

## MATHEMATICAL MODEL OF THE HYDRAULIC VALVE TIMING SYSTEM

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### Abstract

The paper presents a conception and mathematical model of the hydraulic valve timing system. It describes the complex physical phenomena that occur between system components and the hydraulic liquid. Basing on postulates and initial conditions model, one was simulated the movement of elements system resulting from the flow of pressured liquid. The model was simulated in Matlab/Simulink program for different geometrical parameters of the hydraulic system and electric control parameters in order to obtain required lift and timing of valves in YARIS SI 1.3 l engine. The paper presents structure of Simulink model and results of calculation in Simulink. One represented the influence of various parameters and dimensions system on the character of valve lift. The relationship between the diameter of the rod and the liquid significantly affects the valve time-area parameter. The feeding liquid pressure in the system and the re-steering time have the biggest influence on course of valve lift. After preliminary tests the guidelines for the hydraulic valve timing system have been developed. The determined simulation results in Simulink were used to calculate of the device work in the engine computer model created in GT-Power program. The proposed solution was compared with the cam valve timing systems. On the basis of the simulations tests show a big superiority of the hydraulic valve timing system at higher rotational speeds.

**Keywords:** transport, combustion engines, valve timing, hydraulic control

### 1. Tasks of valve timing system

The engine timing system influences on mass flow rate into the cylinder and now is the one of the most developed units in internal combustion engines. Limitation of the exhaust gas components emission and engine parameters are depended on valve timing. Timing systems applied till now in four-stroke engines has a lot of limitation. Applying of camshaft, pushers, valve levers, rods, springs, poppet valves and other elements limits valve lift, influences on velocity and acceleration of moving parts and thus on inertia forces. Valves driven by mechanical system open and close according to a cam profile, which is should be convex for the contact of the cam and pusher. Mechanical limitation of cam design causes, that valves can be opened only in the limit about 240 deg CA. Longer or shorter opening of valve is possible only by applying of special system with changeable profile of cams or by using a camshaft with two or three cams for each valve. Mechanical solutions were applied in Toyota engines (VVTL system) or Honda engines (VTEC). The engine working at different loads and different rotational speeds changes demand on amount of air, particularly in spark ignition engines, where the total excess air coefficient is near one. The mentioned Toyota and Honda solutions belongs to so innovation technique called variable valve timing system. Modern engines should characterize higher total efficiency in whole range of rotational speed, which can be met by changing of valve time-area. Conventional design of the cam profile leads to lower value of this parameter. By applying of electronic control unit it is possible to control valve mechanical systems in order to obtain well engine parameters. Several research works were done on applying hydraulically controlled valve systems, for example in Poland by Zbierski [13].

Complexity of developed model forced to setting up some postulates and initial conditions:

- the system is perfectly sealed, there are only a leakage between the actuator chambers,
- elements of the system are perfectly stiff,
- pressure of the medium has a constant value,
- there is no cavitation,
- working fluid is not pestilential and polluted and fluid has a constant temperature of 90°C,
- there is no hysteresis effect.

## 2. Mathematical model of the hydraulic valve timing system

Kinematics and dynamics of new valve system can be obtained on basis of mathematical equations fluid and mechanical parts motion. This approach can be useful for optimization of whole mechanism with regard of short time of valve opening, small inertia forces and reduction of oil pressure in the hydraulic system.

### 2.1. Modelling of movement resistance of the system

There are movement resistance forces during motion of the distributor's slider and piston rods. There are non-linear sources of moving system components. Three forces of resistance are as follows; inertia force, friction force, hydrodynamic forces. Inertial force is given by formula:

$$F_b = m \cdot a , \quad (1)$$

where:

$m$  - mass of moving element,

$a$  - acceleration of moving element.

In the hydraulic systems moving mass of the element should be increased of mass medium. The oil or other fluid has a mass, whose omission significantly affect on the final result. In the proposed hydraulic system friction forces are induced as a result of contact between the piston seal and the cylinder, piston rod with sealing, slider distributor of the body. They have adversely effect on motion dynamics of the system and for its durability. The described friction force considered in hydraulic system depends on: surface type of working parts, roughness, circularity, concentricity, tolerances, working medium type, radial and axial forces, pressure and temperature. Three components of friction model are used in the simulation, which are given in the below equation:

$$F_t(\dot{y}) = F_v(\dot{y}) + F_c(\dot{y}) + F_s(\dot{y}) = \sigma \dot{y} + \text{sign}(\dot{y}) \left[ F_{c0} + F_{s0} \exp\left(-\frac{\dot{y}}{c_s}\right) \right], \quad (2)$$

where:

$F_t$  - friction force [N],

$F$  - viscous friction force [N],

$F_c$  - Coulomb friction force [N],

$F_s$  - static friction force [N],

$F_{c0}$  - Coulomb parameter of friction forces [N],

$F_{s0}$  - static friction force [N],

$F_c$  - Coulomb friction force [N],

$\sigma$  - coefficient of viscous friction [Ns/m],

$c_s$  - Stribeck velocity [m/s],

$\dot{y}$  - the time derivative of the piston deflection [m/s].

The graph of friction force basis of this formula is show on Fig. 1.

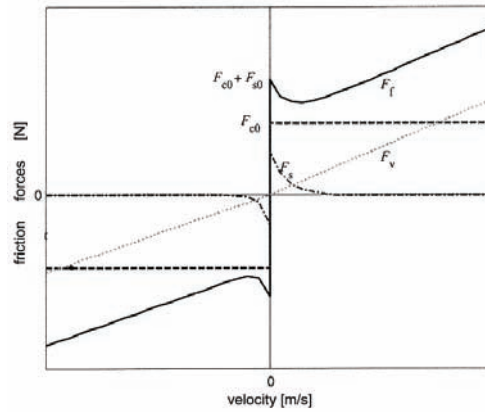


Fig. 1. Graph of friction force [6]

In hydraulic devices operating under high pressure the fluid compressibility has to be taken into consideration. The researchers who had involved with the problem of the hydraulic fluids working parameters had defined many empirical models that describe the change the compressibility module as a function of pressure. In the proposed model an empirical formula is used on liquid compressibility modulus described in [14]:

$$E_a = \frac{1}{2} E_0 \log\left(99 \cdot \frac{p_a}{p_{odn}} + 1\right), \quad (3)$$

where:

$E_a$  - liquid compressibility modulus [Pa],

$E_0$  - Nominal fluid compressibility modulus [Pa],

$p_a$  - pressure [Pa],

$p_{odn}$  - a reference pressure.

Trace of compressibility modulus of liquid in relation to pressure is shown in Fig. 2. For higher pressure this parameter is almost linearly depended from pressure.

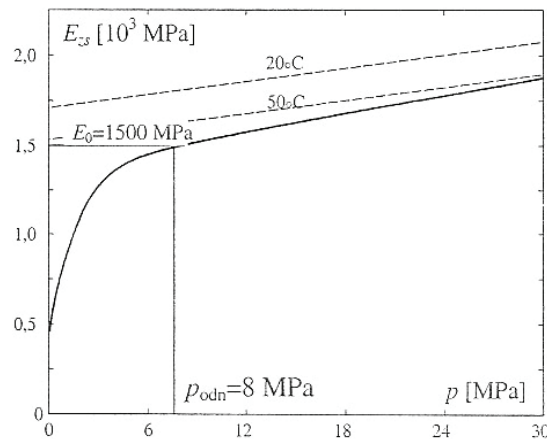


Fig. 2. Compressibility modulus [14]

## 2.2. Modelling the 4/3 slide valve

The slide valve is a device which controls the fluid flow. By changing of the slider position one changes the slot area, enabling the medium flow. This control method found a wide application in position control of hydraulic actuators. The flow rate through the slot of distributor depends on its nature. The flow may be defined as laminar, turbulent or mixed. The first case occurs when low

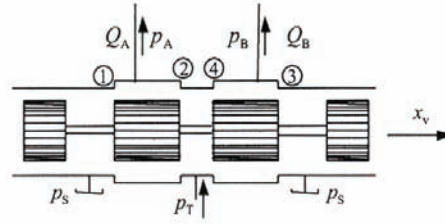


Fig. 3. Diagram of the 4/3 slide valve with zero overlap [6]

and the other at high Reynolds numbers. In this type of the distributor in almost the whole range of its work at turbulent flow takes place. The considered 4/3 slide valve type with zero overlap is shown in Fig. 3. The flow rate can be determined by the formula:

$$Q_A = Q_1 - Q_2 = c_{v1}sg(x_v)sign(p_s - p_A)\sqrt{|p_s - p_A|} - c_{v2}sg(-x_v)sign(p_A - p_T)\sqrt{|p_A - p_T|}, \quad (4)$$

$$Q_B = Q_3 - Q_4 = c_{v3}sg(-x_v)sign(p_s - p_B)\sqrt{|p_s - p_B|} - c_{v4}sg(x_v)sign(p_B - p_T)\sqrt{|p_B - p_T|}, \quad (5)$$

where:

$Q_A$  - flow into the chamber A [ $m^3/s$ ],

$Q_B$  - flow into the chamber B [ $m^3/s$ ],

$p_T$  - outflow pressure [Pa],

$p_A$  - pressure from the line A [Pa],

$p_B$  - pressure from the line B [Pa],

$c_v$  - flow factor [ $m^3/(s N^{0.5})$ ].

However, the function  $sg(x)$  is defined by simultaneous equations:

$$sg(x) = \begin{cases} x & dla \quad x \geq 0, \\ 0 & dla \quad x < 0. \end{cases} \quad (6)$$

### 2.3. Mathematical model of distributor-cylinder system

When we are considering the assumed hydraulic system one should be note some mutual interaction between the modules of the system. Change of location of the slider valve position will affect the movement of the actuator piston rod, but it depends on the flow of the working chamber, the actual position, and consequently the actuator absorptivity, liquid compressibility, the pressure difference in the chambers, the flow rate of liquid outflow, which is regulated by the slider. It is easy to see, that the system has a cascade nature. Changing one single parameter affects changes in other parameters in a certain way.

The equations describing the distributor system - double-acting actuator are purely flow type, differential equations describing the motion or dependence connecting flows, pressures and displacements. The system distributor - double-acting actuator is described by equations:

- the equation of actuator motion:

$$m \frac{d^2 y(t)}{dt} + F_t \frac{dy(t)}{dt} + F_{zew} = A_1 p_1(t) - A_2 p_2(t), \quad (7)$$

- volumetric flow rate between the distributor and actuator chambers:

$$Q(t) = Q_s(t) + Q_h(t) + Q_v(t), \quad (8)$$

where:

$Q$  - flow between the distributor and the actuator [ $m^3/s$ ],

$Q_s$  - flow covering the losses arising out of the liquid compressibility [ $m^3/s$ ],

$Q_h$  - absorption of the actuator [ $m^3/s$ ],

$Q_v$  - the intensity of leakage on the piston [ $m^3/s$ ].

- absorption of actuator:

$$Q_h(t) = A \frac{dy(t)}{dt}, \quad (9)$$

- volumetric flow rate covering the losses arising out of the liquid compressibility:

$$Q_s(t) = \frac{V}{2E_0} \frac{d\Delta p(t)}{dt}, \quad (10)$$

where:

$V$  - volume of liquid in the actuator chamber [m<sup>3</sup>],

$E_0$  - liquid compressibility modulus [Pa],

$\Delta p$  - pressure difference between chambers A and B [Pa],

- intensity of leaks on the piston:

$$Q_v(t) = K_v \Delta p(t), \quad (11)$$

where  $K_v$  is leakage coefficient [m<sup>5</sup>/(Ns)],

- dependence of flow from the piston displacement and pressure in the actuator chamber:

$$Q(t) = K_{Qp} y(t) - K_l \Delta p(t), \quad (12)$$

where:

$K_{Qp}$  - coefficient of flow at zero drop pressure [m<sup>5</sup>/(Ns)],

$K_l$  - linearization of the flow coefficient [m<sup>5</sup>/(Ns)].

### 3. The computer model

Digitization of the mathematical model of a hydraulic valves system drive was done in Matlab/Simulink. It is a universal tool for creating simulations of this type. The computer model simulates the movement of the valve under the influence of pressure on actuator piston rod has a modular construction (Fig. 4). At such complicated simulations, this type of clustering makes the system transparently. It provides the ability to change one segment of the simulation without interfering in other its components.

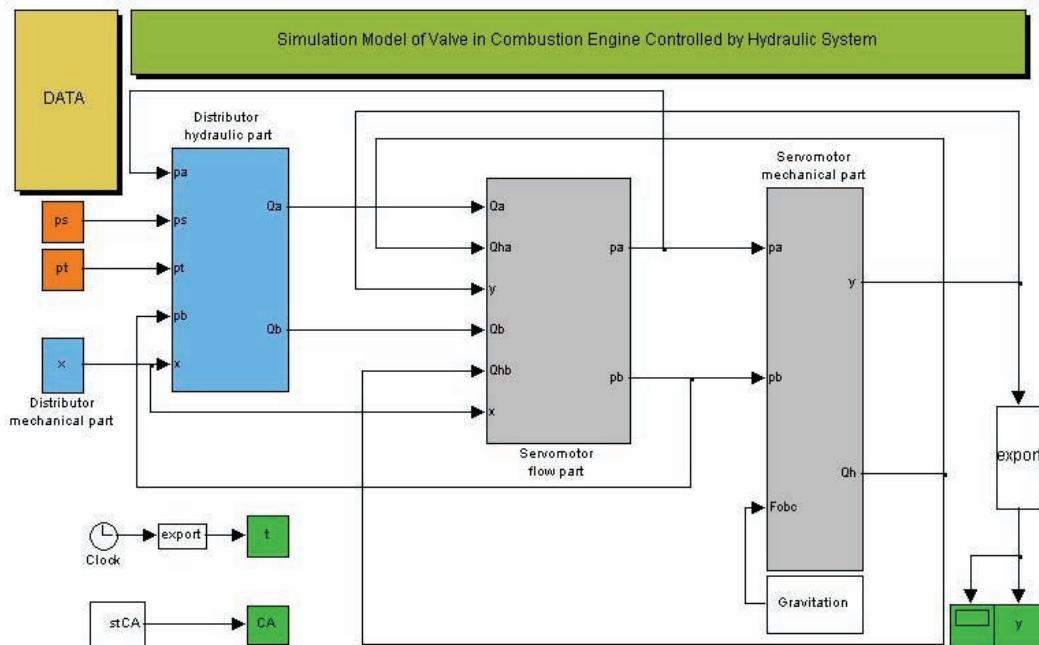


Fig 4. The computer model simulates the movement of the valve in Simulink environment

### 3.1. Divider - the mechanical part

This module provides a displacement of the slider under the influence of electromagnets. The schematic diagram is shown in Fig. 5. The force, which is caused by the action of an electromagnet, is countered damping force and spring. Signals which simulate the forces acting on distributor slider are multiplied by the weight of its moving parts. The received acceleration signal is twice integrated. The result of this operation is signal of which is equivalent of displacement slider -  $x$ .

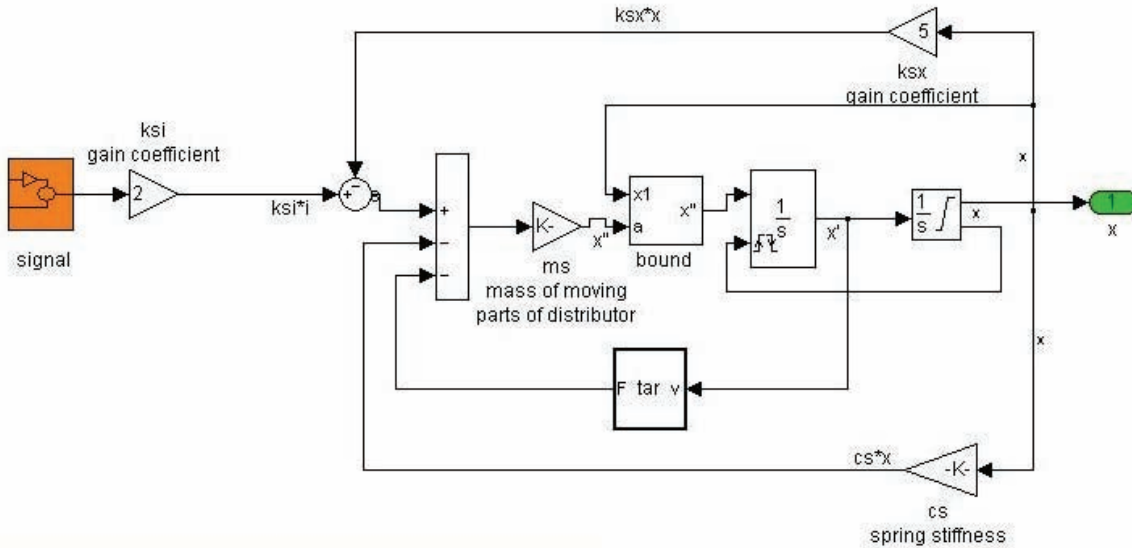


Fig. 5. Module of distributor - the mechanical part

### 3.2. Divider - the flow part

In this part of the computer model of the camless hydraulic valve timing system are determined flow rates occurring in the channels leading to the chambers of the actuator. Diagram of this module is shown in Fig. 6. Input signals are: displacement of the slider -  $x$ , pressure feeding -  $p_s$ , the pressure in the reservoir -  $p_t$ , pressure in chamber A -  $p_a$ , pressure in chamber B -  $p_b$ . Output signals are: flow in the channel chamber A -  $Q_a$ , flow in the channel chamber B -  $Q_b$ .

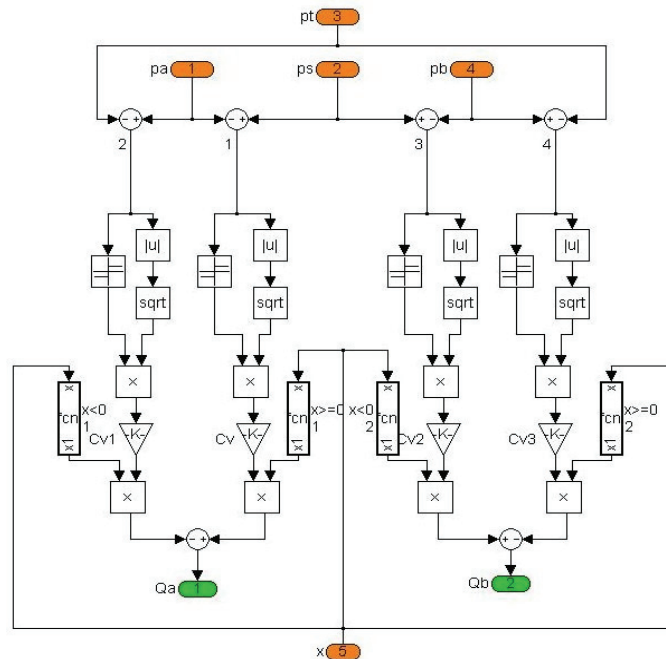


Fig. 6. Module of distributor - the flow part

### 3.3. Actuator - the flow part

Generated in the previous module, the flow rate signals are used to determine the pressure in the chambers of the actuator. Input signals are: displacement of the piston rod –  $y$ , displacement of the slider -  $x$ , flow in the chamber A of actuator -  $Q_a$ , flow in the chamber B of actuator -  $Q_b$  absorption of actuator chamber A -  $Q_{ha}$ , absorption of actuator chamber B –  $Q_{hb}$ . Output signals are: pressure in the chamber A -  $p_a$ , pressure in the chamber B -  $p_b$ . The structure of the module is shown in Fig. 7.

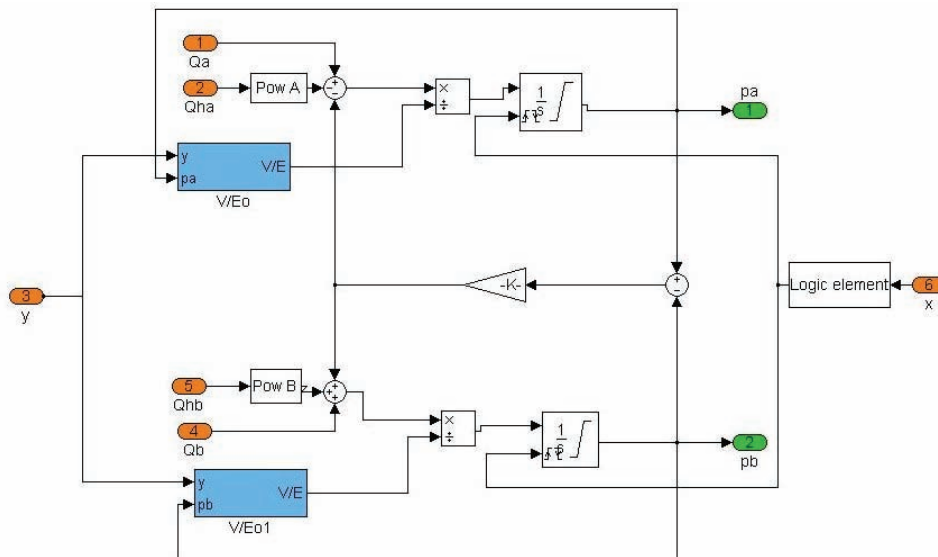


Fig. 7. Module of actuator - the flow part

### 3.4. Actuator - the mechanical part

This subsystem calculates the displacement of the valve (Fig. 8). It uses second-degree differential equation describing the motion of the piston rod under the influence of acting forces.

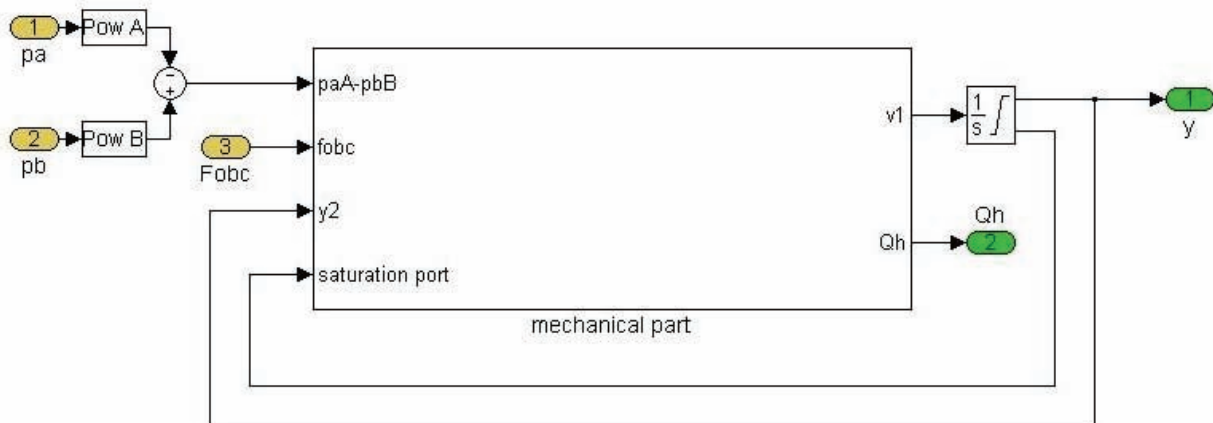


Fig. 8. Module of actuator - the mechanical part

Input signals are: pressure in the chamber A of -  $p_a$ , the pressure the chamber B –  $p_b$ , external load -  $F_{obc}$ . Output signals are: displacement of the piston rod –  $y$ , absorption of actuator chamber -  $Q_h$ . Pressure calculated in the previous module in chambers A and B is multiplied by active surfaces of piston rod. Subsystems “Pow A” and “Pow B” are the blocks generating a force value on the surface of the piston rod based on declared diameter.

Input signals are transmitted to the subsystem “the mechanical part”, where piston velocity and actuator chambers absorptivity signal is generated. It uses elements to simulate effects of safety spring force, friction and amortizations of the piston in the extreme positions.

#### 4. The result of the simulation

The result of the simulation based on the mathematical model is a graph showing valve lift. The sample characteristic of the piston rod displacement is shown in Fig. 9.

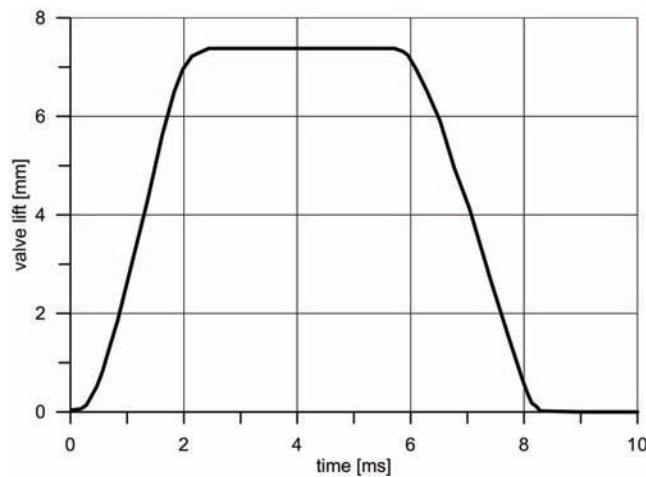


Fig. 9. Characteristics of piston rod displacement

Movement of the valve can be compared to the trapezoidal course. The slope of the piston rod line during lift and sinking is not identical. This is caused mainly by the smaller surfaces of an active side of the piston valve descent. The above diagram shows the amortization system settling the piston in the cylinder. This is best seen when the valve approaching the extreme positions. The correctness of proper work of the proposed model can be determined by analyzing the pressure course in each chamber (Fig. 10). While the pressure increases in the first the chamber at the same time the pressure in second should decrease. The simulation results confirm the above relationship between the chambers of the actuator. The sudden rise of pressure in the chambers is caused by a small volume of the inlet channel and chamber.

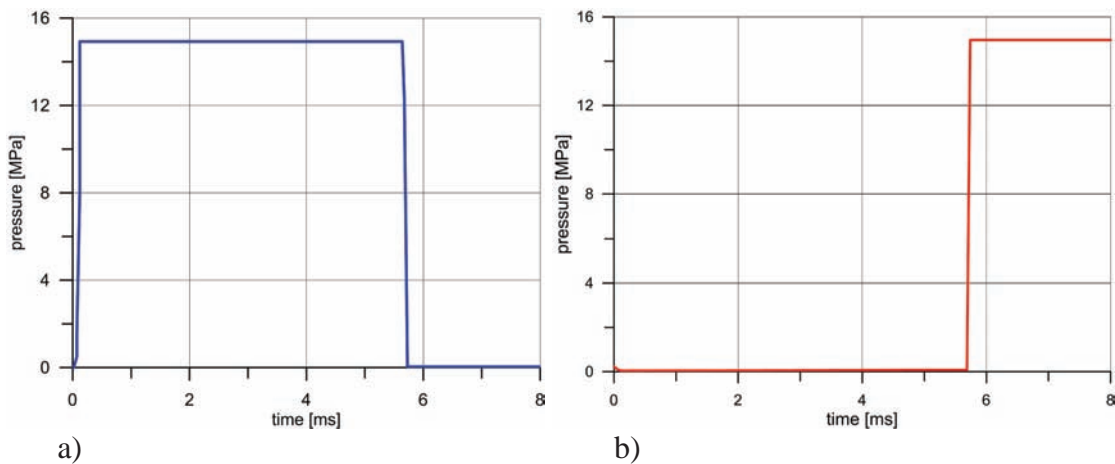


Fig. 10. Characteristics of pressure changes a) the chamber A, b) the chamber B

During the liquid filling the chamber A, the remainder of the working medium in the second chamber causes the pressure peak. This presents a detailed section of Fig. 10b. On the created



computer model series of simulation tests were carried out. They were concerned the effects of change parameters on steering the hydraulic valve timing system. There was examined effect of change switching time on the course of lift and descent of slider valve (Fig. 11 and 12).

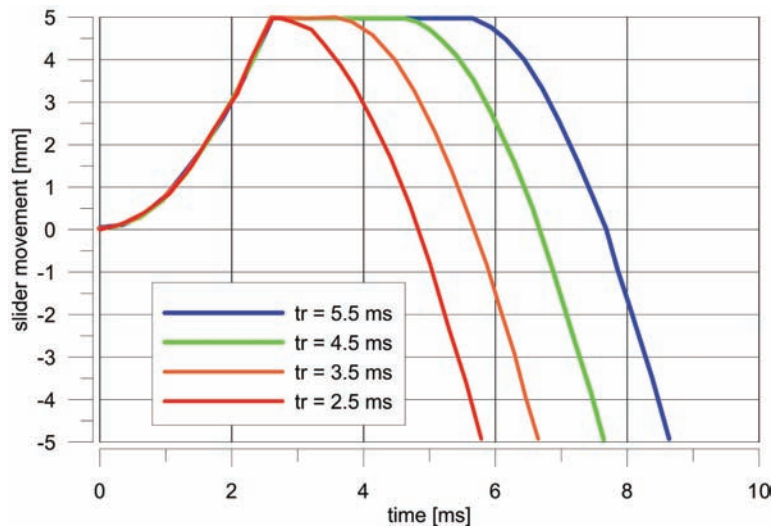


Fig. 11. Characteristics of slider displacement at different distributors switching times

Through proper controlling of the distributor one can mainly regulate the valve closing time. Change the value of switching time distributor does not affect the nature of the valve lift. This parameter can be used as a way to control of time cross-section of the valve. There were also studied influence of an active piston area in time course and nature of the valve lift and descent (Fig. 13). Pressure and distributor switching time were unchanged and amounted 15 MPa and 3.5 ms.

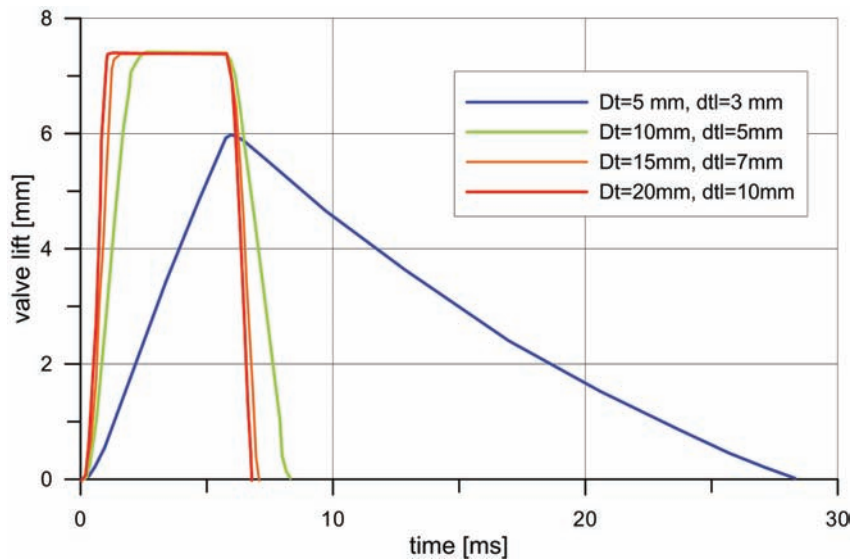


Fig. 13. The graph of the valve's displacement at various diameters of piston rod

Changing of the piston diameter may significantly effect on nature of the valve lift and descent. When this value is too small, it is possible not to achieve full valve lift before distributor's switching time. Increasing the diameter of the piston rod can be an important factor tending to reduce the pressure in the system, however increasing the dimensions of the piston rod is associated with an increased mass of moving parts and the volume of the actuator chambers. Effects of changing pressure on course of valve lift and descent in time also was simulated. Change of pressure is a main parameter controlling the hydraulic valve next to distributor's

steering time. The piston diameter (10 mm) and distributor's steering time (3.5 ms) were constant values during the simulation. Control of the working medium pressure significantly affects on the time and the course of valve lift and descent (Fig. 14).

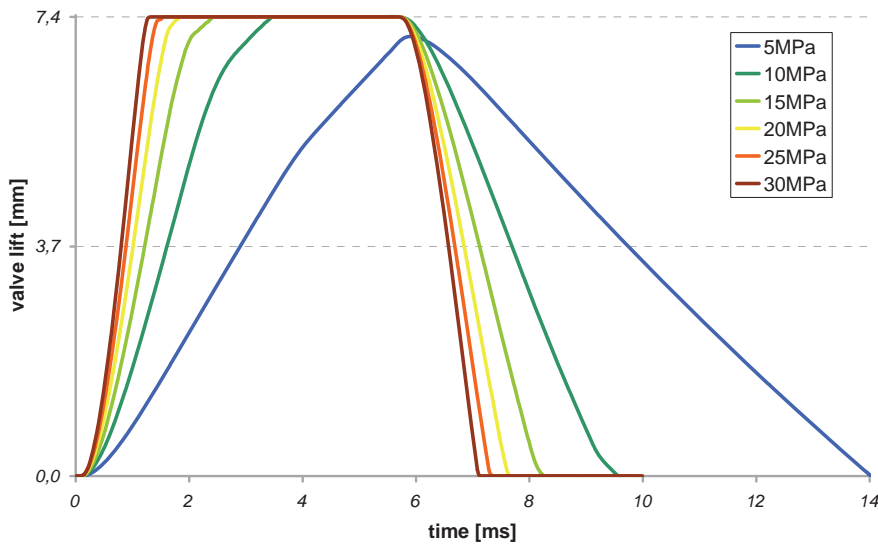


Fig.14. The graph of the displacement valve at different pressures

## 5. Conclusions

1. Mathematical model of fluid flow in the hydraulic valve timing system was given, which enabled to determine valve lift for different fluid pressure and control time.
2. Valve hydraulic drive system enables shaping of valve lift and decrease of valve acceleration in comparison to cam system.
3. Feeding pressure of liquid in the system and re-steering time have the biggest influence on course of valve lift.
4. Valve timing system with hydraulic drive is more complicated and more expensive than conventional system, however, is encouraging for further experimental work and theoretical analysis.
5. Valve braking is required during settle of valve in the seat, because big liquid pressure causes a hard impact of these two parts.

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