OIL AGING INFLUENCE ON THE SLIDE JOURNAL BEARING LUBRICATION

Andrzej Miszczak, Grzegorz Sikora

Maritime University Gdynia, Faculty of Marine Engineering Morska Street 83, 81-225 Gdynia, Poland tel.: +48 58 6901348, +48 58 6798338 fax: +48 58 6901399, +48 58 6798345 e-mail: miszczak@am.gdynia.pl, gsikora@oetker.pl

Abstract

In this study authors solve the fundamental set of equations of the hydrodynamic theory of lubrication, namely are: the continuity equation, conservation of momentum and conservation of energy for the case of stationary slide bearings lubrication with a thixotropic lubricant. Adoption of assumption of steady flow loads in the considered phenomenon to the changes absence of the flow parameters in a short time period i.e. in one hour. In the constitutive equation is assumed that the stress tensor is a function of strain tensor, dynamic viscosity of oil and hydrodynamic pressure. Dynamic viscosity decreases in a long period of time of work f. ex. after 10 000 by 20 000 kilometres. In a thin layer of oil film, density and thermal conductivity was assumed to be constant. Authors define the lubricant's dynamic viscosity as a product of viscosity changes in temperature, pressure and time $\eta = \eta(T) \cdot \eta(p) \cdot \eta(t)$.

In the analysis of hydrodynamic lubrication, Authors consider a journal bearing of finite length, with the smooth sleeve with a full circumferential angle.

Fundamental equations are written in dimensionless form and estimated according to the theory of a thin boundary layer.

Prepared in this way equations of motion can be solved by various methods. Authors propose to solve the motion equations with a method of small parameter.

The small parameter method we define the unknown functions in a form of uniformly convergent power series expanded in the neighbourhood of the small parameters. In most used cases, absolute value of the small parameter is less than unity. These functions are substituted into simultaneous fundamental equations, then the series are multiplied using Cauchy's method. Comparing coefficients with the same exponents of small parameter, simultaneous set of differential equation is acquired, from which next approximations of unknown functions are appointed. With so obtained equations, the equation that allows assigning hydrodynamic pressure and hydrodynamic pressure corrections resulting from taking into account the impact of pressure, temperature and ageing in viscosity changes of the lubricant successively can be assigned.

Keywords: oil ageing, motion equations, the method of small parameter, viscosity changes in time

1. Introduction

Exploitation of the tribosystems makes lubricator, which is mostly mineral oil, modified with a various additives, gradually degrades. The most common effect of degradation is the loss of viscosity and lubricating properties. Such an effect is generally called aging of the oil, that is, among others, changes the dynamic viscosity of lubricator in a function of time. Considered period of time will be a range of several dozen to several hundred days. The decrease of dynamic viscosity of lubricator reduces the capacity force, reduces the minimal height of the oil gap and results in the further changes of operating and exploitation parameters of the bearing.

The purpose of this study is to develop the non-dimensional equations of motion for the lubricating flow in slide journal bearing so that in the next step the velocity vector components and the hydrodynamic pressure and temperature distribution could be determinate in analytical and numerical methods.

2. Physical model of the bearing

Although the temperature difference in the height direction of the oil gap varies in an average at mostly within a few °C, however, changes in temperature gradients in the height direction of the oil gap can be large. Furthermore, the temperature difference between the inner surface of the sleeve and the outer surface of the bearing sleeve is significant. Significant is also the temperature difference along the outer surface of the bearing's sleeve. The temperature on the surface of the bearing's journal, which rotates, equalizes quickly, while the temperature at the inner surface of the sleeve is changing very significantly in the direction of the angle of wrap. Very large temperature changes are also observed on the inner sleeve surfaces along the axis of the shaft. Temperature has a significant impact on the height of the bearing gap. These thermal loads change the oil gap, and this causes changes in the value of hydrodynamic pressure therefore flow and exploitation parameters of the slide journal bearing.

In journal slide bearings, we have to deal with three options of heat transport in the area of slide tribosystem see Fig. 1. As the first option, the transfer of heat from the oil film to the journal or from the journal into the oil film is assumed. As a second option the movement of heat from the oil film to the sleeve and the sleeve housing or from sleeve to the oil film is adopted. A third option is a case of simultaneous transfer of heat from the oil film to the journal and the sleeve, or in the opposite direction.

The heat generated in the oil gap moves also in the direction of the length of the journal, simultaneously searching for the way out trough the front surfaces of the bearing, and also penetrates into machine body and journal.



Fig. 1. Options for heat transport in the bearing: a) from the lubricator to the journal, b) from the lubricator to the sleeve and the sleeve housing, c) from lubricator to the journal and sleeve

The biggest impact on the exploitation parameters of the bearing has the heat transfer by conduction in the area of the slide tribosystem. Influence of forced convection on the bearing capacity is limited and determined by the value of Graetz. Natural convection has usually negligible impact on the exploitation parameters of the bearing.

After considering possible cases, related to the heat aspects occurring in slide journal bearing, for the further analysis is assumed the non-adiabatic and non-isothermal thermodynamic model of slide journal bearing, lubricated with the oil of Newtonian properties. Oil viscosity depends on temperature, pressure and time. For the considerations is adopted cylindrical slide bearing with finite length with a smooth sleeve with a full angle of wrap.

In a thin layer of oil film of oil is assumed constancy of oil density in relation to temperature and independence of thermal conductivity coefficient of oil from thermal changes, because in the range of lubricator film, influence of temperature on density and thermal conductivity of oil is negligible.

To determine the temperature distributions, the energy conservation equation was used.

Heat flows from the oil film to the layers of the material of the journal (sleeve) or in the opposite direction, therefore the Fourier-Newton boundary conditions were used.

Sleeve surface deformation caused by pressure, temperature is not taken into account in this paper.

3. Fundamental equations

Analysis of the hydrodynamic lubrication of the slide journal bearings in non-steady flow contains solution for fundamental equations, such as conservation of momentum, continuity and conservation of energy in the following general form [1-5]:

$$\rho \frac{\mathrm{d} \mathbf{v}}{\mathrm{d} t} = \mathrm{Div} \, \mathbf{S} \quad , \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \qquad (2)$$

div(
$$\kappa$$
 grad T) + div(**v**S) - **v**DivS + $\Omega = \rho \frac{d(c_v T)}{dt}$, (3)

where:

- **S** oil stress tensor with coordinates τ_{ij} for i,j= ϕ ,r,z [Pa],
- T oil temperature[K],
- cv heat capacity of oil in constant volume $[J \cdot kg^{-1} \cdot K^{-1}]$,
- \mathbf{v} oil velocity vector [m·s⁻¹],
- κ thermal conductivity coefficient [W·m⁻¹·K⁻¹],
- ρ oil density [kg·m⁻³],
- Ω heat for volume unit provided from another sources [W·m⁻³]. Classical constitutive references are assumed in form:

$$\mathbf{S} = -\mathbf{p} \, \mathbf{I} + \eta \mathbf{A}_1. \tag{4}$$

Velocity deformation tensor A_1 is defined by the following dependences [3, 5]:

$$\mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^1, \ \mathbf{L} \equiv \text{grad } \mathbf{v}, \tag{5}$$

where:

- **I** unity tensor,
- \mathbf{L} gradient tensor from velocity vector [s⁻¹],
- p hydrodynamic pressure [Pa],
- η dynamic viscosity coefficient [Pa·s].

Basic height h_p of oil gap depends on relative eccentricity λ and non-parallel of journal axis in relation to sleeve axis with angle γ , see Fig. 2.



Fig. 2. Geometry of oil gap of journal slide bearing

This non-parallelism in further part of his paper will be called skew bearing. The total dimensional height of oil gap h_c is a sum of the basic height h_p , thermal and pressure deformation

 u_d , magnetic deformation u_b , deformation caused by the vibrations and unstationarity of operating conditions and dislocation of the middle of journal u_t , roughness and waivness, which can be written as:

$$h_{c}(\phi, z, t) = h_{p}(\phi, z) + u_{d}(\phi, z, t) + u_{b}(\phi, z, t) + u_{f}(\phi, z) + u_{t}(t).$$
(6)

For the equations of motion and energy conservation (1)-(3) are substituted the constitutive relations (4) between the coordinates of the stress tensor $\tau_{\phi\phi}$, τ_{rr} , τ_{zz} , $\tau_{\phi r}$, τ_{rz} and coordinates of the velocity deformation tensor. Skipped are non-stationary parts and heat provided from the outside sources. Skipped are inertia forces in the momentum equations, the parts of forced convection in the equation of conservation energy are also not considered. Such an omission is justified in low- and middle-speed bearings. Assumed is also laminar and stationary flow, so the parts which contain derivatives related to time are skipped. In this way, a full set of motion and conservation of energy equations for the classical stationary flow of the lubricating oil is acquired. This set of equations is written in the form:

- momentum conservation equation in circumference direction ϕ of the journal:

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \phi} + \left(\frac{2}{r} + \frac{\partial}{\partial r}\right) \left[\eta \left(\frac{\partial v_{\phi}}{\partial r} + \frac{1}{r}\frac{\partial v_{r}}{\partial \phi} - \frac{v_{\phi}}{r}\right) \right] + \frac{2}{r}\frac{\partial}{\partial \phi} \left[\eta \left(\frac{1}{r}\frac{\partial v_{\phi}}{\partial \phi} + \frac{v_{r}}{r}\right) \right] + \frac{\partial}{\partial z} \left[\eta \left(\frac{1}{r}\frac{\partial v_{z}}{\partial \phi} + \frac{\partial v_{\phi}}{\partial z}\right) \right], \quad (7)$$

- momentum conservation equation in radius direction r:

$$0 = \frac{\partial}{\partial r} \left[-p + \left(2\eta \frac{\partial v_r}{\partial r} \right) \right] + \frac{1}{r} \frac{\partial}{\partial \phi} \left[\eta \left(\frac{\partial v_{\phi}}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \phi} - \frac{v_{\phi}}{r} \right) \right] + \frac{\partial}{\partial z} \left[\eta \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \right] + \frac{1}{r} \left[2\eta \left(\frac{\partial v_r}{\partial r} - \frac{1}{r} \frac{\partial v_{\phi}}{\partial \phi} - \frac{v_r}{r} \right) \right], \quad (8)$$

- momentum conservation equation in longitudinal direction z:

$$0 = -\frac{\partial p}{\partial z} + \left(\frac{1}{r} + \frac{\partial}{\partial r}\right) \left[\eta \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) \right] + \frac{1}{r} \frac{\partial}{\partial \phi} \left[\eta \left(\frac{1}{r} \frac{\partial v_z}{\partial \phi} + \frac{\partial v_{\phi}}{\partial z}\right) \right] + \frac{\partial}{\partial z} \left(2\eta \frac{\partial v_z}{\partial z}\right), \tag{9}$$

- the continuity equation:

$$\frac{1}{r} \left[\frac{\partial \left(\rho \cdot \mathbf{v}_{\phi} \right)}{\partial \phi} \right] + \frac{1}{r} \left[\frac{\partial \left(r \cdot \rho \cdot \mathbf{v}_{r} \right)}{\partial r} \right] + \left[\frac{\partial \left(\rho \cdot \mathbf{v}_{z} \right)}{\partial z} \right] = 0, \tag{10}$$

- conservation of energy equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\kappa r\frac{\partial T}{\partial r}\right) + \frac{1}{r^{2}}\frac{\partial}{\partial \phi}\left(\kappa\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial z}\right) + \left[-p + \eta\left(2\frac{\partial v_{r}}{\partial r}\right)\right] \cdot \frac{\partial v_{r}}{\partial r} + \\
+ \left[-p + \eta \cdot 2\left(\frac{1}{r}\frac{\partial v_{\phi}}{\partial \phi} + \frac{v_{r}}{r}\right)\right] \cdot \left(\frac{1}{r}\frac{\partial v_{\phi}}{\partial \phi} + \frac{v_{r}}{r}\right) + \left[-p + \eta\left(2\frac{\partial v_{z}}{\partial z}\right)\right] \cdot \frac{\partial v_{z}}{\partial z} + \\
+ \left(\frac{\partial v_{\phi}}{\partial r} + \frac{1}{r}\frac{\partial v_{r}}{\partial \phi} - \frac{v_{\phi}}{r}\right) \cdot \eta\left(\frac{\partial v_{\phi}}{\partial r} + \frac{1}{r}\frac{\partial v_{r}}{\partial \phi} - \frac{v_{\phi}}{r}\right) + \left(\frac{\partial v_{z}}{\partial r} + \frac{\partial v_{r}}{\partial z}\right) \cdot \eta\left(\frac{\partial v_{z}}{\partial r} + \frac{\partial v_{r}}{\partial z}\right) + \\
+ \left(\frac{1}{r}\frac{\partial v_{z}}{\partial \phi} + \frac{\partial v_{\phi}}{\partial z}\right) \cdot \eta\left(\frac{1}{r}\frac{\partial v_{z}}{\partial \phi} + \frac{\partial v_{\phi}}{\partial z}\right) = 0,$$
(11)

where:

 $0 \leq \varphi \leq \Gamma_{\alpha}, \, 0 \leq r \leq h_c, \, -b \leq z \leq +b, \, 0 \leq \Gamma_{\alpha} \leq 2\pi,$

 Γ_{α} - sleeve wrap angle,

b - half of the bearing length [m].

Form of the equations set (7)-(11) makes the estimation of magnitude of each of its segments in the case of lubricant flow in a thin layer of lubricating oil film not possible. To estimate the magnitude of segments in the set of equations (7)-(11) and skip the small units of a higher magnitude, performances for the loss of dimensions and estimation of the sizes of individual segments occurring in the set of equations of conservation of momentum, energy and continuity of the stream has been done. For this purpose, the following dimensional and dimensionless signatures and the numbers [3-5] were assumed:

$$t=t_{o} \cdot t_{1}, \ r=R(1+\psi r_{1}), \ z=bz_{1}, \ h_{c}=h_{c1} \cdot \varepsilon, \ h_{c1}=h_{p1}+u_{d1}+u_{b1}+u_{f1}+u_{t1},$$

$$h_{p}=\varepsilon \cdot h_{p1}, \ u_{d}=\varepsilon \cdot u_{d1}, \ u_{b}=\varepsilon \cdot u_{b1}, \ u_{f}=\varepsilon \cdot u_{f1}, \ u_{t}=\varepsilon \cdot u_{t1}, \ T=T_{o}+T_{o}BrT_{1}, \ p=p_{o}p_{1},$$

$$v_{\phi}=Uv_{1}, \ v_{r}=U\psi v_{2}, \ v_{z}=\frac{U}{L_{1}}v_{3}, \ \eta=\eta_{o}\eta_{1}, \ \kappa=\kappa_{o}\kappa_{1}, \ \rho=\rho_{o}\rho_{1}.$$
(12)

Symbols of the lower index of 1, 2 or 3 indicate the corresponding dimensionless values, while symbols with lower index indicate the specific dimensional values. Dynamic viscosity of the oil depends on temperature, pressure and time are shown in the following dimensionless forms [3-5]:

$$\eta_{I} = \eta_{Ip} \cdot \eta_{IT} \cdot \eta_{It}, \quad \eta_{Ip}(\phi, z) = e^{\varsigma_{p} \cdot p_{o} \cdot p_{I}} = e^{\varsigma_{pI} p_{I}},$$

$$\eta_{IT}(\phi, z, r) = e^{-\delta_{T}(T - T_{o})} = e^{-Q_{Br}T_{I}}, \quad \eta_{IB}(\phi, z) = e^{\delta_{t} \cdot t_{o} \cdot t_{I}} = e^{\delta_{tI}t_{I}},$$

$$Q_{Br} = BrT_{o}\delta_{T}.$$
(13)

Dependence of the viscosity change from time has been adopted in an exponential form on the basis of experimental results from the literature and because of simplicity of experimental analysis of such a form. With a deeper analysis of a specific type it will be possible to use another, more accurate dependence.

Basic dimensionless height of the oil gap varying in the circumferential direction and longitudinal direction has a following form:

$$\mathbf{h}_{p1} = [1 + \lambda \cdot \cos\phi + \mathbf{a}_{\gamma} \cdot \mathbf{z}_{1} \cdot \cos(\phi)], \quad \mathbf{a}_{\gamma} = \frac{\mathbf{L}_{1}}{\Psi} \tan(\gamma). \tag{14}$$

Assumed was also following dimensionless numbers [3-5]:

$$p_{o} \equiv \frac{RU\eta_{o}}{\epsilon^{2}}, \ \psi \equiv \frac{\epsilon}{R} \cong 10^{-3}, \ L_{1} \equiv \frac{b}{R}, \ Re \equiv \frac{U\epsilon\rho_{o}}{\eta_{o}}, \ Ty = Re\sqrt{\psi}, \ Br \equiv \frac{U^{2}\eta_{o}}{\kappa_{o}T_{o}}, \ 0 < Q_{Br} < 1,$$
(15)

where:

- Br dimensionless Brinkman number,
- L₁ dimensionless Bering length,
- QBr- dimensionless coefficient for viscosity changes in temperature T,
- R journal radius [m],
- R' Sauer radius [m],
- Re Reynolds number, for description of type of flow,
- Ty Taylor number, for description of type of flow,
- $U=\omega \cdot R$ dimensional value of circumferential velocity $[m \cdot s^{-1}]$,
- a_{γ} dimensionless coefficient for skew bearing,
- 2b bearing length [m],
- r₁ dimensionless radius coordinate,
- z1 dimensionless longitudinal coefficient,
- δ_t dimensional coefficient for viscosity changes in time [s⁻¹],

 $\delta_{t1} = \delta_t \cdot t$ - dimensionless coefficient for viscosity changes in time,

 $\epsilon = R' R$ - radius clearance [m],

 $\lambda = OO'/\epsilon$ - relative eccentricity,

 ψ - dimensionless value of relative radius clearance,

 ω - angular velocity of bearing journal [s⁻¹].

Dependencies (12)-(15) were substituted into the equations of conservation of momentum, stream continuity and conservation of energy (7)-(11). In this way obtained the set of equations in dimensionless form in which the parts in unity magnitude and negligible small parts in magnitude of relative radial clearance $\psi \approx 10^{-3}$ are visible. Apart from the segments in magnitude of a relative radial clearance, so approximately one thousand times smaller than the value of other segments, obtained a new simplified set of equations in the form:

$$0 = -\frac{\partial p_1}{\partial \phi} + \frac{\partial}{\partial r_1} \left(\eta_1 \frac{\partial v_1}{\partial r_1} \right), \quad 0 = \frac{\partial p_1}{\partial r_1}, \quad 0 = -\frac{\partial p_1}{\partial z_1} + \frac{\partial}{\partial r_1} \left(\eta_1 \frac{\partial v_3}{\partial r_1} \right), \tag{16}$$

$$\frac{\partial \mathbf{v}_1}{\partial \mathbf{\phi}} + \frac{\partial \mathbf{v}_2}{\partial \mathbf{r}_1} + \frac{1}{\mathbf{L}_1^2} \frac{\partial \mathbf{v}_3}{\partial \mathbf{z}_1} = 0, \tag{17}$$

$$\frac{\partial}{\partial \mathbf{r}_{1}} \left(\kappa_{1} \frac{\partial \mathbf{T}_{1}}{\partial \mathbf{r}_{1}} \right) + \left(\eta_{1} \right) \left[\left(\frac{\partial \mathbf{v}_{1}}{\partial \mathbf{r}_{1}} \right)^{2} + \left(\frac{1}{\mathbf{L}_{1}} \frac{\partial \mathbf{v}_{3}}{\partial \mathbf{r}_{1}} \right)^{2} \right] = 0, \qquad (18)$$

where:

 $0 < r_1 < h_{p1}, \ 0 < \varphi < 2\pi, -1 < z_1 < +1, \ \eta_1 \equiv \eta_{1t} (t_1) \eta_{1T} (T_1) \eta_{1p} (p_1), \eta_{1t} (t), \ \eta_{1p} (\phi, z_1), \ \eta_{1T} (\phi, r_1, z_1).$

For further analysis of the equations (16)-(18) is assumed that dimensionless coefficient for heat transfer κ_1 =1 and dimensionless density ρ_1 =1 of lubricant are constant, independent of temperature and pressure [4, 5].

In order to solve the problem of hydrodynamic lubrication, so to designate the functions of wanted values such as: components of velocity vector, temperature and hydrodynamic pressure, used the classical method of small parameter. This method uncouples the nonlinear set of differential equations, developing several linear sets of equations. The first set of equations is allows to determine the flow parameters for classical non-isothermal, Newtonian lubrication without taking into account the impact of pressure and temperature on the viscosity change. In this set of equations, the change of viscosity in time is being taken into account. Other sets of equations makes possible to determine the so-called corrections of the velocity vector components, hydrodynamic pressure and temperature resulting from taking into account influence of the pressure or temperature into viscosity change. Additionally, this method allows to isolate and after that, to analyze influence of temperature, hydrodynamic pressure and time on the value of exploitation parameters. For small dimensionless parameters assumed: dimensionless coefficient for changes of temperature-dependent viscosity Q_{Br} and dimensionless coefficient ς_p which describes the change of dynamic viscosity under influence of hydrodynamic pressure.

Unknown dimensionless functions, so three components of velocity vector of lubricant, hydrodynamic pressure and temperature in the oil gap are presented in a form of uniformly convergent power series expanded in terms of successive powers of small dimensionless parameters Q_{Br} , ς_p . This development has a following form:

$$\begin{aligned} \mathbf{v}_{i} &= \mathbf{v}_{i}^{(0)} + \mathbf{Q}_{Br} \mathbf{v}_{i0}^{(1)} + \ldots + \mathbf{Q}_{Br}^{j} \mathbf{v}_{i0}^{(j)} + \ldots &+ \boldsymbol{\varsigma}_{p} \mathbf{v}_{i1}^{(1)} + \ldots + \boldsymbol{\varsigma}_{p}^{j} \mathbf{v}_{i1}^{(j)} + \ldots \\ \mathbf{p}_{1} &= \mathbf{p}_{1}^{(0)} + \mathbf{Q}_{Br} \mathbf{p}_{10}^{(1)} + \ldots + \mathbf{Q}_{Br}^{j} \mathbf{p}_{10}^{(j)} + \ldots &+ \boldsymbol{\varsigma}_{p} \mathbf{p}_{11}^{(1)} + \ldots + \boldsymbol{\varsigma}_{p}^{j} \mathbf{p}_{11}^{(j)} + \ldots \\ \mathbf{T}_{1} &= \mathbf{T}_{1}^{(0)} + \mathbf{Q}_{Br} \mathbf{T}_{10}^{(1)} + \ldots + \mathbf{Q}_{Br}^{j} \mathbf{T}_{10}^{(j)} + \ldots &+ \boldsymbol{\varsigma}_{p} \mathbf{T}_{11}^{(1)} + \ldots + \boldsymbol{\varsigma}_{p}^{j} \mathbf{T}_{11}^{(j)} + \ldots \end{aligned}$$
(19)

for i=1,2,3 j=1,2....

The function of dynamic viscosity η_{1p} of lubricant (3.25), which depends on the pressure developed in series in relation to dimensionless piezo-coefficient ζ_p which is less than unity. Similarly, a function of dimensionless dynamic viscosity η_{1T} of lubricant, which depends on temperature, developed in series in relation to dimensionless coefficient Q_{Br} , which is less than unity:

$$\eta_{1p} \approx 1 + \varsigma_p p_1^{(0)} + O(\varsigma_p Q_{Br}, Q_{Br}^2, \varsigma_p^2), \ \eta_{1T} \approx 1 - Q_{Br} T_1^{(0)} + O(Q_{Br} \varsigma_p, Q_{Br}^2, \varsigma_p^2).$$
(20)

The series (19), (20) developed in relation to small parameters Q_{Br} , ς_p substitutes for the set of fundamental equations (16)-(18). This series are multiplied with a Cauchy's method, and after that compares the coefficients of the same powers of small parameters Q_{Br} , ς_p . In this way, next partial differential equations are obtained. On basis of these sets of equations, unknown functions should be acquired. The first set of equations, which includes changes of viscosity in time is shown below:

$$0 = -\frac{\partial p_1^{(0)}}{\partial \phi} + \frac{\partial}{\partial r_1} \left[\eta_{1t} \frac{\partial v_1^{(0)}}{\partial r_1} \right], \quad 0 = \frac{\partial p_1^{(0)}}{\partial r_1}, \quad 0 = -\frac{\partial p_1^{(0)}}{\partial z_1} + \frac{\partial}{\partial r_1} \left[\eta_{1t} \frac{\partial v_3^{(0)}}{\partial r_1} \right], \quad (21)$$

$$\frac{\partial \mathbf{v}_1^{(0)}}{\partial \phi} + \frac{\partial \mathbf{v}_2^{(0)}}{\partial \mathbf{r}_1} + \frac{1}{\mathbf{L}_1^2} \frac{\partial \mathbf{v}_3^{(0)}}{\partial \mathbf{z}_1} = 0,$$
(22)

$$\frac{\partial^2 \mathbf{T}_1^{(0)}}{\partial \mathbf{r}_1^2} + \eta_{1t} \left[\left(\frac{\partial \mathbf{v}_1^{(0)}}{\partial \mathbf{r}_1} \right)^2 + \frac{1}{\mathbf{L}_1^2} \left(\frac{\partial \mathbf{v}_3^{(0)}}{\partial \mathbf{r}_1} \right)^2 \right] = 0, \qquad (23)$$

where:

 $0 \le r_1 < h_{c1}, 0 \le \phi < \phi_k, -1 \le z_1 < +1.$

4. Conclusions

This method allows estimating the influence of the various impacts on changing flow and exploitation parameters, especially the impact of oil viscosity changes in time. On the basis of obtained stets of equations, in the next stage of research, using the analytical-numerical method, components of velocity vector, hydrodynamic pressure and temperature distribution and their adjustments resulting from changes in viscosity on temperature and pressure are being obtained.

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