MATHEMATICAL DESCRIPTION OF THE EXTERNAL CHARACTERISTICS OF COMPRESSION-IGNITION ENGINE

Tomasz Stoeck

West Pomeranian University of Technology
Piastów Street 19, 70-310 Szczecin, Poland
tel.: +4891 449 48 13, fax: +4891 449 48 20
e-mail: tstoeck@wp.pl

Abstract

In this paper is presented a calculation procedure based on Lagrange's interpolation formula which has been used for describing the external characteristics of compression-ignition engine of the 359 type. The characteristic curves of specific fuel consumption and exhaust gas smokiness were chosen as an example, which had been obtained during examinations on engine test bed for a drive unit fuelled with four types of fuel. Measurements were carried out on a standard engine test bed which is part of laboratory facilities of the Department of Automotive Vehicles Operation, Western Pomeranian University of Technology in Szczecin. The study object was an unsupercharged four-stroke engine of domestic production of the 359 type. It is a six-cylinder compression-ignition drive unit with direct fuel injection system. Interpolating polynomials determined analytically for respective characteristics have been presented in the graphical form, while experimental examination results necessary for obtaining them have been additionally tabulated. The presented methodology allows simplification of particular experiment stages by giving possibility to estimate analytically the missing data and with preservation of a minimum number of measuring points. With small corrections, it may be used in similar examinations on other engine test beds. Its application should support the planning of particular experiment stages but does not, in any case, replace it since determination of engine characteristic curves is possible through research only.

Keywords: *polynomial interpolation, external characteristic, specific fuel consumption, exhausts gas smokiness*

1. Introduction

In order to learn the operational properties of combustion engine, it is put to versatile examinations on engine test bed. The evaluation of basic operational parameters of a drive unit and their interdependencies is possible owing to the graphical presentation of results in the form of characteristics. They are being made based on direct or indirect measurements, with the latter requiring additional calculations based on functional dependencies connected with the measured quantity. Because of a transparent picture of carried out tests, the characteristic curves most frequently made are those for full power, called also external characteristic curves, which present effective power, torque, mean effective pressure, specific fuel consumption and, depending on the need, other engine operation parameters versus rotational speed. They are being taken at the complete opening of fuel feeding units and hence their importance in case of compression-ignition engines is rather theoretical since in practice the air-fuel mixture combustion would occur with a very heavy smokiness.

One of the necessary conditions for obtaining a correct course of external characteristic is to determine minimum six measuring points [2]. They larger number is being connected most frequently with an increase in the time consumption research process and multiplication of operating costs. This may be particularly noticeable in case of frequently repeated cycles of an experiment, e.g. when changing particular engine adjustment settings, fuelling it with different types of fuel, interfering in intake parameters (use of air filters of different construction, modification in the number of installed magnetic activators), etc. Furthermore, it is recommended to concentrate measuring points in the area of extremes as well as other places of characteristic curves important for the test type. With this end in view, a calculation procedure was developed which, basing on their minimum required number, allows analytical estimation of other data. To describe mathematically the courses obtained experimentally, a method based on Lagrange's interpolation formula was used.

2. Research methods

The test bed was equipped with the following units: compression-ignition engine of the 359 type, hydraulic brake HWZ-3 together with a complete control system, gravimetric fuel gauge and light obscuration smokemeter MDO 2. In successive measuring cycles, the engine was fuelled with fuels with different physicochemical properties, i.e. summer grade diesel oil (ON), low-sulphur diesel oil (EKODIESEL PLUS 50), rapeseed oil (OR), and rapeseed methyl ester (RME) [3, 4]. The obtained study results allowed determination of a number of external characteristic curves of the engine while preserving its factory settings. Because the paper is focused on a mathematical description of the characteristic curves of specific fuel consumption (g_e) and absorption coefficient of infrared radiation (k), their values according to the type of applied fuel are compiled in Tab. 1.

Tab. 1. The values of specific fuel consumption and absorption coefficient for respective fuels

n [1/min]	g_e [g/kWh]				k [1/m]			
	ON	EKODIESEL PLUS 50	OR	RME	ON	EKODIESEL PLUS 50	OR	RME
1200	222.83	225.68	269.29	251.44	1.57	1.83	0.82	0.88
1500	227.56	221.84	271.71	249.97	1.78	2.05	1.01	1.09
1800	219.40	219.86	268.60	245.09	0.76	0.81	0.64	0.67
2100	226.59	224.77	273.53	248.21	0.85	0.78	0.46	0.49
2400	231.48	229.97	277.80	252.63	0.83	0.81	0.35	0.41
2700	238.22	234.06	286.27	264.03	0.88	0.84	0.32	0.44

3. Calculation methods

In order to present the description of the characteristic curves obtained in test bed examinations, one of the basic numerical methods, known as interpolation, was used. This problem is reduced to finding the function y=f(x) which assumes values $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$… $y_n=f(x_n)$ at precisely defined points x_0, x_1, x_2, …, x_n. Having in mind a small number of measuring points (nodes), Lagrange's interpolation formula was applied [1]:

$$W_n(x) = y_0 \frac{(x - x_1)(x - x_2)...(x - x_n)}{(x_0 - x_1)...(x_0 - x_n)} + y_1 \frac{(x - x_0)(x - x_2)...(x - x_n)}{(x_1 - x_0)(x_1 - x_2)...(x_1 - x_n)} +$$
$$... + y_n \frac{(x - x_0)(x - x_1)...(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)...(x_n - x_{n-1})}, \qquad (1)$$

where:
$W_n(x)$ - Lagrange's interpolation polynomial of the n-th degree,
$x_0, x_1, x_2, \dots, x_n$ - measuring points,
$y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …, $y_n=f(x_n)$ - function's values at these points.

Readings were made for the rotational speed of engine crankshaft in six measuring points showed in Tab. 1. To simplify the calculation methodology, [1/s] were adopted as a unit, thus the following sequence was obtained: x_0=20, x_1=25, x_2=30, x_3=35, x_4=40, and x_5=45. According to formula (1), a general equation of the function describing the external characteristic curve of tested engine may be presented as follows:

$$D(x) = \sum_{k=0}^{5} f(x_k) \cdot d_k(x), \tag{2}$$

where:

$$d_k(x) = \frac{(x - x_0)...(x - x_{k-1})(x - x_{k+1})...(x - x_5)}{(x_k - x_0)...(x_k - x_{k-1})(x_k - x_{k+1})...(x_k - x_5)}. \tag{3}$$

By substituting respective measuring points for {k=0; 1; 2; 3; 4; 5} and reducing successive expressions to a common denominator, the following was obtained:

$$d_0(x) = \frac{(x-25)(x-30)(x-35)(x-40)(x-45)}{(-5)(-10)(-15)(-20)(-25)} = \frac{-S_0(x)}{375000}, \tag{4}$$

$$d_1(x) = \frac{(x-20)(x-30)(x-35)(x-40)(x-45)}{(5)(-5)(-10)(-15)(-20)} = \frac{5 \cdot S_1(x)}{375000}, \tag{5}$$

$$d_2(x) = \frac{(x-20)(x-25)(x-35)(x-40)(x-45)}{(10)(5)(-5)(-10)(-15)} = \frac{-10 \cdot S_2(x)}{375000}, \tag{6}$$

$$d_3(x) = \frac{(x-20)(x-25)(x-30)(x-40)(x-45)}{(15)(10)(5)(-5)(-10)} = \frac{10 \cdot S_3(x)}{375000}, \tag{7}$$

$$d_4(x) = \frac{(x-20)(x-25)(x-30)(x-35)(x-45)}{(20)(15)(10)(5)(-5)} = \frac{-5 \cdot S_4(x)}{375000}, \tag{8}$$

$$d_5(x) = \frac{(x-20)(x-25)(x-30)(x-35)(x-40)}{(25)(20)(15)(10)(5)} = \frac{S_5(x)}{375000}. \tag{9}$$

Substitution of the above dependencies into formula (2) allowed presentation of a general equation of the function describing the external characteristic curve of tested engine:

$$\begin{aligned} D(x) = {} & f(x_0) \cdot d_0(x) + f(x_1) \cdot d_1(x) + f(x_2) \cdot d_2(x) + f(x_3) \cdot d_3(x) + \\ & + f(x_4) \cdot d_4(x) + f(x_5) \cdot d_5(x). \end{aligned} \tag{10}$$

As a calculation example, specific fuel consumption curve obtained when fuelling the engine with summer grade diesel oil (ON) was chosen for examination. Having in mind the function values presented in Table 1, the function $D_{ON}(x)$ describing a given external characteristic curve will have the following form:

$$\begin{aligned} D_{ON}(x) = (1/375000)[-222.83 \cdot S_0(x) + 1137.80 \cdot S_1(x) - 2194.00 \cdot S_2(x) + 2265.90 \cdot S_3(x) + \\ -1157.40 \cdot S_4(x) + 238.22 \cdot S_5(x)]. \end{aligned}$$

In order to present a final Lagrange's interpolation polynomial, its respective components $S_0(x)$, $S_1(x)$, $S_2(x)$, $S_3(x)$, $S_4(x)$ and $S_5(x)$ were determined:

$$S_0(x) = x^5 - 175x^4 + 12125x^3 - 415625x^2 + 7046250x - 47250000, \tag{11}$$

$$S_1(x) = x^5 - 170x^4 + 11375x^3 - 373750x^2 + 6015000x - 37800000, \tag{12}$$

$$S_2(x) = x^5 - 165x^4 + 10675x^3 - 337875x^2 + 5222500x - 31500000, \quad (13)$$

$$S_3(x) = x^5 - 160x^4 + 10025x^3 - 307250x^2 + 4605000x - 27000000, \quad (14)$$

$$S_4(x) = x^5 - 155x^4 + 9425x^3 - 281125x^2 + 4113750x - 23625000, \quad (15)$$

$$S_5(x) = x^5 - 150x^4 + 8875x^3 - 258750x^2 + 3715000x - 21000000. \quad (16)$$

Finally, the function describing specific fuel consumption external characteristic curve when fuelling the engine with summer grade diesel oil (ON) assumes the following form:

$$D_{ON}(x) = (1/375000)(67.69x^5 - 11300.75x^4 + 741066.25x^3 - 23804406.25x^2 + 373788662x - 2207467500).$$

Polynomials for other specific fuel consumption curves were determined similarly (Fig. 1), according to the type of fuel used in tests:

- For low-sulphur diesel oil (EKODIESEL PLUS 50):

$$D_{EP50}(x) = (1/375000)(16.83x^5 - 2815.25x^4 + 183863.25x^3 - 5830543.70x^2 + 89447762.00x - 446433750.00).$$

- For rapeseed oil (OR):

$$D_{OR}(x) = (1/375000)(35.83x^5 - 5931{,}25x^4 + 386018.75x^3 - 12312818.70x^2 + 192109387.00x - 1069882500.00).$$

- For rapeseed methyl ester (RME):

$$D_{RME}(x) = (1/375000)(30.49x^5 - 5026.25x^4 + 326106.25x^3 - 10368793.70x^2 + 160972037.00x - 879851250.00).$$

Because all tests were carried out for identical measuring points, there is possibility of direct use of the presented calculation procedure in description of other external characteristic curves of a given engine. This results from the fact that the form of component polynomials (11-16) remains unchanged, while determination of the functions describing other curves is reduced to the replacement and multiplication of applied gradients in the formula (2).

In Fig. 2 is presented the graphical interpretation of interpolating polynomials for absorption coefficient of infrared radiation which, depending on the type of fuel, are as follows:

- For summer grade diesel oil (ON),

$$D_{ON}(x) = (1/375000)(4.96x^5 - 833.00x^4 + 54980.00x^3 - 1778650.00x^2 + 28119525.00x - 172773750.00),$$

- For low-sulphur diesel oil (EKODIESEL PLUS 50)

$$D_{EP50}(x) = (1/375000)(4.91x^5 - 832.00x^4 + 55416.25x^3 - 1808900.00x^2 + 28835775.00x - 178391250.00),$$

- For rapeseed oil (OR)

$$D_{OR}(x) = (1/375000)(x^5 - 171.75x^4 + 11642.50x^3 - 388406.25x^2 + 6346562.50x - 40121250.00),$$

- For rapeseed methyl ester (RME)

$$D_{RME}(x) = (1/375000)(1.16x^5 - 199.25x^4 + 13507.50x^3 - 450493.75x^2 + 7356212.50x - 46488750.00).$$

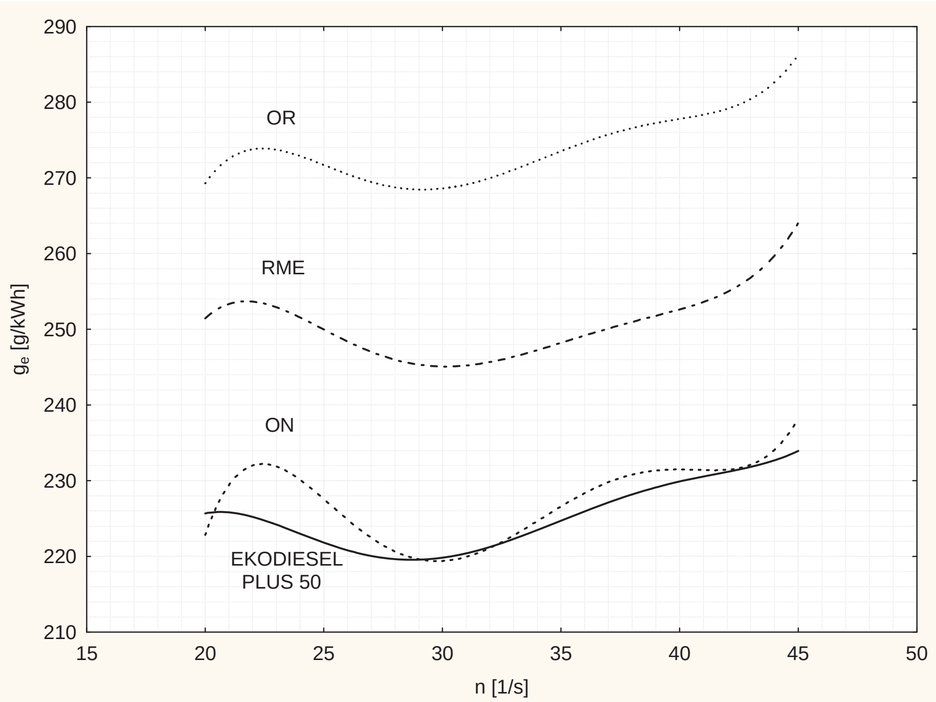

Fig. 1. Graphical interpretation of interpolating polynomials for specific fuel consumption of the 359 type engine

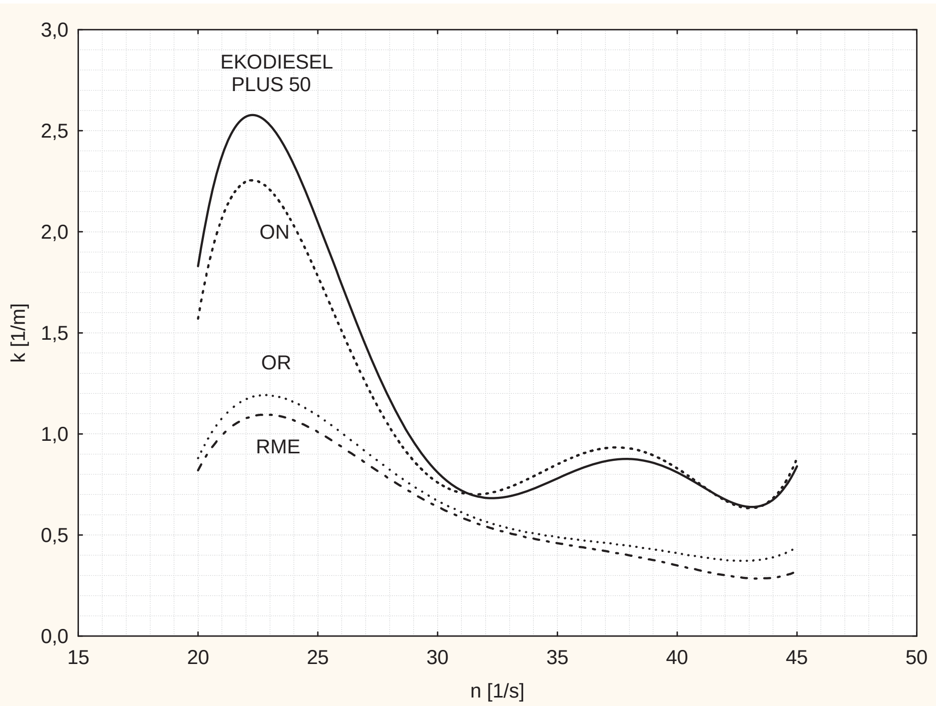

Fig. 2. Graphical interpretation of interpolating polynomials for absorption coefficient of the 359 type engine

4. Conclusions

The calculation methodology presented allows simple description of any external characteristic curve of the 359 type engine as well as, after small modification, other drive units in tests of a similar profile. Its use simplifies experimental procedure to a minimum required number of measuring points since estimation of values for other rotational speeds may be carried out analytically. However, it should be stressed that the use of polynomials based of Lagrange's formula is effective for a small number of nodes. This is because an increase in their number leads to larger complexity of carried out calculations and does not assure reduction of interpolation error. In such a case, better results may be obtained by applying so called spline functions, dividing the examined interval into smaller partitions and carrying out a separate interpolation for each of them with lower degree polynomials.

References

[1] Fortuna, Z., Macukow, B., Wąsowski, J., *Metody numeryczne*, Wydawnictwa Naukowo-Techniczne, pp. 24-39, Warszawa 1993.

[2] Lewicki, J., *Przewodnik do ćwiczeń laboratoryjnych z silników*, Wydawnictwo Uczelniane Politechniki Szczecińskiej, pp. 9-14, Szczecin 1989.

[3] Stoeck, T., *Aspekt ekonomiczny i ekologiczny stosowania paliw rzepakowych do zasilania silnika z zapłonem samoczynnym*, Autobusy - Technika, Eksploatacja, Systemy Transportowe, Nr 1-2, pp. 54-56, Radom 2006.

[4] Stoeck, T., *Wpływ rodzaju paliwa na zadymienie spalin w transporcie miejskim*, Autobusy - Technika, Eksploatacja, Systemy Transportowe, Nr 1-2, pp. 19-23, Radom 2002.