MODELLING OF EPS TYPE STEERING SYSTEMS INCLUDING FREEPLAY AND FRICTION IN STEERING MECHANISM

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Abstract

Electric power steering (EPS) systems are important ingredients of modern cars. Basing on mechanic (steering mechanism), electric (electric motor), electronic (controller) and mechatronic (sensors) components, they are typical servo mechanic devices, which support drivers handling action. The paper presents mathematical modelling of EPS systems treated as multi body mechanisms with power assistance, including strong non-linear phenomena caused by a friction (especially in king-pins) and a feeplay (in gear-box). Mathematical model consists of several variable structure ordinary differential equations with piecewise linear characteristics. Model non-linearities are described with using special piecewise linear luz (…) and tar(...) projections. These projections and their original mathematical apparatus facilitate synthesis and analysis of the model. The model equations are derived from constitutive equations and control rules. This method can be also applied for synthesis of other non-linear multi-body electromechanical systems. The elaborated models of EPS systems have the forms ready to use in simulation programs. Some examples of simulation studies of the open car tests due to sensitivity analysis of steering system model are also presented in the paper.

Keywords: *EPS system, freeplay, friction, piecewise linear model, simulation studies of car open tests*

1. Introduction

A steering system is a main element of a car that realises control tasks created by a driver. For facilitation, mechanical handling steering systems are equipped with power assistance components.

Powering of steering systems can be hydraulic (HPS systems), electro-hydraulic (EHPS systems) or electric (EPS systems). Nowadays cars are equipped more and more frequently with EPS systems. Pure electric solutions have a lot of advantages: supplying independent of a car engine, no liquid medium, possibility of mounting servo components in different places of steering mechanism, possibility of application very sophisticate control algorithms, etc.

Therefore, electric power steering systems come to be a standard equipment of nowadays cars. They displace traditional solutions with hydraulic devices.

The EPS system can be treated as a typical electric servomechanism, which contains mechanisms handled by a driver and supported by an electric motor with automatic controller and sensors. This means that synthesis of EPS systems should be done with all control theory requirements and should be preceded by extensive simulation studies. Such investigations demand mathematical models, which enable also analysis of strong non-linear mechanical effects caused by freeplay and friction, which can be danger for automatic action (problem of limit cycles, instability, stick-slip phenomena and other non-linear effects). Note, freeplay / friction attributes of steering systems are restrictively tested when cars pass periodical tests in diagnostic stations! In addition, stand tests of steering systems of many cars, busses, and trucks show [1] that their angular dead-zone (freeplay) parameter and resistance force (friction) might be very diverse and high even in new vehicles! Therefore, the modelling of EPS type steering systems including freeplay and friction attributes appears as important from theoretical and practical point of view.

Modelling of steering system dynamics has been showed in many papers. Their representative examples have been listed in the monograph [6]. Surprisingly, problems of freeplay/friction steering system non-linearities (e.g. and stick-slip phenomena typical for mechanisms working with static friction action)) have discussed very rarely. Many author’s papers, e.g. [1], [2], [3], [6], [7] contain mathematical models and present results of simulation open car tests including freeplay and friction in steering systems. This paper presents new information on modelling important especially for EPS systems.

2. Substitute structure models of steering systems

The steering system is a spatial multi-body system (MBS) installed on moving car-body. The detail dynamical model of such mechanical structure would be very complicate (many mass elements, which give multiple products of inertia). However, elements of steering mechanism are many times smaller then car steered wheels. In addition, axles of king-pins are near vertical to the plane of a car lateral motion. Therefore, it is reasonable to neglect all internal elements’ inertias as well as all spatial products of inertia of steering mechanism. Spatial movements of mechanism elements change a little the geometry of wheels motions, and this fact can be regarded by kinematic-type characteristics. For better formal compatibility between a primary physical model and its mathematical representative, the steering system mechanism can be treated as MBS-type gear system having only masses rotating on vertical axles (details in p.3).

For modelling the EPS system, the conceptual steering mechanism model should be supplemented by electric and mechatronic components (motor, regulator, sensors etc.). They have very small moments of inertia, and therefore they can be treated as autonomic subsystems, which do not change the mechanical structure of the steering system model. So, they can be expressed by additional functional blocks with rather simple sub-models.

Whether the model of the EPS system can be derived independently of the model of full car and than is joined as input block to vehicle motion dynamic structure? Our intuition suggests that yes (note, a car body moment of inertia is many times grater than moments of inertia of the steering system with steered wheels). Sensitivity investigations [3] confirmed such supposition. They showed that errors caused by ignoring of subtle inertia loops did not exceed several per mils. It means the steering system model can be treated as autonomic sub-model of full car dynamics according to partial modelling conception (Fig. 1).

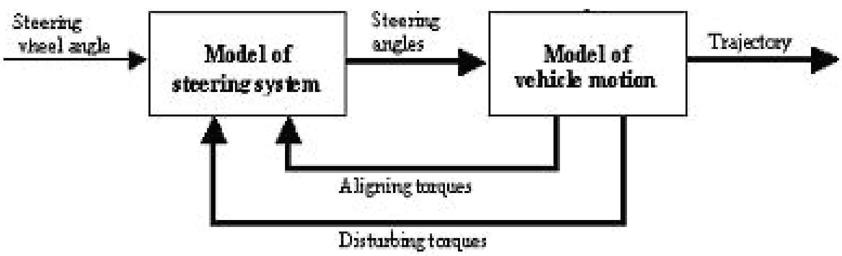

Fig. 1. Conception of decomposition of the car dynamics model into two partial sub-models

3. Model of steering system mechanism

For mathematical modelling of steering system non-linear dynamics the steering system mechanism is treated as MBS-type gear system having only masses rotating on vertical axles Fig. 2 expresses its mechanical structure, which is enough to describe all important mechanical attributes, especially freeplay and friction. This physical model contains not only main elements, but also supplementary mass-less gear wheels and infinitely large stiffness of shafts that facilitate a synthesis of equations of motion. The freeplay concerns the gearbox tooth backlash. The friction (kinetic plus static) action concerns the kingpin bearings as well as the vibration damper.

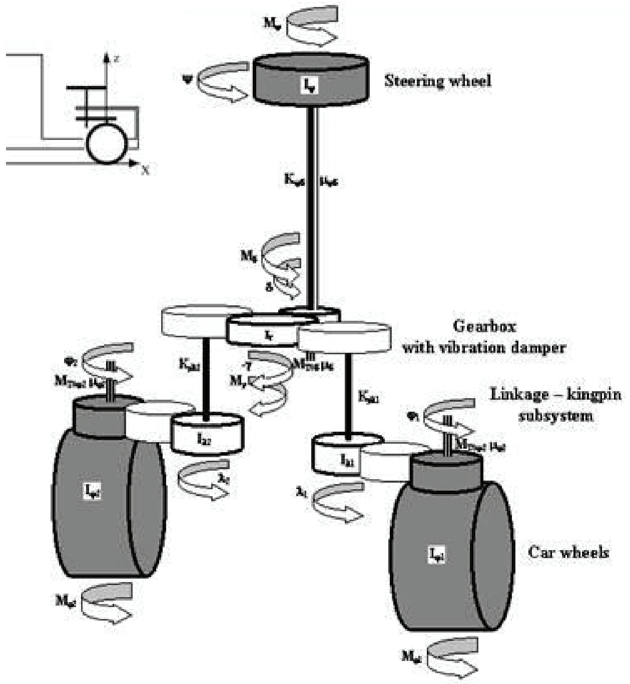

Fig. 2. Steering system mechanism MBS-type equivalent physical model

Notation:

ψ - steering wheel angle,

δ , γ - steering gear input and output angle,

λ_1 ,λ_2 - steering linkage substitutive elements' output angles,

φ_1 ,φ_2 - kingpin (and steered wheels') angles,

M_ψ - steering wheel input torque,

M_δ , M_γ - steering gear assistance torques (two cases),

$M_{\varphi 1}$,$M_{\varphi 2}$ - kingpin external torques (wheels stabilization plus unbalance),

I_ψ - moment of inertia of steering wheel with steering column,

I_δ , I_γ - moment of inertia of gearbox input and output wheel,

$I_{\lambda 1}$,$I_{\lambda 2}$ - moments of inertia of steering linkage substitutive elements,

$I_{\varphi 1}$,$I_{\varphi 2}$ - moments of inertia of kingpins with steered wheels,

$K_{\psi\delta}$ - steering column stiffness coefficient,

$K_{\delta\gamma}$ - gearbox teeth stiffness coefficient,

$K_{\gamma\lambda 1}$,$K_{\gamma\lambda 2}$ - stiffness coefficients of substitutive linkage shafts,

$K_{\lambda 1\varphi 1}$,$K_{\lambda 2\varphi 2}$ - stiffness coefficients of substitutive linkage-kingpin gear subsystem,

$\mu_{\psi\delta}$ - steering column's substitutive material damping coefficient,

μ_δ - damping coefficient of steering mechanism damper,

$\mu_{\varphi 1}$,$\mu_{\varphi 2}$ - damping coefficient of kingpin bearings,

$M_{T0\delta}$ - maximal dry friction torque of gearbox damper,

$M_{T0\varphi 1}$,$M_{T0\varphi 2}$ - maximal dry friction torques of king pin bearings,

$(\delta - p\gamma)_0$ - gear freeplay parameter (1/2 of freeplay seen from steering wheel),

p - gear ratio (for steady state $\delta = p \cdot \gamma$),

n_1 ,n_2 - substitutive gear ratios of left and right part of steering linkage - kingpin subsystem (for steady states $\lambda_1 = n_1 \cdot \varphi_1$, $\lambda_2 = n_2 \cdot \varphi_2$). They can be nonlinear functions of φ_1 and φ_2 .

In utility models one set : $I_\delta = 0$, $I_\gamma = 0$, $I_{\lambda 1} = 0$, $I_{\lambda 2} = 0$, $K_{\lambda 1\varphi 1} \to \infty$, $K_{\lambda 2\varphi 2} \to \infty$, $K_{\delta\gamma} \to \infty$.

To simplify the description of freeplay / friction action, and to obtain a regular form of the model (important for sensitivity analysis [7]), special luz *(…)* and *tar(…)* projections have been applied (Fig. 3). These piecewise linear projections have many interesting properties and create some mathematical apparatus very useful for synthesis and analysis of models of systems with freeplay and friction (and generally for piecewise linear systems). Details are given in [4], [5], [6].

$$luz(x,a) = x + \frac{|x-a| - |x+a|}{2}, \quad tar(x,a) = luz^{-1}(x,a) \quad (-\infty < x < +\infty \text{ and } a \ge 0). \qquad (1), (2)$$

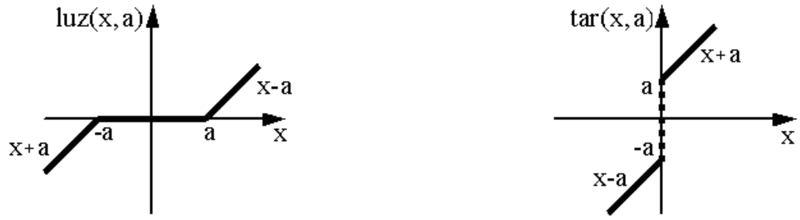

Fig. 3. Topology of luz(…) and tar(…) projections

A mathematical model and its derivation are described in the monograph [6]. The model consists of four variable structure ordinary differential equations which contain piecewise linear components expressing by *luz(…)* and *tar(…)* projections. Because of editorial limitations, here only general form of the final model is showed.

$$\ddot{\psi}(t) = f_\psi\left(\dot{\psi}(t), \dot{\delta}(t), \psi(t), \delta(t), M_\psi(t)\right), \qquad (3)$$

$$\dot{\delta}(t) = f_\delta\left(\dot{\psi}(t), \psi(t), \delta(t), \varphi_1(t), \varphi_2(t), M_\delta(t), M_\gamma(t)\right), \qquad (4)$$

$$\ddot{\varphi}_1(t) = f_{\varphi 1}\left(\dot{\varphi}_1(t), \delta(t), \varphi_1(t), \varphi_2(t), M_{\varphi 1}(t), M_\gamma(t)\right), \qquad (5)$$

$$\ddot{\varphi}_2(t) = f_{\varphi 2}\left(\dot{\varphi}_2(t), \delta(t), \varphi_1(t), \varphi_2(t), M_{\varphi 1}(t), M_\gamma(t)\right). \qquad (6)$$

Input signals are: steering driver's signal ($\psi(t)$ - for kinetic form of steer or $M_\psi(t)$ - for dynamic one), power assistance signal ($M_\delta(t)$ - when powering is on steering shaft, or $M_\gamma(t)$ - when powering is below steering gear), sums of aligning and destructive signals ($M\varphi_1(t)$ - for left wheel, $M\varphi_2(t)$ - for right wheel). Output signals (which are input signals for vehicle motion model) are steered wheels angles (φ_1 and φ_2), as well as any different variables of the model.

This model is proper also for unsymmetrical steering system construction, and for unsymmetrical external excitations (different for left and right wheels). A variable structure of the model (equations (5),(6)) express stick-slip phenomena caused by the dry friction with stiction.

The model enables very sophisticated and extensive simulation studies - for mixed (kinetic/dynamic) forms of steering signals, for different levels of amplitudes of signals.

Thank to different places of additional torque signals, the model is useful for analysis EPS systems having different configurations (column, pinion or rack).

4. Model of steering system electric power assistance

The electric power assistance is accomplished as a typical modern servo system with electric motor, electronic controller and mechatronic sensor (or sensors). The electric motor as an actuator gives a torque signal, which supports driver's action. It is steered by the electronic controller (also digital processor) on the basis of sensor (sensors) voltage signal (signals). The mechatronic sensor measures a strain of the steering mechanism shaft. If the EPS system is equipped in two sensors (e.g. Toyota Prius [8]) the additional sensor measures also a derivative of the steering shaft strain.

We look for the mathematical models, which describe the most important power assistance dynamic properties and are corresponding to our steering mechanism model. Note: the model of steering system mechanism enable two variables of additional torques ($M_\delta(t)$ or $M_\gamma(t)$), and angles ($\psi(t)$ and $\delta(t)$) that can be used to form (by the difference $\varepsilon(t) = \psi(t)-\delta(t)$) the strain of the steering system shaft – for modelling the sensor signal. Here we will discuss a version with $M_\delta(t)$.

A simplified general block diagram of the EPS system is presented on Fig. 4.

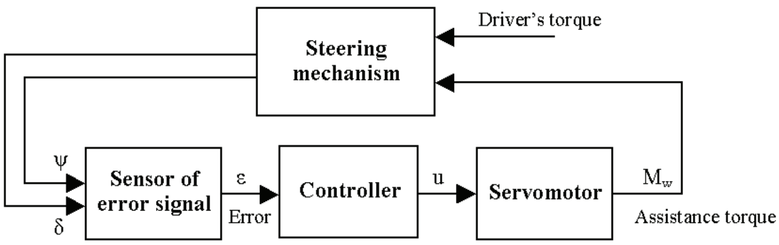

Fig. 4. Block diagram of EPS system (here with one sensor)

Mathematical modelling of electric power assistance can be done by typical simplified dynamical descriptions for each element. These sub-models should have rather simple forms but sufficient for expression a power assistance action during simulation / sensitivity studies focused on freeplay / friction steering mechanism problems.

<u>Sensor:</u>

The sensor (torque sensor) is a small mechatronic device which takes advantage of physical effects as piezoelectricity, LC-resonance, etc. It changes a steering mechanism shaft strain on a voltage signal. According to control system theory, a well-matched sensor should have a small time constant, so its functional dynamical model has a static-type form. For small excitations, it can be expressed by the "error signal" static linear formula:

$$\varepsilon(t) = K_\varepsilon\left(\psi(t) - \delta(t)\right), \tag{7}$$

where: K_ε - gain coefficient of sensor.

<u>Controller:</u>

The controller is an electronic device, which generates steering voltage signal $u(t)$ according to its regulation algorithm on the basis of error voltage sensor signal $\varepsilon(t)$. This algorithm can have different forms depending on control system synthesis method and apparatus. When the algorithm is PID-type (standard for regulation systems), its simplified formula is:

$$u(t) = K_P \cdot \varepsilon(t) + \int_0^t K_I \cdot \varepsilon(\tau)d\tau + K_D \cdot \dot{\varepsilon}(t), \tag{8}$$

where: K_P, K_I, K_D - coefficients of PID controller.

<u>Servomotor:</u>

The EPS system actuator is an electric DC motor. This subsystem can be expressed by a typical substitute diagram.(Fig. 5), which is necessary for derivation its model equations.

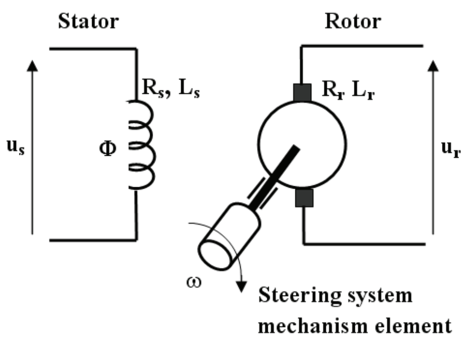

Fig. 5. Substitute physical model of DC electric motor

Notation:
u_s, u_r - stator and rotor applied voltages,
i_s, i_r - stator and rotor currents,
Φ_s, Φ_r - stator and rotor magnetic fluxes,
Φ_m - rotor to stator coupling magnetic flux (with Φ_s),
E_r, - rotor counter-electromotive force (counter voltage),
M_r - induced electromotive torque of rotor,
R_s, R_r - stator and rotor resistances,
L_s, L_r - stator and rotor inductances,
z_s, z_r - stator and rotor numbers of turns of windings,
C_r, - constant of counter voltage,
C_m - constant of induced electromotive torque,
ω - rotor angular velocity,
t - time.

Note: When powering is "on the top" of steering mechanism: $M_\delta(t) = M_r(t)$ and $\omega(t) = \dot{\delta}(t)$.

Equation of stator circuit: $$R_s i_s(t) + z_s \frac{d\Phi_s(i_s(t))}{dt} = u_s(t). \quad (9)$$

Equation of rotor circuit: $$R_r i_r(t) + z_r \frac{d\Phi_r(i_r(t))}{dt} = u_r(t) - C_r \Phi_m(i_s(t))\omega(t). \quad (10)$$

Equations of torques: $$M_r(t) = C_m \Phi_s(i_s(t)) i_r(t) \text{ and } \omega(t) = \dot{\delta}(t). \quad (11),(12)$$

Non-linear magnetic characteristics $\Phi_s(i_s)$, $\Phi_r(i_r)$, $\Phi_m(i_s)$ are smooth, so their linearization is possible around the steady state $u_s = u_{s0}$, $i_s = i_{s0}$, $u_r = u_{r0}$, $i_r = i_{r0}$, $\omega = \omega_0$.

In steady state conditions, time derivatives are zeros, and the steady state point fulfils:

$$R_s i_{s0} = u_{s0}, \; R_r i_{r0} = u_{r0} - C_r \Phi_m(i_{s0})\omega_0, \; M_{r0} = C_m \Phi_s(i_{s0}) i_{r0}. \quad (13), (14), (15)$$

Setting: $z_s \left.\frac{\partial \Phi_s(i_s)}{di_s}\right|_{i_{s0}} = L_s$, $z_r \left.\frac{\partial \Phi_r(i_r)}{di_r}\right|_{i_{r0}} = L_r$, $C_m \left.\frac{\partial \Phi_s(i_s)}{\partial i_s}\right|_{i_{s0}} = L_m$ (L_s, L_r, L_m - inductances),

$$C_r \left.\frac{\partial \Phi_m(i_s)}{\partial i_s}\right|_{i_{s0}} = k_e, \quad C_r \Phi_m(i_{s0}) = k_\omega, \quad C_m \Phi_s(i_{s0}) = k_m,$$

after linearization, one obtains equations for small excitations around steady state point which express detail dynamical model of the electric motor:

$$R_s i_s(t) + L_s \frac{i_s(t)}{dt} = u_s(t), \quad (16)$$

$$R_r i_r(t) + L_r \frac{i_r(t)}{dt} = u_r(t) - k_e \omega_0 i_s(t) - k_\omega \omega(t), \quad (17)$$

$$M_r(t) = L_m i_{r0} i_s(t) + k_m i_r(t). \quad (18)$$

Suppose that powering is steered by the signal $u(t) = u_r(t)$ (steering from rotor circuit). In such case $u_s(t) = 0$ (stator circuit without voltage activation). Then $i_s(t) = 0$, and the model simplifies to

$$R_r i_r(t) + L_r \frac{i_r(t)}{dt} = u_r(t) - k_\omega \omega(t), \quad M_r(t) = k_m i_r(t). \quad (19), (20)$$

The equations (19), (20) give a new one ordinary equation:

$$T_r \dot{M}_r(t) + M_r(t) = K_r(u_r(t) - k_\omega \omega(t)), \quad (21)$$

where: $K_r = \dfrac{k_m}{R_r}$ - gain coefficient, $T_r = \dfrac{L_r}{R_r}$ - time constant.

This electric motor model has two inputs u_s, $k_e\omega$, and one output M_r. But for functioning as an actuator in a SISO-type control system we can achieve relation $u_s(t) >> k_e\omega(t)$. Then the model simplifies to the SISO-type form presented on fig.6, and is expressed by the first order equation:

$$T_r\dot{M}_r(t) + M_r(t) = K_r u_r(t)\,. \tag{22}$$

Note: Modern power assisted steering systems have variable (depending of a car speed V) amplification. For a low speed (parking-like manoeuvres) strong power assistance is applied whereas for a high speed (road-like driving) the power assistance is reduced for stabilization of steering wheel movements. Such adaptation function can be easy realized in our system. The output electromotive torque maximum value can be actively limited (and adapted to temporary vehicle speed) if the electric motor is equipped with an active current limiter, (diode limiter independently steered by additional voltage signal from speed sensor). In such case the final differential model equation is supplemented by a saturation function and piecewise linear functions (Fig. 6). They can be also described analytically with using the *luz(…)* projection.

$$\overline{M}_r(t) = M_r(t) - luz\big(M_r(t), M_{r0}(V)\big), \qquad \text{where:} \tag{23}$$

$$M_{r0}(V) = M_{r01} + \frac{M_{r01} - M_{r02}}{V_1 - V_2}\big(luz(V, V_1) - luz(V, V_2)\big). \tag{24}$$

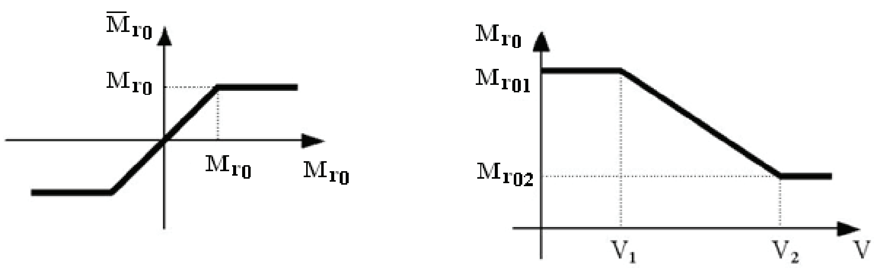

Fig. 6. Characteristics of the electric motor model

The model of the power assistance with adaptation is joined together with the model of steering system mechanism. According to our assumptions, when the power assistance is placed in front of a steering mechanism gear, $M_\delta(t) = \overline{M}_r(t)$ (then $M_\gamma(t) = 0$).

5. Application of EPS steering system model

The elaborated model has been partially experimentally verified and numerically tested [6], and now is ready to use in extensive simulation / sensitivity studies focused on freeplay / friction steering system problems. Such unique investigations have been undertaken.

In these studies the steering system model collaborated with the vehicle motion model describing with details a dynamical motion of a passenger car (see the papers, e.g. [2], [6]). Simulations have concerned typical open road tests according to ISO and ECE regulations as well as to more sophisticate their combinations. During these tests, the freeplay / friction parameters have been varied. Example simulation / sensitivity results for combined ISO test (ISO 7401 ramp input on the steering wheel, then ISO 4138 circular steady state motion, and finally steering wheel released, all with a constant speed 80 km/h) are presented on Fig. 7. The example results (here a time history of the left wheel angle) concern the 2WS vehicle with the EPS system. As we can see, an influence of varied parameters (especially of freeplay) on the results is evident. Comparative simulations [6] show that application of EPS system servo components to steering system mechanism influent a lot on its dynamics as well as on its freeplay / friction sensitivity.

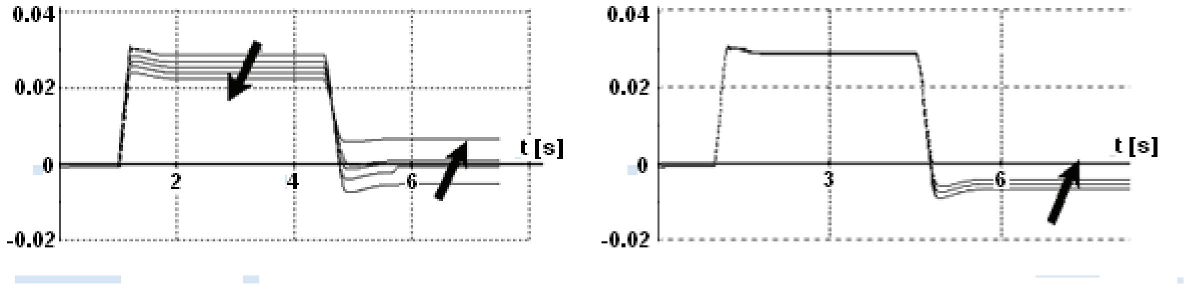

Fig. 7. Time history of the left wheel angle [rd]. The arrows show trends when the parameter increases: A) freeplay parameter $(\delta\text{-}p\gamma)_0$ changes from 0 rd to 0,0.27 rd. Here $M_{T0\varphi 1}$ = 2.7Nm (nominal), B) friction parameter $M_{T0\varphi 1}$ changes from 1.35 Nm to 4.05Nm. Here $(\delta\text{-}p\gamma)_0$ = 0 rd (nominal)

6. Final remarks

- The elaborated model of the EPS steering system enables very sophisticate simulation studies concerning non-linear effects caused by freeplay / friction in steering system mechanisms.
- The EPS power assistance elements change dynamic properties of the steering system and influent on its sensitivity on freeplay / friction in its mechanism.
- Modelling and simulation / sensitivity studies seem to be very important for synthesis of special so called robust controllers especially designed for systems working in the presence of freeplay and friction.
- The simplified version of the steering system model can be used as a reference model in robust controller algorithms.

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