APPLYING THE MARKOV DECISION PROCESSES TO MODEL CHANGES TO THE MAINTENANCE STATES OF AN OBJECT

Bogdan Landowski

University of Technology and Life Sciences
Machine Maintenance Department
Prof. S. Kaliskiego Street 7, 85-791 Bydgoszcz, Poland
tel.: +48 52 3408495, fax: +48 52 3408495
e-mail: lbogdan@utp.edu.pl

Abstract

Operation and maintenance of technical objects is related to occurrence of various events, the effects of which affect the operation and maintenance process course, and particularly the courses of their use and service processes. Occurrence of those events may be of both determined and random nature. Assessing, analysing and forecasting the operation and maintenance process course, in complex technical systems, are connected with the problems of modelling the operation and maintenance processes of technical objects. Those processes are random ones which depend on one another. The paper presents an example of the Markov decision process to model changes of the analysed operation and maintenance states of technical objects. The investigation object is a real operation and maintenance system of urban transport buses in a middle-sized agglomeration (about 400 k residents). Supporting a decision maker, in the process of making decisions concerning performance of the operation and maintenance process of the means of transport, may be carried out by analysing the results of the investigations of the operation and maintenance process model. The investigations of that type are to determine values of the selected measures of technical and economic efficiency of the process being carried out for the estimated values of the model parameters. The values of the model parameters were estimated on the basis of the analysis of the results of the investigations performed in the analysed system of urban bus transport. A change to the values of the model parameters may reflect a change of influence of internal and external factors on behaviour of the system and the operation and maintenance process of the means of transport being carried out in it. Mathematical models of the operation and maintenance processes are intrinsically simplified, therefore practical conclusions resulting from investigating those models should be formulated carefully.

Keywords: *Markov decision process, modelling, operation and maintenance process, transport system, urban public transport*

1. Introduction

It seems that simulation of changes to the maintenance states of a TO (Technical Object) and forecasting, on this basis, behaviour of the maintenance system as well as determining and analysing values of the selected decision indices may facilitate control of a complex maintenance system [6, 7]. The indices related to the serviceability assurance subsystem, and concerning the number of repairs, repair labour consumption, demand for specialised equipment, etc. may be included among the indices of this type [4, 8]. It is also important to forecast changes to the values of the indices being analysed.

The paper presents chosen assumptions of a simplified (due to the nature of the elaboration) model of the TO maintenance process on the basis of which it is possible to simulate changes to the maintenance states. A possibility of applying the Markov decision process to describe changes to the maintenance states with distinguishing the type of the damaged system (component) of the TO in the repair state has been presented.

The type of the damaged system (component) of the TO has an influence on such features as: time of repairing the TO (the time for which a vehicle stays in the serviceability assurance subsystem), repair labour consumption (measured by the number of man-hours), demand for specialised diagnostic and repair stands, time of exclusion of the TO from use, necessity to include a reserve TO in order to perform tasks, costs of bringing the serviceability state back, others.

The share of the costs related to the performance of the actions by the serviceability assurance subsystem in all the costs of the transport system operation is significant and it mostly amounts to from 15 to 40 percent. The important component of the costs are the costs of the vehicle current repairs.

Determination of the values of the indices being characteristic for the process under analysis is carried out on the basis of a computer simulation of the Markov decision process, being a mathematical model of the maintenance process of technical objects.

2. Investigation object

The investigation object is a broadly defined maintenance system of technical objects. An example of the investigation object, being the basis to illustrate all the considerations, is a real maintenance system of an urban bus transport system in an urban agglomeration (about 400 k residents). This system is one of the subsystems of the urban transport system.

It is assumed in the paper that in the selected set of the technical objects being operated and maintained within the maintenance system it is possible to distinguish n separable subsets of homogenous objects from the point of view of the investigation purpose. The subsets of the technical objects created that way are called categories [3]. Further considerations refer to one category of the objects.

By identification of the maintenance system of buses in an urban transport system and the maintenance process being carried out in it the following maintenance states of a bus, significant for analysing the work of the system under investigation have been distinguished [3, 4, 6]:

- State of using a technical object (performance of a transport task),
- State of a technical object’s waiting for corrective services performed in the environment of the maintenance system (waiting for so called unit of technical emergency service),
- State of corrective services of a technical object performed in the environment of the maintenance system,
- State of a technical object’s waiting for corrective services performed in the maintenance system,
- State of corrective services of a technical object performed in the maintenance system,
- State of a technical object’s waiting for pre-repair diagnostics,
- State of pre-repair diagnostics of a technical object,
- State of a technical object’s waiting for post-repair diagnostics,
- State of post-repair diagnostics of a technical object,
- State of servicing on the day of using a technical object,
- State of a technical object’s waiting for undertaking performance of a transport task (after bringing the serviceability state back a technical object does not perform the scheduled transport task due to the method of organizing the transport tasks – e.g. in the maintenance system of buses of an urban transport system, specified by the existing schedule of transport tasks so called “timetable”),
- State of a technical object’s waiting due to the environment unserviceability,
- State of organisational standstill of a technical object (no transport tasks – e.g. in the maintenance system of buses of an urban transport system resulting from the schedule of transport tasks, including a night break in performance of the transport tasks).

In order to illustrate the considerations, the following bus maintenance states, out of the aforementioned ones, have been analysed:

S_1 - state of using – the state in which a bus together with an operator perform the transport tasks they have been entrusted with,

S_2 - state of servicing in the serviceability assurance subsystem, which occurs when, for instance there is a damage that cannot be removed outside the service station by the units of technical emergency service,

S_3 - state of waiting for performance of transport tasks (so called standstill at a bus depot when the transport task schedule does not specify any task performance).

The buses being maintained in the investigation object have been disassembled into the following systems:

- Steering system U_1,
- Suspension system U_2,
- Electric installation system U_3,
- Bodywork system U_4,
- Drive train U_5,
- Wheels and steering system U_6,
- Power supply system U_7,
- Cooling system U_8,
- Pneumatic system U_9,
- Engine system U_{10},
- Braking system U_{11}.

Such a form of disassembling a vehicle due to the method of recording data on the damages used in the investigation object has been applied. The data concerning, among other things, the number of damages to the distinguished bus systems (results of investigations for sixty vehicles for the period of two years) were used in a computational example. The results of the investigations cover a damage removed both in the serviceability assurance subsystem (state S_2) and in the environment of the maintenance system (due to performance of actions of the units of technical emergency service).

3. Mathematical model – Markov decision process

It has been assumed that the process of changes to a certain system (process) is described by a stochastic process $\{X_t, t\in T\}$, $t \geq 0$ with a finite space of states $S = \{1, 2, 3, \ldots, n\}$. It has been assumed in the paper that the states of the considered stochastic process correspond to the maintenance states of a technical object (a bus).

a_{ik}, $(i\in S, k\in N)$ has been used to denote an alternative k undertaken when entering the state i. A finite set A_i of the alternatives (decisions) corresponds to each state, $i\in S$.

The sets of alternatives for each state do not have to be equivalent. There may be states for which the set of alternatives is a single element set, so called non-decision states. Some references describe the non-decision states as the states for which the set of alternatives is an empty set [4]. It seems that adoption of single element sets of alternatives is more consistent from the point of view of description of execution of the of the process $\{X_t, D_t\}$, $t\geq 0$, being analysed below.

It has been assumed that the elements of the set A_i, $i\in S$ are the elements a_{ik}, $(i\in S, k\in N)$, that is $A_i = \{a_{i,1}, a_{i,2}, \ldots, a_{i,\bar{\bar{i}}}\}$, where $\bar{\bar{i}}$ stands for the power of the set A_i. The set of all the subsets of alternatives is denoted as A, that is $A = \bigcup_{i\in S} A_i$.

In a general case performance, when the process $\{X_t, t\in T\}$ enters the state $i\in S$, of a specific alternative may have an influence on the next state $j\in S$ of the process and the features of the state i (distribution and parameters of the time of staying in the state, costs or profits generated by the system in that state, etc.).

The alternatives may represent specific a methods of proceeding, acting, events, taken decisions, etc. which may be allocated to the state of the process being modelled. In a real maintenance state these may be various methods of servicing, repairing, surveying, scopes of performed actions or using an object, e.g. different transport routes, on which a vehicle is used. Adoption of a specific alternative may have an influence on: the costs borne, incomes received, on frequencies and types of damages to an object, duration of the maintenance states, state sequences, etc.

It is assumed in the paper that the alternatives are the types of the damaged systems of a technical object. It has been assumed that a set of alternatives with the power higher than one is allocated only to one maintenance state S_2 (state of servicing).

The stochastic process $\{D_t, t\in T\}$, $t \geq 0$ with a finite set of alternatives A describes the method of selecting the alternatives $a\in A$. A change of the process state $\{D_t, t\in T\}$ takes place at the moments t of the process state changes $\{X_t, t\in T\}$. At the moments t_n, $n \in N$ of the process state changes $\{X_t, t\in T\}$ the alternative $a \in A$ is selected (of course, if at the moment t_n the state of the process $\{X_t, t\in T\}$ is the state i then, $a \in A_i$).

The process $\{X_t, D_t\}$, $t \geq 0$ with a finite set of the states S and a finite set of the alternatives A is called the stochastic decision process. By performing this process from the initial moment t_0 to the moment t_n the sequence of the states and decisions $h_{t_n} = \{i_{t_0}, a_{t_0}, i_{t_1}, a_{t_1}, \ldots, i_{t_n}, a_{t_n})$ is received which is called the history of the process to the moment t_n.

Next it has been assumed that the analysed stochastic decision process $\{X_{t_n}, D_{t_n}\}_{n=1}^{\infty}$ is the Markov decision process. The set of possible executions of the Markov decision process is the set $W = \{S \times D\}^{\infty}$. It has also been assumed that the probability of selecting the alternative $a_{t_n} \in A$ depends only on the state $i_{t_{n-1}} \in S$, and does not depend on the history $h_{t_{n-1}}$ of the process.

In this case the sequence of the process states $\{X_t, t\in T\}$ is a non-homogeneous Markov chain [1, 2].

In order to define the analysed stochastic decision process $\{X_t, D_t\}$, $t \geq 0$ it is also needed to describe:

- Method of selecting the process alternatives $\{D_t, t\in T\}$,
- Initial distribution of the process $\{X_t, t\in T\}$,
- Conditional probability of process state changes $\{X_t, t\in T\}$,
- Random variables which stand for the durations of the process states $\{X_t, t\in T\}$.

In a significant simplification the rule describing the method of selection, when entering the state $i\in S$, of the alternative $a\in A_i$ is called a strategy. The method of selecting an alternative being performed when entering the process state may be random or determined. The method of selecting the alternatives adopted from the point of view of the paper purpose has been described in the computational example.

$p = [p_1, p_2, \ldots, p_n]$ where $\sum_{i\in S} p_i = 1$, and $p_i \geq 0$ for $i \in S$ were used to denote initial distribution of the process $\{X_t, t\in T\}$. By providing the value p_i of the elements of the initial distribution vector the probability that the process $\{X_t, t\in T\}$ is at the moment $t = 0$ in the state i is determined.

The probability of a state change in one step of the process $\{X_t, t\in T\}$ from the state i, $i \in S$ to the state j, $j\in S$ when taking, at the entry to the state i, the alternative $a\in A_i$ is denoted as p_{ij}^{a}.

The condition is met $\sum_{j\in S} p_{ij}^{a} = 1$, i $p_{ij}^{a} \geq 0$, for all $i, j \in S$ and $a \in A_i$.

To each state $i \in S$ and alternative $a \in A_i$ it is possible to allocate the stochastic matrix $P^{(i,a)}$ describing conditional probabilities of transition p_{ij}^{a}. The set of the matrices allocated to the state $i \in S$ has the power equal to $\bar{\bar{i}}$ (power of the set A_i).

The matrix $P^{(i,a)}$ is a matrix combined of stochastic rows which describe the probabilities of transition from the state demoted with the row number to all the remaining states. The element p_{ij}^{a}, on the crossing of the row with the number i and the column with the number j in the matrix $P^{(i,a)}$, is the probability of transition from the state $i \in S$ to the state $j \in S$, provided that the alternative $k \in A_i$ was applied when entering the state $i \in S$.

The random variable which stands for the state duration $i \in S$ of the process $\{X_t, t\in T\}$, when the next state is $j \in S$ and when entering the state i the decision was taken $a \in A_i$ with the distribution described by the distribution function $F_{ij}^{a}(t)$ is denoted with the symbol T_{ij}^{a}.

In order to simplify, further considerations assume that $F_{ij}^{a}(t) = F_{i}^{a}(t) = F_{ia}(t)$, i,j ∈ S, a ∈ A_i. It means that the duration of the state i∈S does not depend on the next state in which the process will be. The function $F_{ia}(t)$ is the distribution function of distribution of duration of the state i ∈ S, provided that the decision a was taken when entering this state.

The random variable which stands for duration of the state i ∈ S, with the distribution determined by the distribution function $F_{ia}(t)$ is denoted with the symbol T_{ia}.

4. Computational example

The model of the process of changes of maintenance states of the vehicles being operated and maintained in the investigation object is the described stochastic decision process $\{X_t, D_t\}$, t≥0.

The states i∈S of the process $\{X_t, t\in T\}$ correspond to the distinguished maintenance states S_i, i = 1, 2, 3 of a bus in the analysed example. The states i=1,3 of the process $\{X_t, t\in T\}$ are non-decision states, it means that the subsets of alternatives A_1 and A_3 are single element ones. In those states the alternatives are only formal (maintaining consistency of the formula) and have no influence on the changes of the analysed process $\{X_t, D_t\}$, t≥0. In the state i=2 the alternatives a ∈ A_2 of the process $\{D_t, t\in T\}$ correspond to the codes of the damaged distinguished systems of a bus and reflect the damages to the systems (components) of a vehicle. The set A_2 contains the following elements $A_2 = \{a_{2,1}, a_{2,2}, ..., a_{2,11}\}$ Interpretation of the entry, at the moment t, to the state i=2 of the process $\{X_t, t\in T\}$ and occurrence of the alternative $a_{2,3}$ of the process $\{D_t, t\in T\}$ is as follows: there was a damage to the bus, and the damaged system is the system denoted with the code U_3.

The rule of selecting the alternatives a in the state i=2 is determined by the distribution of probability of occurrence of the analysed alternatives. It has been assumed that q_{ik}, i = 2, k = 1, 2, ..., 11 stands for the probability of occurrence of the alternative a_{ik}.

$q_i = [q_{i1}, q_{i2},\dots, q_{ik}]$ where $\sum_{i=2,k=1,2,...,11} q_{ik} = 1$, and $q_{ki} \geq 0$ for i = 2, k = 1, 2, ..., 11 was used to denote the vector of distribution of occurrence of alternatives in the state i = 2. In the considered example the elements of this vector stand for probabilities of occurrence of damage to the specific type of system.

Simulation of changes of the maintenance states of a single technical object taking into account the type of the damaged system (component) is to simulate execution of the described stochastic decision process being a model of the process of changes of the maintenance states of the vehicles being operated and maintained in the investigation object.

A program has been developed which makes it possible to simulate the stochastic decision process. The data necessary to perform the simulation are the data being indispensable to determine the described process $\{X_t, D_t\}$, t≥0.

The data indispensable to perform simulation experiments should be determined on the basis of the results of the real maintenance investigations performed in the investigation object.

In case it is possible to make such an assumption that the damages to the analysed systems (components) of a technical object do not depend on one another, the vector q_i of the probability distribution may be determined on the basis of frequency of occurrence of damages to the analysed systems.

In order to illustrate the considerations the simulation experiments were performed for the considered example.

Essential data:

- Space of the process states $\{X_t, t\in T\}$: S = {1,2,3}, (distinguished maintenance states of a bus),
- Space of the process decision $\{D_t, t\in T\}$: A={A_2}, $A_2 = \{a_{2,1}, a_{2,2}, ..., a_{2,11}\}$, (codes of the distinguished systems of a bus).

To simplify the consideration the following assumptions were made:

- $p_{ij}^{a} = p_{ij}$ for i,j $\in$S, a $\in$A, which means that the transitions between the process states {X_t, t$\in$T} do not depend on the selected alternative and there is only one matrix of transitions P=[p_{ij}] i,j $\in$S between the states of this process (in one step),
- Random variables T_{ia} for i = 1, 3, a$\in$A have gamma distributions with various parameters,
- Staying in the state is related to receiving revenues (state 1) and bearing expenses by the system in which the objects are operated and maintained.

The values of the matrix elements P=[p_{ij}] i,j $\in$S were estimated on the basis of the preliminary maintenance studies.

The initial distribution p = [p_1, p_2,…, p_n], n=3, of the process {X_t, t$\in$T} in case of simulation of a large number of the states for the analysed example has no significant meaning. The values p_i of the elements of the initial distribution vector adopted for the calculations are hypothetical. To eliminate influence of the wrongly estimated values of the initial distribution vector, a fragment of performance of the process, omitting its initial phase was adopted for the analysis.

The values of the probabilities q_{ik}, i = 2, k = 1, 2, ..., 11 of occurrence of the alternative a_{ik} were determined on the basis of the results of the investigations concerning damages to the buses. To simplify the description, the alternatives a_{2k}, k = 1, 2, ..., 11 are denoted below with the code k.

It should be assumed that the values of the parameters of distributions of random variables T_{ia} i $\in$S, a$\in$A used in the example are hypothetical. The values of the parameters were estimated on the basis of the set of data of a small amount.

Calculations were performed for one category of objects covering 60 vehicles and the period of two years. The selected calculation results are presented in the Tab. 1-3.

Tab. 1. Durations of repairs and numbers of damages to the distinguished systems of a bus

System code	Duration of the state S_2		Number of entries to the state
	Average [h]	Stand. deviation [h]	
U_1	4.49	0.46	140
U_2	3.14	0.50	257
U_3	0.20	0.09	2795
U_4	0.26	0.09	1770
U_5	5.51	0.53	111
U_6	0.72	0.65	291
U_7	1.67	0.52	237
U_8	0.48	0.49	771
U_9	1.16	0.44	954
U_{10}	6.14	5.99	505
U_{11}	4.12	2.09	438

5. Conclusions

The purpose of the considerations was, among other things, to present possibilities of applying the Markov decision processes for mathematical modelling of the system and process of vehicle maintenance. A possibility of using models of this type to analyse a transport system and support decision makers in that system, e.g. by forecasting behaviour of the vehicle maintenance system after changing the control requests has been presented.

The results of the simulation experiments performed let us state that the model is susceptible to a change of the value of its input parameters.

The analysis of the results of the simulation experiments shows a significant stability of the calculation results obtained (for the same data).

Tab. 2. Numbers of entries and durations of the analysed states

State code	Object code	Lw	EX	s
1	OT1	733	18.02	0.52
2	OT1	148	1.04	2.25
3	OT1	627	5.49	0.51
1	OT2	730	17.99	0.49
2	OT2	139	1.54	2.94
3	OT2	634	5.45	0.49
…	…	…	…	…
1	OT60	729	18.01	0.49
2	OT60	122	1.04	1.90
3	OT60	650	5.44	0.49

Lw - number of entries to the state
EX - average value of the state duration [h]
s - standard duration of the state duration [h]

Tab. 3. Average value of the daily income generated by the system due to performance of the transport tasks by the particular objects

Object code	Income [PLN]
OT1	4.3
OT2	4.06
OT3	4.48
OT4	4.35
OT5	3.97
OT6	4
OT7	4.29
OT8	4.51
OT9	4.41
…	…
OT60	4.29
Average	4.25
Stand. deviation	0.17

Mathematical models of the maintenance processes, performed in complex systems, are intrinsically a significant simplification of the real processes. The consequence of the above is a necessity to carefully formulate conclusions resulting from investigations of those models [5]. However, it seems that the analysis of the results of the investigations of those models, for the values of the model parameters, determined on the basis of the maintenance studies performed in a real transport system, makes it possible to formulate both qualitative and (to the limited extent) quantitative conclusions and opinions.

The considered example of the model of maintenance process of buses in an urban transport system described in the category of the maintenance states of objects is characterised by a significant simplification (due to the nature of the elaboration). However, the presented method of constructing models of that type and their analyses show that they may be used to provide preliminary forecasts of the system behaviour. There is a possibility to perform analyses concerning both technical and economic aspects. The set of the indices that may be determined includes subsets of the indices concerning: readiness, repair times, effectiveness of performance of the transport tasks, costs and others.

Performance of the research experiments for various categories of objects makes it also possible to evaluate the selected technical and economic aspects of replacing objects with new ones and evaluation of usefulness of various types of objects in the specific maintenance system.

The model has been created in such a way to assure a possibility of using it in the widest possible class of problems related to maintenance of technical objects.

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