STATIC ANALYSIS AND STABILITY OF THE GRILLAGE

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Abstract

Steel floor structures for which the grillage is a computational model, are used as carrying systems in building engineering, e.g. in buildings intended for warehouses. The linear and nonlinear static analysis was carried out as well as the verification of grillage sensitivity to the effect of stability loss.
Updating of structure geometry description with every step of load increment and updated Lagrange description was applied, and for solution of equilibrium equations the Newton-Raphson method was used with matrix updating with every step of load increment and after each iteration.
At dimensioning of the cross-section should be still added conditions connected with internal forces, so-called limit state of load capacity, which requires to fulfill additional conditions, e.g. related to local stability, geometric imperfections, warps and so on, of individual fragments of beam's cross-section, causing increase of rigidity of the grillage and thus carrying away possibility that nonlinear work of the structure occurs under the right conditions of using. Accomplished assessment of the grillage's sensibility to additional compressive loads Fx gives quantitative information relating to their maximum values, so as to fulfil the condition of the limit deflection as well.

1. Introduction

In the work, the calculations of the grillage as the carrying steel floor structure are presented for a building intended for warehousing of E1 category, i.e. storage areas, including storage of books and other documents.
The linear analysis has been applied and nonlinear, by which the result obtained is closer to real state occurring in the structure.
The incremental method has been used in the work, in which calculations are carried out by stages, by increasing the load by some increment at every stage and finding responding displacement increment.

2. Linear static calculations

First, linear static analysis has been carried out for the grillage of dimensions (12 x 6) m, (Fig. 1). It was assumed that the beams were H-section beams HEB 300, of steel material 18G2AV, and of computational tensile, compression and bending strength f_d =370MPa. Characteristic load q_k was assumed according to [6] $q_k = 7{,}5$kN/m^2 for floors loaded with storage of E1 category, i.e. storage areas, including storage of books and other documents.
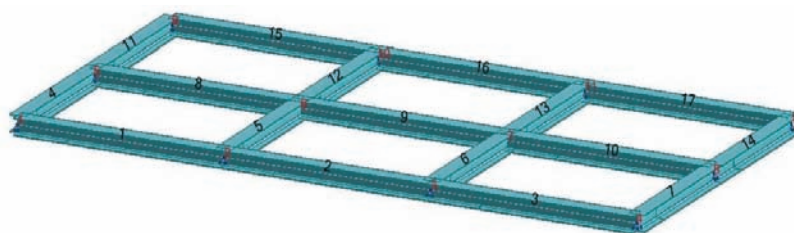

Fig. 1. Structure of the grillage

Solution of the grillage has been carried out by Robot program [5], based on finite element method, which has produced normal stress pattern shown in Fig. 3 and Fig. 4.
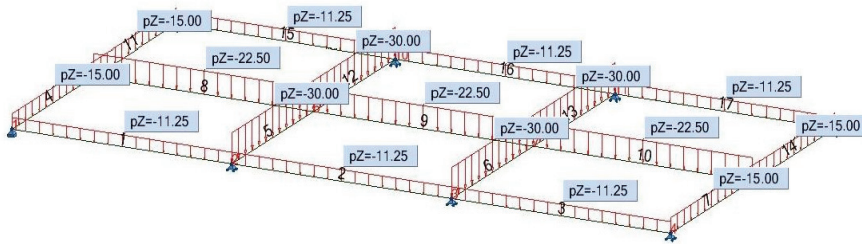

Fig. 2. Loading of the grillage.

3. Nonlinear static calculations

When changing the type of supports in the nodes from slidable bearings to fixed bearings, normal tensile forces appear in the solution and then there is the problem of nonlinear static analysis of the grillage, resulting from nonlinear geometric relations.
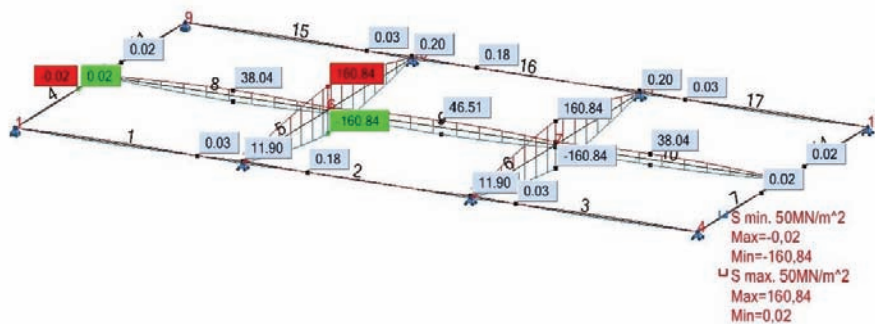

Fig. 3. Normal stress pattern resulted from bending moments

$$\varepsilon_x^z = \frac{du}{dx} - z\frac{d^2w}{dx^2} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2.$$ (1)

Additionally in the nonlinear solution appear normal forces Fx, shearing force Fy and bending moment Mz.
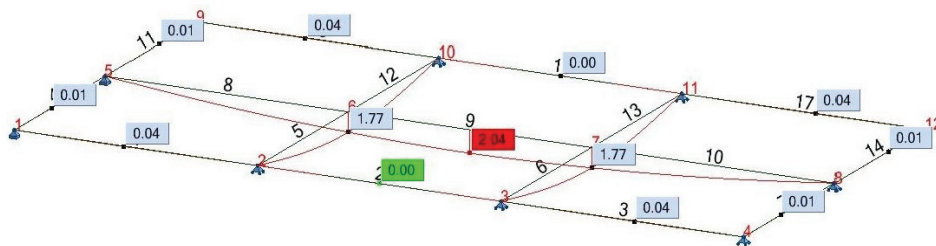

Fig. 4. Deformation of the grillage

It results from use of nonlinear geometric relations, taking into account the influence of deflection on longitudinal displacement (normal force Fx) and of non-dilatational strain angle (shearing force Fy and bending moment Mz connected with this force).

Comparing the results of linear and nonlinear analyses, it may be noticed that for internal forces there are slight changes in the values, that is:
- ➢ for shearing force Fz – reduction by 0,2% of the value,
- ➢ for twisting moment Mx – no change in the value,
- ➢ for bending moment My - reduction by 0,3% of the value,
- ➢ for maximum deflection w_{max} - no change in the value;

which shows that the character of grillage's behaviour is considerably close to linear for the applied load of $q_k = 7.5$ kN/m^2.

In designing practice, standard conditions must be fulfilled. One of these conditions is that maximum deflection cannot exceed the limit deflection, which results from the limit state of using. For the grillage is (Fig. 4)

$$w_{max} = 2.04cm \ < f_{gr} \ = \frac{l_d}{250} = \frac{600}{250} = 2.4cm, \tag{2}$$

and fulfilling of this condition determines high rigidity of the grillage and thus work range of the grillage close to linear.
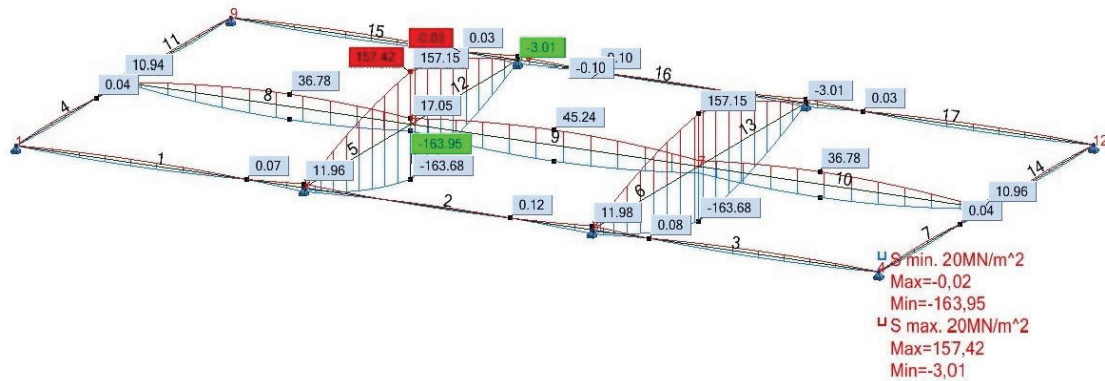

Fig. 5. Normal stress pattern resulted from normal force and bending moments
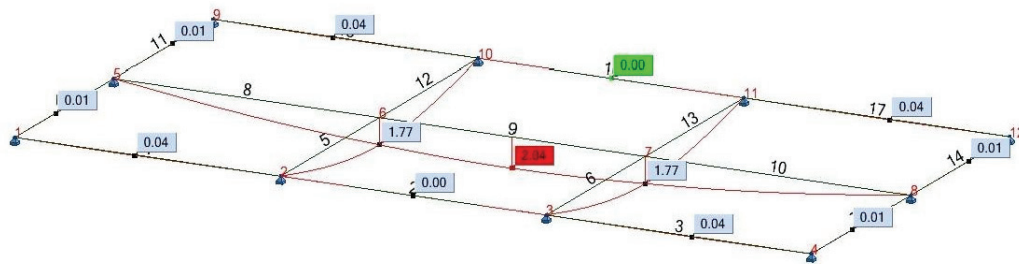

Fig. 6. Deformation of the grillage

For dimensioning of steel structures should be still added conditions connected with internal forces, so-called limit state of load capacity, which requires to fulfil additional conditions, e.g. related to local stability, geometric imperfections, warps and so on, of individual fragments of beam's cross-section. It also limits possibility that nonlinear work of the structure occurs under the right conditions of using.

Then, equilibrium curve of the grillage has been determined, i.e. relationship showing deflection as a function of load q_k – Fig. 7, Table 1.

Diagrams of deflection-load in the range of applied loads q_k differ from linear solution to a small extent, even for high stresses, close to the limit of failure strength R_m=560MPa. For the grillage structure, it means that calculations of linear theory are credible in the process of designing it.

4. Nonlinear analysis of stability of the grillage

In order to estimate sensitivity of grillage structure in situation of occurring additional axial load (Fig. 8) causing normal compressive forces, at bending state acting under usable load q_k at the same time, the analysis of nonlinear stability has been carried out.

From the run of dependence of deflection on normal compressive force F_x (Table 2), a clearly nonlinear character of grillage work is seen. However, because of the condition that maximum deflection cannot exceed the limit deflection, which results from the limit state of using

$$w_{\max} \leq f_{gr} = 2.4cm, \tag{3}$$

the maximum axial forces cannot exceed the value of about $F_x = 1900\text{kN}$, as then $w = w_{\max} = 2.4\text{cm}$, from nonlinear solution of stability.
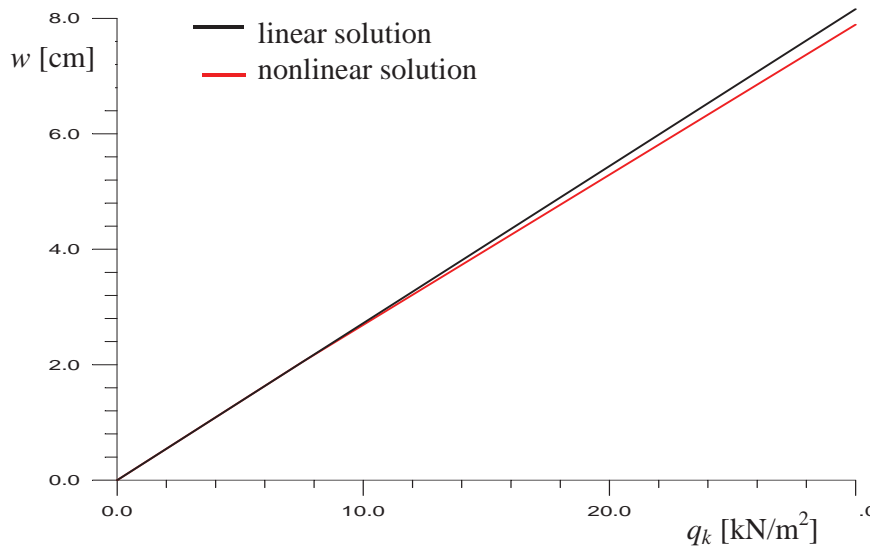

Fig. 7. Dependence of deflection w on load q_k of the grillage

Tab. 1. Results of nonlinear analysis of the grillage

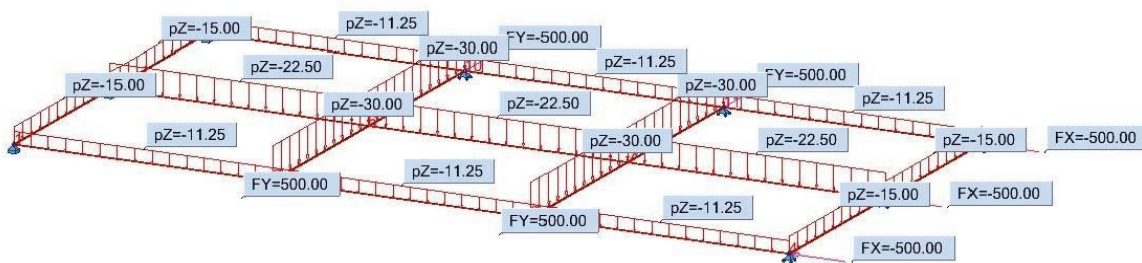| q_k [kN/m²]	w [cm]	F_X [kN]	σ_{Fx} [MPa]	M_Y [kNm]	M_Z [kNm]	σ_{My} σ_{Mz} [MPa]	σ^g σ^d [MPa]
7.5	**2.04**	**-48.68**	**-3.27**	**269.03**	**0.20**	**± 160.33** **± 0.36**	**157.42** **-163.95**
15.0	3.99	-97.27	-6.53	526.44	0.41	± 313.73 ± 0.71	307.92 -320.97
22.5	5.94	-145.86	-9.79	783.85	0.61	± 467.13 ± 1.07	458.41 -477.99
30.0	7.89	-194.44	-13.05	1041.26	0.81	± 620.54 ± 1.42	608.91 -635.01
Linear solution							
7.5	2.04	0.0	0.0	269.89	0.0	± 160.84	± 160.84

Fig. 8. Loading of the grillage for nonlinear analysis of stability

From the linear solution, even at maximum axial forces $F_x = 5000\text{kN}$ (Fig. 9), the deflection is $w = 2.29\text{cm}$, which fulfils the condition of limit state $2.29 < 2.4$. But from the nonlinear solution, deflection is $w = 3.37\text{cm}$, which means that the condition of dimensioning $3.37 > 2.4$ is not fulfilled. Therefore, when axial compressive forces occur for a structure apart from bending state, then nonlinear solution should be carried out, as behaviour of the structure is nonlinear (Fig. 9).

Tab. 2. Results of nonlinear analysis of stability of the grillage

q_k [kN/m²]	w [cm]	F_X [kN]	σ_{Fx} [MPa]	M_Y [kNm]	M_Z [kNm]	σ_{My} σ_{Mz} [MPa]	σ^g σ^d [MPa]
7.5	2.12	500.0	33.45	281.43	2.08	± 167.72 ± 2.39	203.57 -136.66
7.5	2.21	1000.0	66.67	283.91	2.39	± 175.15 ± 3.46	245.28 -119.94
7.5	2.42	2000.0	134.0	322.10	5.39	± 191.96 ± 5.65	331.62 -63.61
7.5	2.68	3000.0	200.2	355.57	7.88	± 219.91 ± 7.92	428.03 -27.63
7.5	2.99	4000.0	266.9	395.97	10.06	± 235.98 ± 1.87	504.74 29.02
7.5	3.37	5000.0	334.4	445.69	13.04	± 265.61 ± 12.90	612.91 55.89
Static linear solution							
7.5	2.29	5000.0	335.0	269.89	6.46	± 160.84 ± 11.32	507.19 162.86

In the case of analyzed grillage, only usable loads take place which results in bending state only. The axial compressive forces have been applied in calculations only for estimation of sensibility of the grillage to the effect of stability loss.

5. Dynamic stability of the grillage

The equations of motion have been formulated by finite element method, i.e. to the static equilibrium equations the consistent mass matrix has been added

$$M\ddot{w} + (K + K(\sigma))w = P(t) = F_x = st .\qquad(4)$$

The equations of motion have been solved by Newmark direct integration method. From the solution of the equations of motion may be observed that the first phase of response is lack of deflections, until the moment when F_x forces get the values close to critical of the problem of stability loss with bifurcation.
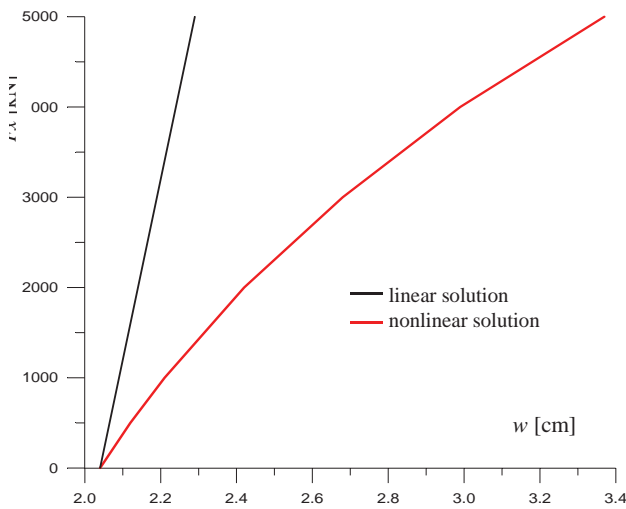

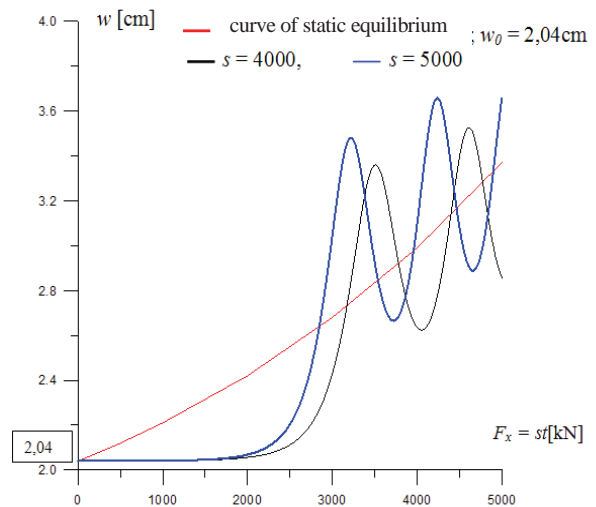*Fig. 9. Dependence of deflection w on load Fx of the grillage under load of q_k=7.5[kN/m²]*

*Fig.10. Runs of dynamic deflections **w** under load Fx = st of the grillage*

The second phase of response is vibrations with constant amplitude around the curve of the state of static equilibrium obtained from the analysis of stability loss of the grillage bending problem.

6. Summary

Linear and nonlinear static analysis and nonlinear verification of sensibility of the grillage to the stability effect has been carried out.

Comparing the results of linear and nonlinear solutions of bending state it may be stated that the values differ to small extent, which shows that the grillage behaves in the way close to linear character of work for applied load. The reason for it is high rigidity of the grillage in relation to applied loads, and such high rigidity results from fulfilling the standard conditions for dimensioning of structure. One of these conditions requires that maximum deflection does not exceed the limit deflection, which results from the limit state of using. Fulfilling of this condition just imposes so high rigidity and thus almost linear range of the grillage work.

In the process of designing the grillage, the condition determining its rigidity is the condition of the limit state of using, i.e. maximum deflection can not exceed the value of limit deflection. Linear and nonlinear solutions in this range are practically the same.

At dimensioning of the cross-section should be still added conditions connected with internal forces, so-called limit state of load capacity, which requires to fulfil additional conditions, e.g. related to local stability, geometric imperfections, warps and so on, of individual fragments of beam's cross-section, causing increase of rigidity of the grillage and thus carrying away possibility that nonlinear work of the structure occurs under the right conditions of using.

Accomplished assessment of the grillage's sensibility to additional compressive loads Fx gives quantitative information relating to their maximum values, to fulfil the condition of the limit deflection as well.

References

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