapon systems engaged,

K_g – probability that the aircraft is ready for the sortie as the need be,

$\overline{K}_g$ – probability that the pilot is ready for the sortie,

R – probability that the aircraft performs the sortie in a reliable manner,

$\overline{R}$ – reliability of the pilot.

The so-called close-range operational environment of the weapon system under the assessment subject to estimation based on the analysis of external circumstances (e.g. settlements reached by decision-makers, situations that have occurred, operational circumstances, etc.) generates probability P of the need to perform a sortie.

The following events should be given careful consideration because of the possibility that the increment in effect may occur:

$(1-P)$ – there is no need to perform a sortie,

$P(1-K_g)$ – a need for a sortie has appeared but there is no aircraft ready (i.e. no increment in effect will occur),

$PK_g(1-\overline{K}_g)$ – a need for a sortie has appeared, the aircraft is ready but the pilot is not,

$PK_g\overline{K}_g(1-P_1)$ – there is a need for a sortie, the aircraft and the pilot are ready, but the aircraft is not suitable to perform the task (sortie) of this type (i.e. no increment in effect will occur),

$PK_g\overline{K}_gR(1-R)$ – there is a need for a sortie, the aircraft and the pilot are ready, the aircraft is suitable to perform the task (sortie) but it has proved unreliable, which has made the increment in effect stay beyond the reach,

$PK_g\overline{K}_gP_1R(1-\overline{R}_1)$ – the aircraft will perform the sortie, since it is ready together with the pilot, it has proved reliable, but the pilot has proved unreliable,

$P K_g \overline{K}_g P_1 R \overline{R}$ – the aircraft will perform the sortie, since it has proved ready together with the pilot, it has also proved suitable for the task and operated in a reliable manner, which means the increment in effect is possible.

Probabilities of the above-determined events should sum up to unity, i.e.:

$$(1-P)+P(1-K_g)+PK_g(1-\overline{K}_g)+PK_g\overline{K}_g(1-P_1)+PK_g\overline{K}_gP_1(1-R)+$$
$$+ P K_g \overline{K}_g P_1 R(1-\overline{R}) + PK_g P_1 R\overline{R} = 1 . \tag{1}$$

Let us then consider the problem of a formal description of the increment in effects against the (total) flying time of the aircraft. Let us assume the operational use of the aircraft has already taken some time, i.e. some number of sorties has been flown. Another assumption to be made is that the effects gained within this time are z. Let $E[\Delta\overline{z}]$ be the expected value of the increment in the number of targets destroyed in the course of one sortie by means of some air warfare agents.

With the relationship (1) applied, one can describe the possibility of the increment in effect in time τ with the following difference equation:

$$U(z,t+\tau)=(1-PK_g\overline{K}_gP_1R\overline{R})u(z,t)+PK_g\overline{K}_gP_1R\overline{R}\,u(z-E[\Delta\overline{z}],t), \tag{2}$$

where:

$u(z,t)$ – the probability density function of increment in effects of aircraft operation for the (total) flying time t.

$E[\Delta\overline{z}]$ – the expected value of the increment in effect owing to the application of warfare agents.

Let us rearrange equation (2) into a partial differential equation. The following expansions have to be done to do this:

$$u(z,t+\tau)\cong u(z,t)+\frac{\partial u(z,t)}{\partial t}\tau , \tag{3}$$

$$u(z-E[\Delta z],t)\cong u(z,t)-\frac{\partial u(z,t)}{\partial z}E[\Delta\overline{z}])=\frac{1}{2}(E[\Delta\overline{z}])^2\frac{\partial^2 u(z,t)}{\partial z^2}. \tag{4}$$

Having substituted the above written expressions into equation (2), we arrive at what follows:

$$u(z,t)+\frac{\partial u(z,t)}{\partial t}\tau=(1-PK_g\overline{K}_gP_1R\overline{R})\ u(z,t)+PK_g\overline{K}_gP_1R\overline{R}\ (u(z,t)-E[\Delta\overline{z}]\frac{\partial u(z,t)}{\partial z}$$
$$+\frac{1}{2}(E[\Delta\overline{z}])^2\ \frac{\partial^2 u(z,t)}{\partial z^2}). \tag{5}$$

After the ordering we get:

$$\frac{\partial u(z,t)}{\partial t}=-\frac{PK_g\overline{K}_gP_1R\overline{R}E[\Delta\overline{z}])}{\tau}\ \frac{\partial u(z,t)}{\partial z}+\frac{1}{2}\frac{PK_g\overline{K}_gP_1R\overline{R}(E[\Delta\overline{z}])^2}{\tau}\ \frac{\partial^2 u(z,t)}{\partial z^2}. \tag{6}$$

Let us now denote coefficients in equation (6) in the following way:

$$b=\frac{PK_g\overline{K}_gP_1R\overline{R}E[\Delta\overline{z}]}{\tau} , \tag{7}$$

$$a=\frac{PK_g\overline{K}_gP_1R\overline{R}(E[\Delta\overline{z}])^2}{\tau} , \tag{8}$$

where:

b – the aircraft's mean increment in effect per a flying-time unit,

a – the aircraft's mean square of increment in effect per a flying-time unit.

Hence, equation (6) can be written down in the following way:

$$\frac{\partial u(z,t)}{\partial t} = -b\frac{\partial u(z,t)}{\partial z} + \frac{1}{2}a\frac{\partial^2 u(z,t)}{\partial z^2}. \tag{9}$$

On the other hand, the searched for solution to equation (9) takes the following form [4]:

$$U(z,t) = \frac{1}{\sqrt{2\pi at}}\, e^{\frac{(z-bt)^2}{2at}}. \tag{10}$$

In the case under consideration, function (10) describes distribution of effects of the aircraft's operation against the aircraft's (total) flying time.

3. Expected values of effects

Making use of the density function (10), let us find the expected value of effects for some definite flying time *t* of the aircraft on the basis of the following equation:

$$E_t[z] \cong \int_0^\infty z\, u(z,t)\, dz = bt. \tag{11}$$

Hence, the expected value of effects in the flying-time interval (*0,t*) is:

$$E_t[z] = \frac{PK_g\overline{K}_g P_1 R\overline{R}E[\Delta\overline{z}]}{\tau}\, t. \tag{12}$$

Because of losses in the aircraft fleet due to hostile counteractions, i.e. to damages resulting in the aircraft loss, the (total) flying time *t* is a random variable. Let:

Q – be the probability of the aircraft loss (destruction) in the course of one sortie due to both the hostile counteraction and destructive damages resulting from the unreliable operation of the aircraft,

λ – be the aircraft loss intensity that equals

$$\lambda = \frac{Q}{\tau}. \tag{13}$$

The average (total) flying time $\overline{T}_t$ (the expected value) of the aircraft within the time interval (0,*t*) is determined with the following relationship (with account taken of possible losses):

$$\overline{T}_t = -te^{-\lambda t} + \frac{1}{\lambda}(1-e^{-\lambda t}). \tag{14}$$

Hence, the expected value of effects of the aircraft's operation is:

$$\overline{E}_t[z] = \frac{PK_g\overline{K}_g P_1 R\overline{R}E[\Delta\overline{z}]}{\tau}\left[\frac{1}{\lambda}(1-e^{-\lambda t}) - te^{-\lambda t}\right]. \tag{15}$$

or

$$\overline{E}_t[z] = PK_g\overline{K}_g P_1 R\overline{R}\,E[\Delta\overline{z}]\,\overline{n}. \tag{16}$$

where:

$\overline{n}$ - the average number of flights throughout the lifetime, which equals:

$$\overline{n} = \frac{\frac{1}{\lambda}(1-e^{-\lambda t}) - te^{-\lambda t}}{\tau}. \tag{17}$$

On the other hand, the expected value of effects throughout the total operational lifetime (for $t \to \infty$) is:

$$\overline{E}_\infty[z] = -\frac{PK_g\overline{K}_g P_1 R\overline{R}E[\Delta\overline{z}])}{\tau}\frac{1}{\lambda} = \frac{PK_g\overline{K}_g P_1 R\overline{R}E[\Delta\overline{z}])^2}{\tau}\frac{\tau}{Q} = \frac{PK_g\overline{K}_g P_1 R\overline{R}E[\Delta\overline{z}])}{Q}. \tag{18}$$

When the above-gained formula (18) is analysed, one should remember and take account of the fact that Q, i.e. the probability of the aircraft loss never takes value equal to zero in the course of performing a combat task (i.e. a sortie). What may result if $Q = 1$ is maximum one take-off for a sortie and then the effects will be closely related to that sortie. To practically use final formulae (15) or (16), it is still necessary to determine the explicit form of the $E[\Delta\bar{z}]$.

4. Determination of the increment in effect with available air warfare agents in the course of one sortie

It is generally known that results of applying air warfare agents depend mainly on the hit accuracy, characteristics of destructive effects of armament sections (warheads) of missiles, the quantity/amount of warfare agents applied, and vulnerability/susceptibility of attacked hostile targets to destruction [3, 6].

Let us discuss some definite case to illustrate how to determine the effectiveness of warfare agents upon the increment in effect $E[\Delta\bar{z}]$. We assume the aircraft to be furnished with four guided air-to-air missiles (AAMs) designed to destroy aerial targets. The aircraft is also capable of launching two missiles in a single-salvo mode. Let us also assume the aircraft attacks two aerial targets and is capable of performing maximum two independent attacks at targets to be destroyed. In the assumed situation the aircraft can destroy either 'no' ('zero'), or one, or two hostile targets. Another assumption is that the probability of destroying the target with one missile is:

$$\bar{p} = drk\,, \tag{19}$$

where:

d – probability of the on-board launch-ready preparation (in-flight arming) of the missiles,

r – reliability of the missile from the launch up to reaching the target,

k – probability that the target is destroyed by the missile providing the missile has reached the target (the area of strike of the warhead).

Therefore, probability of destroying one target with two missiles at one attack is:

$$q = 1 - (1 - drk)^2\,, \tag{20}$$

where $(1 - drk)$ – probability that the target is not destroyed with one missile.

Probability of destroying 'no' ('zero') targets in two attacks are:

$$q_0 = P\{\Delta\bar{z} = 0\} = (1 - q)^2\,. \tag{21}$$

Probability of destroying one target in two attacks is:

$$q_1 = P\{\Delta\bar{z} = 1\} = q(1 - q) + (1 - q)\,q = 2q\,(1 - q)\,, \tag{22}$$

whereas probability of destroying two targets in two attacks is:

$$q_2 = P\{\Delta\bar{z} = 2\} = q^2\,. \tag{23}$$

Hence,

$$E[\Delta\bar{z}] = q_0 \cdot 0 + q_1 \cdot 1 + q_2 \cdot 2\,. \tag{24}$$

In a similar way one can determine $E[\Delta\bar{z}]$ for more complex arrangements of weapon systems. The analysis of forms of up to the present gained final dependences (mathematical formulae) only confirms the commonly known fact that if considerable combat effects are expected, the following requirements should be satisfied [5, 7]:

1. High values (close to unity) of the following probabilities should be provided:
   - suitability,
   - availability of the aircraft and the pilot,
   - in-flight reliability of the aircraft and the pilot;
2. Single-sortie effects should be maximised by means of applying effective weapon systems and suitable aircraft equipment (systems).

3. The aircraft should be operated in such a way as to minimise probability of destroying the friendly aircraft.

With both data reflecting actual operational rates of aircraft subject to the assessment available and data on battlefield conditions and requirements, one can predict with the above-described method the effects of combat applications of these aircraft in particular sorties, in the course of operations, or throughout the aircraft lifetimes (service lives).

5. Final remarks

The above-presented description delivers formulae for the assessment of effectiveness, which take the following forms:

a) for a single sortie by the aircraft:

$$E_1[z] = PK_g \overline{K}_g P_1 R \overline{R}\, E[\Delta \overline{z}], \tag{25}$$

b) for the aircraft's performance within some time interval, e.g. in the course of an air mission:

$$E_t[z] = PK_g \overline{K}_g P_1 R \overline{R}\, E[\Delta \overline{z}]\overline{n}, \tag{26}$$

c) for the assessment of effects throughout the total operational life-time (service life) of the aircraft:

$$E_\infty[z] = PK_g \overline{K}_g P_1 R \overline{R}\, E[\Delta \overline{z}] \cdot \frac{1}{Q}, \tag{27}$$

where:

$$E[\Delta \overline{z}] = q \cdot 0 + q_1 \cdot 1 + q_2 \cdot 2, \tag{28}$$

$$\overline{n} = \frac{\frac{1}{\lambda}(1 - e^{-\lambda t}) - t\, e^{-\lambda t}}{\tau}. \tag{29}$$

In the case the data on the warfare agents are subject to change, the dependence for the increment in effect $E[\Delta \overline{z}]$ should be adequately modified.

If new rates of aircraft operation are introduced and types of warfare agents changed, the dependences should be appropriately adapted. One should note that results/findings of field tests could also be used to estimate rates of effectiveness.

References

[1] Habayeb, A. R., *System Effectiveness*, Pergamon Press, 1987.

[2] Kececioglu, D., *Maintainability, Availability and Operational Readiness Engineering Handbook*, DEStech Publications, Inc, 2002.

[3] Loroch, L., Tomaszek, H., *An introduction to the assessment of operational effectiveness of military aircraft*, Zagadnienia Eksploatacji Maszyn 1(141), pp. 67-80, Radom 2005.

[4] Risken, H., *The Fokker-Planck Equation. Methods of Solution and Applications*, Springer Verlag, Berlin /Heidenberg/ New York/Tokio 1984.

[5] Sikorski, M., Tomaszek, H., *A study on the effectiveness of selected aeronautical components and systems*, Wydawnictwo ITWL, Warszawa 1982.

[6] Tomaszek, H., *Air weapons systems*, Wydawnictwo WAT, Warszawa 1995.

[7] Tomaszek, H., Wróblewski, H., *The essentials of the assessment of operational rates of air weapons systems*, Dom Wydawniczy Bellona, Warszawa 2001.