

## VELOCITY DISTRIBUTION IN SLIDE JOURNAL BEARING GAP BY LAMINAR UNSTEADY LUBRICATION

Pawel Krasowski

Gdynia Maritime University, Faculty of Marine Engineering  
Morska Street 81-87, 81-225 Gdynia, Poland  
tel.: +48 58 6901331, fax: +48 58 6901399  
e-mail: pawkras@am.gdynia.pl

### Abstract

*This paper shows results of numerical solutions the modified Reynolds equations for laminar unsteady oil flow in slide journal bearing gap. Laminar unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in time. Above perturbations occur mostly during the starting and stopping of machine. During modelling crossbar bearing operations in combustion engines, bearing movement perturbations from engine vertical vibrations causes velocity flow perturbations of lubricating oil on the bearing race and on the bearing slider in the circumferential direction. This solution example applies to isothermal bearing model with infinity length. Lubricating oil used in this model has Newtonian properties and constant dynamic viscosity. Perturbations connected with unsteady lubricating oil velocity in the circumferential direction on the slide bearing and on the slider of bearing were taken into consideration. Results are presented in the dimensionless hydrodynamic pressure and velocity distribution diagrams. Received solutions were compared with the solution received by the stationary lubrication flow in the slide journal bearing, which were made with the same parameter assumption by constant dynamic oil viscosity. Isothermal bearing model is similar to friction node model by steady-state heat load conditions.*

**Keywords:** laminar unsteady lubrication, journal slide bearing, pressure, velocity perturbation

### 1. Introduction

This article refer to the unsteady, laminar flows issue, in which modified Reynolds number  $Re^*$  is [4, 5] smaller or equal to 2. Laminar, unsteady oil flow is performed during periodic and unperiodic perturbations of bearing load or is caused by the changes of gap height in the time. Above perturbations occur mostly during the starting and stopping of machine. Lubricated oil disturbance velocity the pin and on the bearing shell was also consider in the article. Reynolds equation system describing Newtonian oil flow in the gap of transversal slide bearing was discussed in the articles [3, 4]. Velocity perturbations of oil lubricated flow on the pin can be caused by torsion pin vibrations during the rotary movement of the shaft. Perturbations are proportional to torsion vibration amplitude, frequent constraint and to pin radius of the shaft. Oil velocity perturbations on the shell surface can be caused by rotary vibration of the shell together with bearing casing. This movement can be considered as kinematical constraint for whole bearing friction node. Isothermal bearing model can be approximate to bearing operation in friction node under steady-state thermal load conditions for example bearing in generating set on ship.

### 2. Reynolds equation and hydrodynamic pressure

The unsteady, laminar and isotherm flow Newtonian oil in journal bearing gap is described for modified Reynolds equation [1, 2] from Newtonian oil with constant and variable dynamic viscosity depended for pressure. In considered model we assume small unsteady disturbances and in order to maintain the laminar flow, oil velocity  $V_i^*$  and pressure  $p_1^*$  are total of dependent quantities  $\tilde{V}_i$ ;  $\tilde{p}_1$  and independent quantities  $V_i$ ;  $p_1$  from time [3, 5] according to equation (1).

$$V_i^* = V_i + \tilde{V}_i, \quad p_1^* = p_1 + \tilde{p}_1, \quad i = 1, 2, 3. \quad (1)$$

Unsteady components of dimensionless oil velocity and pressure are [4] in following form of infinite series:

$$\begin{aligned} \tilde{V}_i(\varphi, r_1, z_1, t_1) &= \sum_{k=1}^{\infty} V_i^{(k)}(\varphi, r_1, z_1) \exp(jk\omega_0 t_0 t_1) \quad i=1, 2, 3, \\ \tilde{p}_1(r_1, z_1, t_1) &= \sum_{k=1}^{\infty} p_1^{(k)}(r_1, z_1) \exp(jk\omega_0 t_0 t_1), \end{aligned} \quad (2)$$

where:

$\omega_0$  - angular velocity perturbations in unsteady flow,

$j = \sqrt{-1}$  imaginary unit.

Components of oil velocity  $V_\varphi, V_r, V_z$  in cylindrical co-ordinates  $r, \varphi, z$  have presented as  $V_1, V_2, V_3$  in dimensionless [1] form:

$$V = UV_1, \quad V_r = \psi UV_2, \quad V_z = \frac{U}{L_1} V_3, \quad (3)$$

where:

$U$  - peripheral journal velocity  $U = \omega R$ ,

$\omega$  - angular journal velocity,

$R$  - radius of the journal,

$\psi$  - dimensionless radial clearance ( $10^{-4} \leq \psi \leq 10^{-3}$ ),

$2b$  - length of bearing,

$L_1$  - dimensionless bearing length,

$\varepsilon$  - radial clearance:

$$\psi = \frac{\varepsilon}{R}, \quad L_1 = \frac{b}{R}. \quad (4)$$

Putting following quantities [1], [4]: dimensionless values density  $\rho_1$ , hydrodynamic pressure  $p_1^*$ , time  $t_1$ , longitudinal gap height  $h_1$ , radial co-ordinate  $r_1$  and co-ordinate  $z_1$ .

$$\begin{aligned} \rho &= \rho_0 \rho_1, \quad p = p_0 p_1, \quad t = t_0 t_1, \\ z &= b z_1, \quad r = R(1 + \psi r_1), \quad h = \varepsilon h_1. \end{aligned} \quad (5)$$

Rule of putting dimensionless velocity and pressure quantities in unsteady and steady part of the flow stays similar. Following symbols with bottom zero index signify density, dynamic viscosity, pressure and time describe characteristic dimension values assigned to adequate quantities. Reynolds number  $Re$ , modified Reynolds number  $Re^*$  and Strouhal number  $St$  are in form [1]:

$$p_0 = \frac{U \eta_0}{\psi^2 R}, \quad Re = \frac{U \rho_0 \psi R}{\eta_0}, \quad Re^* = \psi Re, \quad St = \frac{\varepsilon}{U t_0}. \quad (6)$$

In work [1], [3] are presented general equations for dimensionless components circumferential velocities  $V_i^{(k)}$  and longitudinal velocities  $V_3^{(k)}$  in form:

$$\begin{aligned} V_i^{(k)} &= \frac{j}{\eta_1 A_k} \frac{\partial p_1^{(k)}}{\partial x_i} \left\{ 1 - \frac{\exp(r_1 \sqrt{j A_k}) + \exp[(h_1 - r_1) \sqrt{j A_k}]}{1 + \exp(h_1 \sqrt{j A_k})} \right\} + \\ &+ \frac{1}{k^2} \left\{ V_{i0} \frac{\exp[(r_1 - h_1) \sqrt{j A_k}] - \exp[-(r_1 - h_1) \sqrt{j A_k}]}{\exp(-h_1 \sqrt{j A_k}) - \exp(h_1 \sqrt{j A_k})} + V_{ih} \frac{\exp(-r_1 \sqrt{j A_k}) - \exp(r_1 \sqrt{j A_k})}{\exp(-h_1 \sqrt{j A_k}) - \exp(h_1 \sqrt{j A_k})} \right\}, \end{aligned} \quad (7)$$

where:

$$A_k = k \frac{\rho_1}{\eta_1} Re St \omega_0 t_0, \quad k = 1; 2; 3; \dots \quad i=1, 3, \quad x_1 = r_1; \quad x_3 = z_1. \quad (8)$$

Dimensionless quantities  $V_{i0}, V_{ih}$  are components perturbations in circumferential velocities

( $i=1$ ) and longitudinal ( $i=3$ ) on the journal ( $r_1 = 0$ ) and the sleeve ( $r_1 = h_1$ ). Dimensionless components  $V_1^{(k)}$  unsteady flow for circumference of oil velocity and  $V_3^{(k)}$  for longitudinal of oil velocity we assume on the oscillating journal and sleeve surface the following boundary conditions:

$$\begin{cases} V_1^{(k)}( ; r_1 = 0; z_1) = \gamma_V \frac{V_{10}}{k^2}, & \begin{cases} V_3^{(k)}( ; r_1 = 0; z_1; t_1) = \gamma_V \frac{V_{30}}{k^2}, \\ V_3^{(k)}( ; r_1 = h_1; z_1; t_1) = \gamma_V \frac{V_{3h}}{k^2}. \end{cases} \end{cases} \quad (9)$$

Quantity  $\gamma_V$  are factor of scale for velocity perturbations, dependent for acceptant of term series (2). Velocity perturbation  $U_0$  in direction  $\varphi$  on the journal and velocity perturbation  $U_h$  on the sleeve, velocity perturbation  $V_0$  in direction  $z_1$  on the journal and  $V_h$  on the sleeve have following dimensionless form:

$$\begin{aligned} V_{10} &\equiv \frac{U_0}{U}, & V_{1h} &\equiv \frac{U_h}{U}, \\ V_{30} &\equiv \frac{V_0}{L_1 U}, & V_{3h} &\equiv \frac{V_h}{L_1 U}. \end{aligned} \quad (10)$$

Presented for formulas (7) components velocity perturbations include derivatives components of pressure perturbation  $p_1^{(k)}$  by coordinates  $\varphi$  and  $z_1$ . Reynolds equations [1, 3, 4] for pressure components  $p_1^{(k)}$  by laminar unsteady oil flow have form:

$$\begin{aligned} &\frac{\partial}{\partial} \left( \frac{h_1^3}{\eta_1} \frac{\partial p_1^{(k)}}{\partial} \right) \cos(k\omega_0 t_0 t_1) + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left( \frac{h_1^3}{\eta_1} \frac{\partial p_1^{(k)}}{\partial z_1} \right) \cos(k\omega_0 t_0 t_1) = -12\gamma_h \frac{\omega_0}{\omega} \frac{h_1}{k} \sin(k\omega_0 t_0 t_1) + \\ & - \frac{6}{k^2} \gamma_V \left\{ \frac{\partial}{\partial} [h_1(V_{10} + V_{1h})] + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} [h_1(V_{30} + V_{3h})] - 2 \left( V_{1h} \frac{\partial h_1}{\partial} + \frac{1}{L_1^2} \frac{\partial h_1}{\partial z_1} \right) \right\} \cos(k\omega_0 t_0 t_1) + \\ & + \frac{1}{2k^2} \gamma_V \left\{ \frac{\partial}{\partial} [A_k h_1^3 (V_{10} + V_{1h})] + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} [A_k h_1^3 (V_{30} + V_{3h})] \right\} \sin(k\omega_0 t_0 t_1). \end{aligned} \quad (11)$$

Reynolds equation describing total dimensionless pressure  $p_1^*$  (sum steady and unsteady components) in oil journal bearing gap [1] by unsteady, laminar, isotherm Newtonian flow along with disturbances of peripheral velocity  $V_{10}$  on the journal and  $V_{1h}$  on the sleeve and disturbances of velocity on journal length  $V_{30}$  on the journal and  $V_{3h}$  on the sleeve has following form:

$$\begin{aligned} &\frac{\partial}{\partial} \left( h_1^3 \frac{\partial p_1^*}{\partial} \right) + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left( h_1^3 \frac{\partial p_1^*}{\partial z_1} \right) = 6 \frac{\partial h_1}{\partial} + \\ & + \frac{1}{2} \frac{\rho_1}{\eta_1} \text{Re}^* n \gamma_V \left\{ \frac{\partial}{\partial} [h_1^3 (V_{10} + V_{1h})] + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} [h_1^3 (V_{30} + V_{3h})] \right\} \sum_{k=1}^{\infty} A_{(k)} + \\ & - 6\gamma_V \left\{ \frac{\partial}{\partial} [h_1 (V_{10} + V_{1h})] + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} [h_1 (V_{30} + V_{3h})] - 2 \left( V_{1h} \frac{\partial h_1}{\partial} + \frac{1}{L_1^2} V_{3h} \frac{\partial h_1}{\partial z_1} \right) \right\} \sum_{k=1}^{\infty} B_{(k)} \end{aligned}$$

for  $0 \leq \varphi \leq \varphi_e$ ;  $0 \leq r_1 \leq h_{p1}$ ;  $-1 \leq z_1 \leq 1$ ;  $0 \leq t_1 \leq t_k$ ;  $p_1^* = p_1^*(\varphi; z_1; t_1)$ . (12)

Sum for series  $\sum_{k=1}^{\infty} A_k$  and  $\sum_{k=1}^{\infty} B_k$  in right side of Reynolds equation (12) are results from conservation of the momentum solutions and were define in work [1, 2].

$$\begin{aligned} \sum_{k=1}^{\infty} A_{(k)} &= \sum_{k=1}^{\infty} \frac{\sin(k\omega_0 t_0 t_1)}{k} = \begin{cases} \frac{\pi - \omega_0 t_0 t_1}{2}, & 0 < t_1 < 1, \\ 0, & t_1 = 0; 1, \end{cases} \\ \sum_{k=1}^{\infty} B_{(k)} &= \sum_{k=1}^{\infty} \frac{\cos(k\omega_0 t_0 t_1)}{k^2} = \frac{1}{4} \left[ (\pi - \omega_0 t_0 t_1)^2 - \frac{\pi^2}{3} \right], \quad 0 \leq t_1 \leq 1. \end{aligned} \quad (13)$$

The equation solution (12) for bearing with infinity length determine unsteady dimensionless hydrodynamic pressure function  $\tilde{p}_1$  in following [2] form:

$$\tilde{p}_1 = \frac{1}{2} \rho_1 \text{Re}^* n V_V (V_{10} + V_{1h}) \left( -h_{1e}^3 \int_0^{\frac{d}{h_1^3}} \right) \sum_{k=1}^{\infty} A_{(k)} - V_V p_{10} (V_{10} - V_{1h}) \sum_{k=1}^{\infty} B_{(k)}. \quad (14)$$

Pressure  $p_{10}$  is located in the oil gap by steady flow and by constant oil dynamic viscosity.

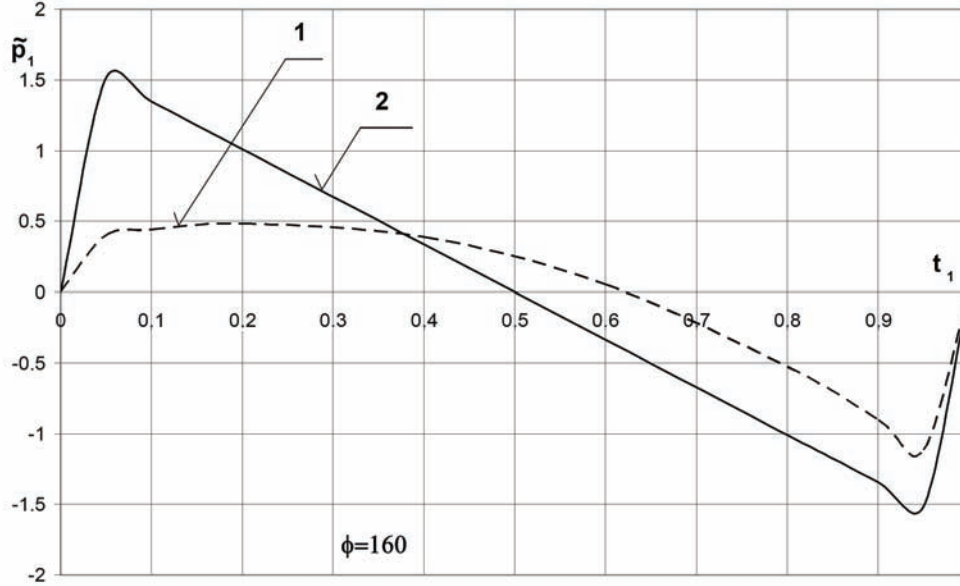


Fig. 1. Pressure distributions  $\tilde{p}_1$  in place  $\varphi=160^\circ$  in the time  $t_1$  by velocity perturbations: 1)  $V_{10}=0.05$ ;  $V_{1h}=0$ ; 2)  $V_{10}=0.05$ ;  $V_{1h}=0.05$

Pressure perturbation  $\tilde{p}_1$  course in point  $\varphi=160^\circ$  presented Fig.1 by following circumferential velocity perturbations:

1. velocity perturbations on the journal  $V_{10}= 0.05$  and on the sleeve  $V_{1h}=0$ ,
2. velocity perturbations on the journal  $V_{10}= 0.05$  and on the sleeve  $V_{1h}= 0.05$ .

Two velocities perturbations will analysed in this article.

### 3. Velocity distribution in bearing gap

We analyse cylindrical bearing infinite length with circumferential velocity perturbations on the journal  $V_{10}$  and on the sleeve  $V_{1h}$ . Circumferential velocity perturbations are caused by torsional vibrations shaft (on the journal) or circumferential displacement frame bearing (on the sleeve). In the further numerical analysis relation time  $t_0$  was taken into account as a propagation period of axial velocity perturbation of lubricating oil. Express by a formula (7) are in form:

$$A_k = k \frac{\rho_1}{\eta_1} \text{Re} \text{St} \omega_0 t_0 = k \frac{\rho_1}{\eta_1} \text{Re}^* n, \quad k = 1;2;3..... \quad (15)$$

In case where oil velocity perturbations are caused by forced vibrations of engine then the number  $n$  in equation (5) define multiplication of perturbation frequency  $\omega_0$  to angular velocity of engine crankshaft  $\omega$ . Multiplication factor  $n$  is equal to number of cylinder  $c$  in two-stroke engine ( $s=2$ ) or in four-stroke engine ( $s=4$ ) to number of cylinders  $c/2$ :

$$n = \frac{\omega_0}{\omega} = \begin{cases} c, & s = 2, \\ \frac{c}{2}, & s = 4. \end{cases} \quad (16)$$

Solution Reynolds equation (11) for cylindrical bearing infinity length and Reynolds boundary conditions where film oils ended  $\varphi=\varphi_e$  are in form:

$$\begin{aligned} \frac{\partial p_1^{(k)}}{\partial} \cos(k\omega_0 t_0 t_1) = & -\frac{6\eta_1}{k^2} \gamma_V (V_{10} - V_{1h}) \frac{h_1 - h_{1e}}{h_1^3} \cos(k\omega_0 t_0 t_1) + \\ & + \gamma_V \frac{\rho_1}{2k} \text{Re}^* n (V_{10} + V_{1h}) \frac{h_1^3 - h_{1e}^3}{h_1^3} \sin(k\omega_0 t_0 t_1), \end{aligned} \quad (17)$$

where:  $h_1$  and  $h_{1e}$  height of gap

$$h_1(\varphi) = 1 + \lambda \cos \varphi; \quad h_{1e} = h_1(\varphi_e). \quad (18)$$

Velocity circumferential perturbation (7) are in formula:

$$\begin{aligned} \tilde{V}_1 = & \frac{6\eta_1}{\rho_1 \text{Re}^* n} \gamma_V (V_{10} - V_{1h}) \frac{h_1 - h_{1e}}{h_1^3} \sum_{k=1}^{\infty} \frac{P(k)}{k^3} \cos(2k\pi t_1) + \\ & - \frac{\gamma_V}{2} (V_{10} + V_{1h}) \frac{h_1^3 - h_{1e}^3}{h_1^3} \sum_{k=1}^{\infty} \frac{P(k)}{k^2} \sin(2k\pi t_1) + \\ & + \gamma_V V_{10} \sum_{k=1}^{\infty} \frac{A(k)}{k^2} \cos(2k\pi t_1) + \gamma_V V_{1h} \sum_{k=1}^{\infty} \frac{B(k)}{k^2} \cos(2k\pi t_1). \end{aligned} \quad (19)$$

Components of series  $P(k)$ ,  $A(k)$  and  $B(k)$  from formula (19) are in following forms:

$$\begin{aligned} P(k) = & \frac{\sin(\text{sh}_1 X_k) \sinh[(1-s)h_1 X_k] + \sin[(1-s)h_1 X_k] \sinh(\text{sh}_1 X_k)}{\cosh(\text{sh}_1 X_k) + \cos(\text{sh}_1 X_k)}, \\ A(k) = & \frac{-\cosh(\text{sh}_1 X_k) \cos[(2-s)h_1 X_k] + \cosh[(2-s)h_1 X_k] \cos(\text{sh}_1 X_k)}{\cosh(2h_1 X_k) - \cos(2h_1 X_k)}, \\ B(k) = & \frac{\cosh[(1+s)h_1 X_k] \cos[(1-s)h_1 X_k] - \cosh[(1-s)h_1 X_k] \cos[(1+s)h_1 X_k]}{\cosh(2h_1 X_k) - \cos(2h_1 X_k)}, \\ s = & \frac{r_1}{h_1}; \quad \gamma_V = \frac{6}{\pi^2}; \quad X_k = \sqrt{k \frac{\rho_1}{2\eta_1} \text{Re}^* n} \quad k=1,2,3,\dots \end{aligned} \quad (20)$$

Additional symbol  $s$  marking dimensionless parameter height of gap ( $0 \leq s \leq 1$ ). In numerical calculation example oil with constant density was assume, what is equivalent to quantity  $\rho_1$ . In presented calculation way an expression value is assumed  $n\rho_1 \text{Re}^* = 12$ , what is approximately

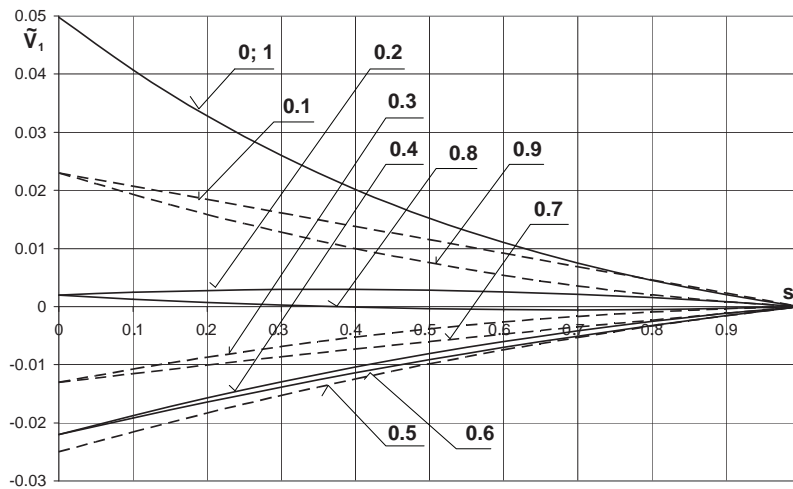


Fig. 2. Velocity distribution  $\tilde{V}_1$  in the circumferential direction, by velocity perturbations  $V_{10} = 0.05$ ,  $V_{1h} = 0$  in gap height  $s$ , in the time  $t_1$ : 0; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1

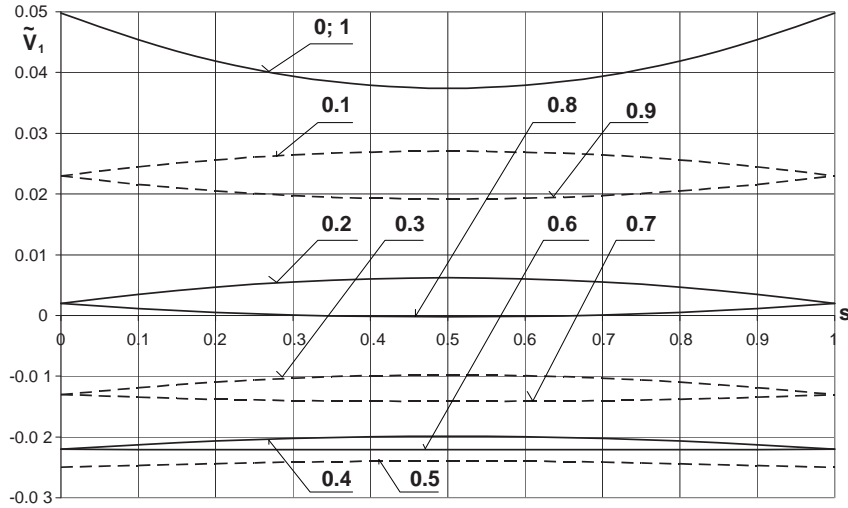


Fig. 3. Velocity distribution  $\tilde{V}_1$  in the circumferential direction by velocity perturbations  $V_{10} = 0.05$ ,  $V_{1h} = 0.05$ , in gap height  $s$ , in the time  $t_1$ : 0; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1

equivalent to force over first frequency torsion vibrations force of six cylinder engine shaft. Examples apply to bearing with constant dependent eccentricity  $\lambda=0.6$ . Numerical examples velocity distributions are by two circumferential velocity perturbations.

Determine a distributions of circumferential velocity in gap height  $s$  by gap crosswise section of coordinate  $\varphi=160^\circ$ . The results velocity distribution in the time  $t_1$  show Fig. 2 and 3. Velocity perturbation are values different from initial perturbations  $V_{10}$ ,  $V_{1h}$ . They are smaller than initial

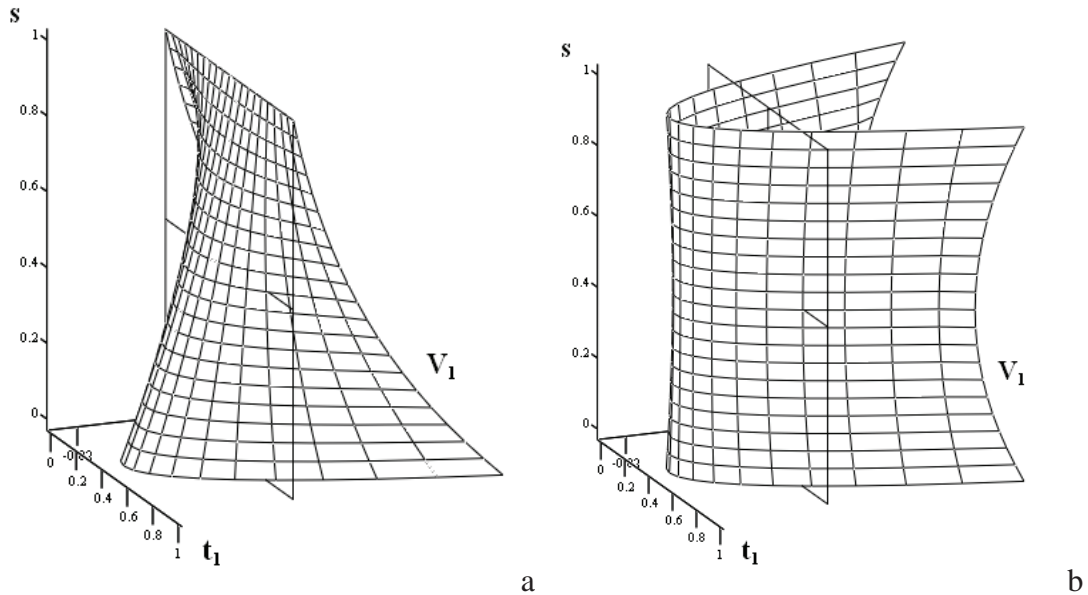


Fig. 4. Velocity distribution  $\tilde{V}_1$  in the circumferential direction in gap height  $s$  in the time  $t_1$  by velocity perturbations a)  $V_{10} = 0.05$ ,  $V_{1h} = 0$ ; b)  $V_{10} = 0.05$ ,  $V_{1h} = 0.05$

values, the values of the opposite sign achieve, and the amplitude of changes are larger than initial disorders (in the example about 25%). Graphs velocity distributions from Fig. 2 are unsymmetrical in the time  $t_1$ . Graphs velocity distributions from Fig. 3 are symmetrical in the time  $t_1$ . Causes are pressure distributions [2] in the time  $t_1$  from Fig. 1. In Fig. 4 presented three-dimensional graphs distributions of velocity perturbation  $\tilde{V}_1$  for description two case velocity



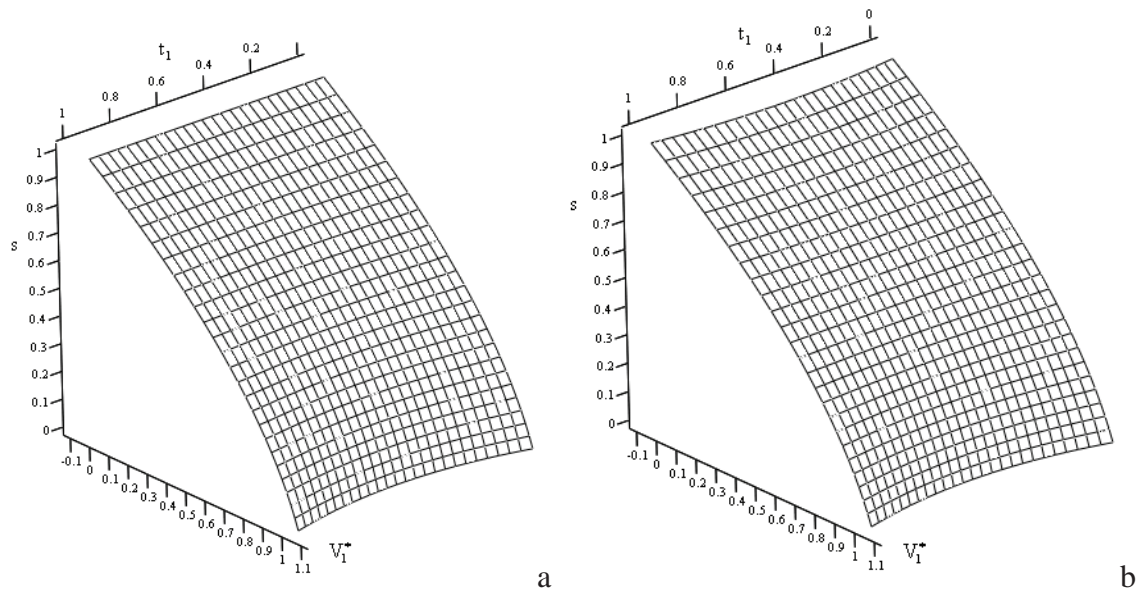


Fig. 5. Velocity distribution  $V_1^*$  in the circumferential direction in place  $\varphi=160^\circ$ , in gap height  $s$ , in the time  $t_1$  by velocity perturbations a)  $V_{10} = 0.05$ ,  $V_{1h} = 0$ ; b)  $V_{10} = 0.05$ ,  $V_{1h} = 0.05$

perturbations in dimensionless gap height  $s$  (vertical axis) in the time  $t_1$ . Velocity perturbation are presented values for horizontal axis  $V_1$ . Additional in Fig. 4 mark frame zero plane, when perturbations of velocity are zero. In Fig. 5 presented three-dimensional graphs distributions of velocity  $V_1^*$  for description two case velocity (a and b) in the circumferential direction in dimensionless gap height  $s$  in the time  $t_1$  by formula (2) in place  $\varphi=160^\circ$ .

#### 4. Conclusions

Presented Reynolds Equation solution for unsteady laminar Newtonian flow of lubricated oil to enable initial opinion to hydrodynamic pressure and velocity distribution as a basic slide bearing operating parameter. Unsteady velocity perturbation on the journal and sleeve effect on hydrodynamic pressure and velocity distribution in lubricated gap. Pressure variation in bearing have periodical character equal to periodical velocity perturbation time and this variations value and character depend on type of perturbation. Pressure increase and decrease and velocity distributions is not symmetrical during the perturbation time. Author is aware of simplifications that were assumed in presented model which apply to Newtonian oil and to isothermal bearing model. Despite that presented calculation example apply to bearing with infinity length, obtained conclusions can be useful to pressure and velocity distribution by laminar, unsteady lubrication of cylindrical slide bearing with finite length.

#### References

- [1] Krasowski, P., *Modelling of laminar unsteady and unsymmetrical oil flow in slide journal bearing gap*, Tribologia 5, pp. 1425-1436, 2002.
- [2] Krasowski, P., *Pressure and velocity distribution in slide journal bearing gap by laminar unsteady lubrication*, Journal of KONES, Powertrain and Transport, Vol. 16, No. 2, pp. 239-246, Warsaw 2009.
- [3] Teipel, I., Waterstraat, A., *The Reynolds equation for lubrication under unsteady conditions*, Proceedings The IX Canadian Congress of Applied Mechanics, University of Saskatchewan, 497-502, 1983.

- [4] Wierzcholski, K., *Mathematical methods in hydrodynamic theory of lubrication*, Technical University Press, Szczecin 1995.
- [5] Wierzcholski, K., *Teoria niekonwencjonalnego smarowania łożysk ślizgowych*, Technical University Press, Szczecin 1995.