AVAILABILITY OF A TECHNICAL OBJECT DETERMINED IN A FINITE TIME INTERVAL

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Abstract

In the article, a method for determination of the technical object availability, $G^{OT}(\tau)$ *in a finite time interval* τ*, defined by the time of the transport task duration, has been discussed. All of the study has been presented on the example of a selected system of technical object operation – a system of transport means operation, in a big urban complex (more than 400 thousand inhabitants). Direct performance of the transport tasks in the analyzed system is the responsibility of an executive system consisting of elementary subsystems of the type operator- transport means. Availability of transport means elementary subsystems is crucial for efficient accomplishment of transport tasks. The presented method involves determination of the technical object availability, on the basis of a mathematical model of the operation process which takes place in the studied system, taking into consideration the technical object reliability characteristics. For this purpose, significant states of the transport means operation have been determined and they have been classified in terms of the availability criterion. Basing on this, an event and mathematical models of transport means operation process have been developed, with the semi Markov homogenous model serving as a mathematical model of the studied operation process. Also, values of the analyzed characteristics have been determined.*

Keywords: *transport system, operation process, semi-Markov model, availability*

1. Introduction

In general, the technical object availability (of an element or a system) can be defined as a feature which characterizes it in terms of its capability to achieve or maintain the availability state (enabling performance of the transport task). The concept of availability applies to such systems which are due to react fast in emergencies that is: the army, police, emergency service, fire brigade, and transport systems as well. In these systems, in case of an emergency, a human or a group of humans, together with the technical objects they are in charge of, take immediate actions in order to deal with it [1, 11, 12].

Efficiency of the transport means system operation depends on proper performance of the task it is supposed to deal with. It is the direct responsibility of elementary subsystems of the type: operator – transport means, being components of the executive subsystem of the transport means operation system. One of the factors largely affecting the possibility of appropriate performance of the transport task is availability of a particular elementary subsystem which is determined for time interval τ, intended for its completion.

If the transport means operation process model, for the analyzed system, is a stochastic process being the Markov or semi Markov process, then availability $K^{OT}(t)$ of a single technical object (transport means), determined on the basis of the Markov operation process model, in time t, can be established basing on boundary probabilities of being in states of the so developed operation process model. In order to determine availability of a single technical object (transport means) on the basis of Markov or semi Markov operation process, the process states must be divided into states belonging to a set of availability states SG and states belonging to a set unavailability states SNG. The set of the technical object availability states SG consists of states in which the technical object and its operator are available and well equipped to perform the task or will be made available and equipped in time shorter than the time which is meant for it. The set of the technical object unavailability states S_{NG} consists of states in which the object and its operator are beyond the operation system (available or unavailable) and also when an unavailable and/or unequipped object is within the operation system.

Then, availability $K^{OT}(t)$ of a single technical object, determined in time t, is defined as a sum of time probabilities $p_i(t)$ of the object being in states belonging to the set of availability states S_G, according to dependences [2, 3, 11, 12]

$$K^{OT}(t) = \sum_{i} p_i(t), \ i \in S_G . \tag{1}$$

For time $t \to \infty$, the function described by dependence (1) reaches the boundary value called a boundary coefficient of availability and is defined as a sum of boundary probabilities p_i^* of the process being in states belonging to the set of availability states S_G:

$$K^{OT} = \lim_{t \to \infty} K^{OT}(t) = \sum_{i} p_i^*, \ i \in S_G . \tag{2}$$

Whereas, availability $G^{OT}(t,\tau)$ of a single technical object (transport means) used in an executive subsystem of a transportation system, in time interval τ, is determined as a product of availability $K^{OT}(t)$ and reliability $R^{OT}(t,\tau)$ of a single technical object. Hence, after having taken into consideration formulas (1) and (2), it is expressed by dependencies [1, 11, 12]:

$$G^{OT}(t,\tau) = K^{OT}(t) \cdot R^{OT}(t,\tau), \tag{3}$$

or

$$G^{OT}(t,\tau) = \sum_{i} p_i(t) \cdot R^{OT}(t,\tau), \ \ i \in S_G , \tag{4}$$

where $R^{OT}(t,\tau)$ - function of reliability of a single technical object (transport means).

Function $G^{OT}(t,\tau)$ for time $t \to \infty$ can approach a boundary value called a stationary value, hence:

$$G^{OT}(\tau) = \lim_{t \to \infty} G^{OT}(t,\tau) = \lim_{t \to \infty} \left[K^{OT}(t) \cdot R^{OT}(t,\tau)\right] = K^{OT} \cdot R^{OT}(\tau), \tag{5}$$

or

$$G^{OT}(\tau) = \sum_i p_i^* \cdot \frac{1}{E(T^{OT})} \cdot \int_\tau^\infty R^{OT}(x)dx, \quad i \in S_G, \tag{6}$$

where $E(T^{OT})$ - expected value of availability time for a single technical object (transport means).

In this paper, for determination of availability of a single technical object (transport means) used in an executive subsystem of a transport system, a mathematical model of transport means operation process has been developed with the use of the semi-Markov theory processes.

2. Mathematical model of transport means operation process

A mathematical model of the transport means operation process has been developed on the basis of the event model of this process, built basing on an analysis of the space of sates and events occurring in an operation system of transport means (city buses), used in the analyzed, real transportation system. In result of a multi-state process of technical objects operation, significant states of the operation process have been determined as well as possible transitions between these states. On this basis, a graph of the operation process state changes has been built. This graph is demonstrated in Fig. 1.

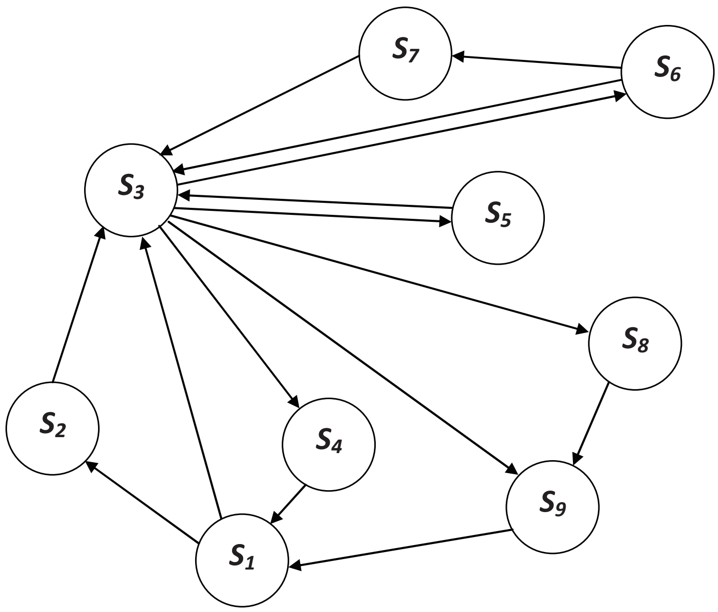


Fig. 1. Directed graph of transport means operation process imaging; S_1 – standby at the bus depot, S_2 – assuring availability at the bus depot, S_3 – transport task performance, S_4 – refuelling between rush hours, S_5 – assuring availability by an emergency service without losing a ride, S_6 – assuring availability with losing a ride,
S_7 – waiting for transport task performance after being made available by an emergency service unit, S_8 – emergency exit, S_9 – making the technical object available at stations of the subsystem for assuring serviceability

Using the semi-Markov processes for mathematical modelling of the operation process, the following assumptions were accepted:

- the modelled operation process has a finite number of states S_i, $i = 1,2,\ldots,9$,
- random process $X(t)$, being a mathematical model of the operation process, is a homogenous process,
- in time $t = 0$ the process is in state S_3 (S_3 is the initial state).

The semi-Markov homogenous process is uniquely defined when its initial distribution and nucleus are given [2, 3, 5, 7]. From the accepted assumptions, and on the basis of the directed graph shown in Fig. 1, the initial distribution $p_i(0)=P\{X(0)=i\},\ \ i=1,2,...,9$ has the form:

$$p_i(0)=\begin{cases}1 & if \quad i=3,\\ \\ 0 & if \quad i\neq 3,\end{cases} \tag{7}$$

however, the nucleus of process *Q(t)*:

$$Q(t)=\begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t)\\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0\\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t)\\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

where:

$$Q_{ij}(t)=P\{X(t_{n+1})=j,t_{n+1}-t_n\leq t|X(t_n)=i\},\ \ i,j=1,2,...,9, \tag{9}$$

means that the semi-Markov process state and its duration time depend only on the previous state, and do not depend on earlier states and their duration times, where t_1, t_2, …, t_n, … are random times, such that $t_1 < t_2 < … < t_n < …$

and

$$Q_{ij}(t)=p_{ij}\cdot F_{ij}(t), \tag{10}$$

where:

$$p_{ij}=\lim_{t\to\infty}p_{ij}(t), \tag{11}$$

p_{ij} - stands for conditional probability of transition from state S_i to state S_j,

$$p_{ij}(t)=P\{X(t)=j|X(0)=i\}, \tag{12}$$

and

$$F_{ij}(t)=P\{t_{n+1}-t_n\leq t|X(t_n)=i,X(t_{n+1})=j\},\ \ i,j=1,2,...,9, \tag{13}$$

is an integral probability distribution function Θ_{ij} denoting duration time of S_i state, providing that S_j state will be the next one.

The semi-Markov mathematical model of the operation process is a stochastic process $X(t)$, with a finite set of states S. If $X(t) = i$, then the process is in the i-th state ($i\in S$). Hence, the probability of process $X(t)$ being in the i-th state is described in the following way:

$$p_i(t)=P\{X(t)=i\},\ \ i=1,2,...,9\,. \tag{14}$$

For time $t\to\infty$, we obtain boundary values (stationary) of time probabilities $p_i(t)$ of being in the states of process $X(t)$, that is:

$$p_i^*=\lim_{t\to\infty}p_i(t),\ \ i=1,2,...,9\,. \tag{15}$$

In order to determine the values of boundary probabilities p_i^* of being in the semi-Markov process model states of transport means operation, there have been built: matrix P of the state change probabilities and matrix Θ of conditional times of $X(t)$ process states duration, on the basis of a directed graph, presented in Fig. 1:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (16)$$

$$\Theta = \begin{bmatrix} 0 & \overline{\Theta}_{12} & \overline{\Theta}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Theta}_{34} & \overline{\Theta}_{35} & \overline{\Theta}_{36} & 0 & \overline{\Theta}_{38} & \overline{\Theta}_{39} \\ \overline{\Theta}_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{63} & 0 & 0 & 0 & \overline{\Theta}_{67} & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\Theta}_{89} \\ \overline{\Theta}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (17)$$

Boundary probabilities p_i^* of being in the semi-Markov process states, were determined on the basis of the limit theorem for the semi-Markov process states [2, 3]:
If the Markov chain entered into the semi-Markov process, with a finite set of states S and a nucleus of a continuous type, contains one class of recurrent positive states, such that for each state $i \in S, f_{ij} = 1$ *and positive expected values* $E(\Theta_i), i \in S$ *are finite, then there exist boundary probabilities*

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)}, \quad (18)$$

where
probabilities $\pi_i, i \in S$ make up a stationary distribution of the Markov entered chain, which also satisfies the system of linear equations

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1. \quad (19)$$

On the basis of data obtained from experimental tests carried in a real system of municipal bus transportation, values of elements of matrix P (16) and Θ (17), unconditional values of the process states duration times and values of the stationary distribution entered into the Markov chain process were estimated. Next, with the use of MATHEMATICA program, values of the semi-Markov process boundary distribution were determined (values of boundary probabilities of the object being in the process states). The results are presented in Table 1. Data from the experimental tests came from 182 technical objects (buses of municipal transportation system) used in a selected real system of municipal transportation, in the period from 04.2009 to 12.2009.

The experimental tests were conducted with the use of a passive experiment method in natural conditions of the studied technical objects operation.

Tab. 1. Values of probabilities p_i^* of the semi-Markov process boundary distribution

$p_1^* = 0.30296$	$p_2^* = 0.00042$	$p_3^* = 0.54054$
$p_4^* = 0.00652$	$p_5^* = 0.00049$	$p_6^* = 0.00215$
$p_7^* = 0.00085$	$p_8^* = 0.00272$	$p_9^* = 0.14334$

3. Availability of technical objects (transport means) determined in finite time interval τ

In order to determine the transport means availability $G^{OT}(\tau)$ in a finite time interval τ, according to dependencies (5) and (6), it is necessary to establish formulas defining:

- availability K^{OT} of a single technical object (transport means), determined on the basis of the semi-Markov operation process model,
- reliability $R^{OT}(\tau)$ of a single technical object (transport means) in time interval τ.

The formula describing availability K^{OT} of a single transport means was developed on the basis of the semi-Markov model of the operation process in which the operation states were divided into states of the object availability S_G and states of unavailability S_{NG} to perform the assigned task. In this model the following states of the object availability to complete its tasks have been distinguished:

state S_1 - standby at the bus depot,
state S_3 - performance of the transport task,
state S_4 - refuelling between the rush hours,
state S_7 - waiting for the task performance to be started after being made fit for use by the emergency service.

In the considered model, state S_4 was classified to states of availability on the basis of the accepted assumption that the technical object equipment in state S_4 is performed during time intended for this purpose so as not to disrupt performance of the task, thereby, without the necessity of its being replaced by a standby object. Then, taking into consideration formula (2), availability K^{OT} of a transportation system technical objects, determined basing on the semi-Markov model of the operation process, with the use of MATHEMATICA program, is defined by the following dependence

$$K^{OT} = \frac{(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta}_1+\overline{\Theta}_3+p_{34}\cdot\overline{\Theta}_4+p_{36}\cdot p_{67}\cdot\overline{\Theta}_7}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta}_1+p_{12}\cdot\overline{\Theta}_2\right)\right]+\overline{\Theta}_3+p_{34}\cdot\overline{\Theta}_4+p_{35}\cdot\overline{\Theta}_5+\left[p_{36}\cdot\left(\overline{\Theta}_6+p_{67}\cdot\overline{\Theta}_7\right)\right]+p_{38}\cdot\overline{\Theta}_8+(p_{38}+p_{39})\cdot\overline{\Theta}_9}. \quad (20)$$

In order to determine reliability $R^{OT}(\tau)$ of the transport means in time interval τ, the type of random variable distribution of availability time T^{OT} (time between failures) was defined for the analyzed transport means. Verification of the accepted null hypotheses H_0 concerning consistence of the examined random variable empirical distribution with selected theoretical distributions was carried out by means of χ^2 test (Pearson test). The following three types of theoretical distributions were chosen for verification: exponential, gamma, normal and log-normal. The studies were carried out for two significance level values $\alpha = 0.05$ and $\alpha = 0.01$. Results of the accepted hypotheses verification have been presented in Tab. 2.

Having analyzed the results presented in Tab. 2, it can be said that there is no reason to reject hypothesis H_0 concerning the consistence of the studied random variable (time of serviceability

T^{OT}) of the empirical distribution with exponential distribution (at the significance level for both 0.05 and 0.01). On this basis there was assumed that reliability of transport means is described by a function with exponential distribution.

Tab. 2. Verification results of accepted hypothesis H_0 with the use of test of goodness to fit χ^2

	Distribution type							
	exponential		gamma		normal		log-normal	
χ_0^2	6.93		451.79		983.07		593.92	
v	10		8		9		8	
α	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
$\chi^2_{1-\alpha}$	18.31	23.21	15.51	20.09	16.92	21.67	15.51	20.09
h_0	1	1	0	0	0	0	0	0

χ^2_0 – calculated value of statistics χ^2. $\chi^2_{1-\alpha}$ – critical value of statistics χ^2 for the accepted significance level α. v – number of freedom degrees χ^2. α – for the accepted significance level. h_0 = 0 – rejection of hypothesis H_0. h_0 = 1 – no grounds for rejection of hypothesis H_0

Then, formula (6), defining transport means availability in a finite time interval τ, will have the following form:

$$G^{OT}(\tau) = K^{OT} \cdot \lambda^{OT} \cdot \int_{\tau}^{\infty} e^{-\lambda^{OT} \cdot x} dx = \sum_{i} p_i^* \cdot e^{-\lambda^{OT} \cdot \tau}, \quad i \in S_G, \tag{21}$$

where:

$\lambda^{OT} = \dfrac{1}{E(T^{OT})}$ - failure intensity for a single technical object (transport means).

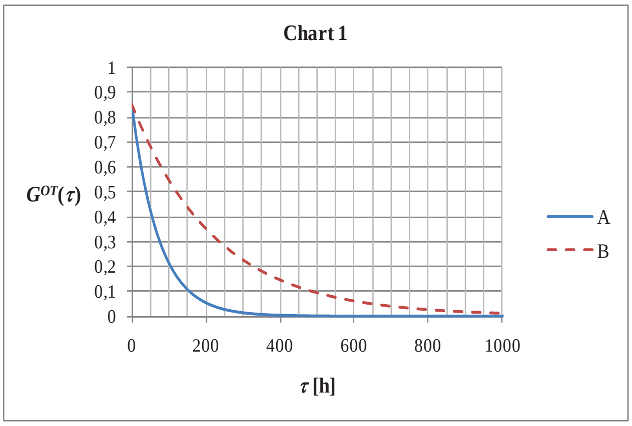
For experimental data, there have been determined values of transport means availability $G^{OT}(\tau)$, in relation to length of time interval τ intended for the task to be performed, for two cases (presented in Fig. 2):

- when all failures of transport means used in the studied transportation system (curve A, for $\lambda_A^{OT} = 0{,}0138$ [h^{-1}]), were taken into consideration,
- with taking into consideration only those transport means failures which occurred during performance of transport tasks, in result of which there can occur disruptions to complete the assigned transport tasks, involving a necessity to do repairs during the task performance. In case of these failures it is necessary to replace the damaged objects with standby ones in order to provide the system with a possibility of the task completion (curve B, for $\lambda_A^{OT} = 0{,}0044$ [h^{-1}]).

In a real transportation system, time interval τ intended for the transport task to be completed is predefined (on the basis of the transport task schedule). Maximal values of the task performance time for transport means in a municipal bus transportation system are contained in a time interval up to 16 [h]. Below, there have been presented charts of the technical object $G^{OT}(\tau)$ availability for a general case (chart1) and an enlarged part of chart 1, for time interval τ from 0 to 16 [h] (chart 2).

4. Conclusion

The studied method makes it possible to determine the technical object availability referred to as the probability with which the technical object, that started to perform a task in time t, will have completed it in due time interval τ. Depending on the accepted assumptions and the technical object operation modelling simplification degree, this method can be applied both with the use of a mathematical apparatus concerning the theory of Markov and semi-Markov processes. Due to its universal character, the proposed method can be applied for determination of availability of technical objects used in systems other than the transportation ones.



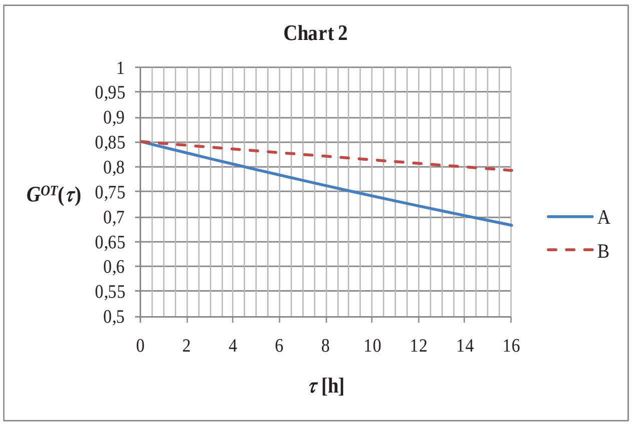


Fig. 2. Values of availability $G^{OT}(\tau)$ *of transport means used in a municipal bus transportation system*

References

[1] Borgoń, J., Jaźwiński, J., Sikorski, M., Ważyńska-Fiok, K., *Niezawodność statków powietrznych*, Wydawnictwo ZETOM, Warszawa 1992.

[2] Grabski, F., *Semi-markowskie modele niezawodności i eksploatacji*, Polska Akademia Nauk, Instytut Badań Systemowych, Badania Systemowe, T. 30, Warszawa 2002.

[3] Grabski, F., Jaźwiński, J., *Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki*, WKiŁ, Warszawa 2009.

[4] Iosifescu, M., *Skończone procesy Markowa i ich zastosowanie*, PWN, Warszawa 1988.

[5] Jaźwiński, J., Grabski, F., *Niektóre problemy modelowania systemów transportowych*, Instytut Technologii Eksploatacji, Warszawa-Radom 2003.

[6] Kowalenko, I. N., Kuzniecow, N. J., Szurienkow, W. M., *Procesy stochastyczne. Poradnik*, PWN, Warszawa 1989.

[7] Kulkarni, V. G., *Modelling and analysis of stochastic systems*, Chapman & Hall, New York 1995.

[8] Leszczyński, J., *Modelowanie systemów i procesów transportowych*, Wydawnictwo Politechniki Warszawskiej, Warszawa 1994.

[9] Migawa, K., *Semi-Markov model of the availability of the means of municipal transport system*, Zagadnienia Eksploatacji Maszyn, 3(159), Vol. 44, Radom 2009.

[10] Praca zbiorowa pod red. J. M., Migdalskiego, *Inżynieria niezawodności. Poradnik*, Wydawnictwo ZETOM, Warszawa 1992.

[11] Ważyńska-Fiok, K., *Podstawy teorii eksploatacji i niezawodności systemów transportowych*, Wydawnictwo Politechniki Warszawskiej, Warszawa 1993.

[12] Żurek, J., *Problemy gotowości techniki lotniczej*, Roz. 13, Praca zbiorowa, Problemy badań i eksploatacji techniki lotniczej, Tom 2, Wydawnictwo ITWL, Warszawa 1993.