

# PRESSURE AND VELOCITY DISTRIBUTION IN SLIDE JOURNAL PLANE BEARING LUBRICATED WITH MICROPOLAR OIL

Paweł Krasowski

Gdynia Maritime University  
Morska Street 81-87, 81-225 Gdynia, Poland  
tel.: +48 58 6901331, fax: +48 58 6901399  
e-mail: pawkras@am.gdynia.pl

## Abstract

Present paper shows the results of numerical solution Reynolds equation for laminar, steady oil flow in slide plane bearing gap. Lubrication oil is fluid with micropolar structure. Materials engineering and tribology development helps to introduce oils with the compound structure (together with micropolar structure) as a lubricating factors. Properties of oil lubrication as of liquid with micropolar structure in comparison with Newtonian liquid, characterized are in respect of dynamic viscosity additionally dynamic couple viscosity and three dynamic rotation viscosity. Under regard of build structural element of liquid characterized is additionally microinertia coefficient. In modelling properties and structures of micropolar liquid one introduced dimensionless parameter with in terminal chance conversion micropolar liquid to Newtonian liquid. The results shown on diagrams of hydrodynamic pressure, velocity and velocity of microrotation distribution in dimensionless form in dependence on coupling number  $N^2$  and characteristic dimensionless length of micropolar fluid  $\Lambda_1$ . Differences were showed on graphs in the schedule of the longitudinal velocity oils after the height of the gap in the flow of the micropolar and Newtonian liquid. In presented flow, the influence of lubricating fluid inertia force and the external elementary body force field were omitted. Presented calculations are limited to isothermal models of bearing with infinite breadth.

**Keywords:** micropolar lubrication, journal plane bearing, hydrodynamic pressure, velocity, velocity of microrotation

## 1. Introduction

Presented article take into consideration the laminar, steady flow in the crosswise slide plane bearing gap. Non-Newtonian fluid with the micropolar structure is a lubricating factor. Exploitation requirements incline designers to use special oil refining additives, to change viscosity properties. As an experimental studies show, most of the refining lubricating fluids, can be included as fluids of non-Newtonian properties with microstructure [3, 4, 6]. They belong to a class of fluids with symmetric stress tensor that we shall call polar fluids, and include, as a special case, the well known Navier-Stokes model. Physically, the micropolar fluids may represent fluids consisting rigid randomly orientated spherical particles suspended in a viscous medium, where the deformation of fluid particles is neglected [4]. Presented work dynamic viscosity of isotropic micropolar fluid is characterized by five viscosities: shearing viscosity  $\eta$  (known at the Newtonian fluids), micropolar coupling viscosity  $\kappa$  and by three rotational viscosities bounded with rotation around the coordinate axes. This kind of micropolar fluid viscosity characteristic is a result of essential compounds discussed in works [3-5]. Regarding of limited article capacity please read above works. In presented flow, the influence of lubricating fluid inertia force and the external elementary body force field were omitted [4, 5, 9].

## 2. Reynolds equation, hydrodynamic pressure

Basic equation set defining isotropic micropolar fluid flow is described following equations [2-5]: momentum equation, moment of momentum equation, energy equation, equation of flow continuity.

Incompressible fluid flow is taken into consideration with constant density skipping the body force. We assume also, that dynamic viscosity coefficients which characterize micropolar fluid are constant. According to above velocity flow field is independent from temperature field and the momentum equation, moment of momentum equation and equation of flow continuity are part of closed system of motion equations.

Lubricating gap is characterized by following geometric parameters: maximal gap height  $h_0$ , minimal gap height  $h_e$ , gap length  $L$  and gap breadth  $b$  (Fig. 1). In presented model the following assumption were made: lubricating gap dimensions along it's width of mating surfaces remain identical. Lubricating gap height after gap length was described in cartesian co-ordinate system by the following dimensionless form:

$$h_1(x_1) = \varepsilon - (\varepsilon - 1)x_1 \quad \text{for} \quad 0 \leq x_1 \leq 1. \quad (1)$$

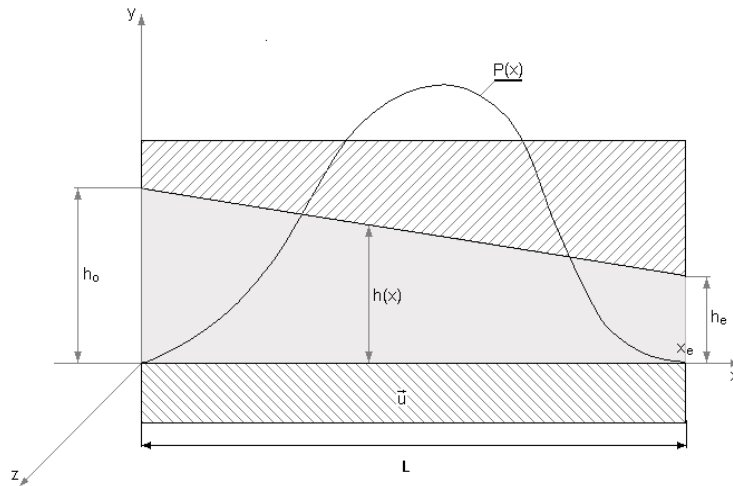


Fig. 1. Geometry schema of the slide journal plane bearing gap

Dimensionless values [2-4] that characterize lubricating gap are: length coordinate  $x_1$ , gap height coordinate  $h_1$  and gap convergence coefficient  $\varepsilon$ :

$$x_1 = \frac{x}{L}, \quad h_1 = \frac{h}{h_e}, \quad \varepsilon = \frac{h_0}{h_e}. \quad (2)$$

The constant viscosity of micropolar oil, independent from thermal and pressure condition in the bearing. Quantity of viscosity coefficient depends on shearing dynamic viscosity  $\eta$ , which is decisive viscosity in case of Newtonian fluids. Reference pressure  $p_0$  is also described with this viscosity, in order to compare micropolar oils results with Newtonian oil results. In micropolar oils decisive impact has quantity of dynamic coupling viscosity  $\kappa$  [1, 3, 4]. In some works concerning bearing lubrication with micropolar oil, it's possible to find the sum of the viscosities as micropolar dynamic viscosity efficiency. In presented article coupling viscosity was characterized with coupling number  $N^2$ , which is equal to zero for Newtonian oil:

$$N = \sqrt{\frac{\kappa}{\eta + \kappa}}, \quad 0 \leq N < 1. \quad (3)$$

Quantity  $N^2$  in case of micropolar fluid, define a dynamic viscosity of coupling share in the oil dynamic viscosity efficiency. From the dynamic rotational viscosities [1] at the laminar lubrication, individual viscosities are compared to viscosity  $\gamma$ , which is known as the most important and it ratio to shearing viscosity  $\eta$  is bounded to characteristic flow length  $\Lambda$ , which in case of Newtonian flow assume the zero quantity. Dimensionless quantity of micropolar length  $\Lambda_1$  and micropolar length  $\Lambda$  is defined [1]:

$$\Lambda = \sqrt{\frac{\gamma}{\eta}}, \quad \Lambda \Lambda_1 = h_e. \quad (4)$$

Dimensionless micropolar length  $\Lambda_1$  in case of Newtonian oil approach infinity. Basics equations for micropolar flow, after writing out in rectangular arrangement of co-ordinates are presented in article [2], where individual phases leading to Reynolds equation for laminar, stationary lubricating process in dimensionless form are mentioned.

Reynolds equation can be presented in dimensionless form [1], [8] using the method of changing into these values:

$$\frac{\partial}{\partial x_1} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial x_1} \right) + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial z_1} \right) = 6 \frac{dh_1}{dx_1}, \quad (5)$$

for  $0 \leq x_1 \leq 1$ ,  $0 \leq y_1 \leq h_1$ ,  $-1 \leq z_1 \leq 1$ ,

where:

$$\Phi_1 = h_1^3 + 12 \frac{h_1}{\Lambda_1^2} - 6 \frac{N h_1^2}{\Lambda_1} \coth\left(\frac{h_1 N \Lambda_1}{2}\right). \quad (6)$$

The dimensionless values for pressure  $p_1$ , bearing width  $L_1$  and remaining co-ordinates  $y_1$  and  $z_1$  are described as follows:

$$p = p_0 p_1, \quad L_1 = \frac{b}{L}, \quad z = b z_1, \quad y = h_e y_1. \quad (7)$$

Reference pressure  $p_0$  caused by linear velocity  $U$  of slide bearing was assumed in (7) taking into consideration dynamic viscosity of shearing  $\eta$  and the lubricating gap height  $h_e$  by relative play  $\psi$  in form :

$$p_0 = \frac{U \eta}{\psi L}, \quad \psi = \frac{h_e}{L}, \quad 10^{-4} \leq \psi \leq 10^{-3}. \quad (8)$$

Below solutions (6) for infinity breadth bearing is presented. In this solution the Reynolds boundary conditions, applying to zeroing of pressure at the beginning ( $x_1=0$ ) and at the end ( $x_1=1$ ) of the oil film ended. The pressure distribution function in case of the micropolar lubrication has a form:

$$p_1(x_1) = 6 \int_0^{x_1} \frac{h_1 - C_1}{\Phi_1(\Lambda_1, N, h_1)} dx_1, \quad C_1 = \frac{\int_0^1 \frac{h_1}{\Phi_1} dx_1}{\int_0^1 \frac{1}{\Phi_1} dx_1}. \quad (9)$$

In the boundary case of lubricating Newtonian fluid, pressure distribution function is  $p_{1N}(x_1)$ :

$$p_{1N}(x_1) = 6 \int_0^{x_1} \frac{h_1 - 1}{h_1^3} dx_1, \quad p_{1N} = \frac{6(\varepsilon - 1)(1 - x_1)x_1}{(\varepsilon + 1)(\varepsilon - \varepsilon x_1 + x_1)^2}. \quad (10)$$

Example numerical calculation was made for the infinity breadth bearing with convergence coefficient  $\varepsilon$ :  $\varepsilon_{opt} = 1 + \sqrt{2}$  end  $\varepsilon=1.4$  appointed the continuous and intermittent line.

Analyzing the influence of coupling number  $N^2$  and the influence of dimensionless micropolar length  $\Lambda_1$  on hydrodynamic pressure distribution in the bearing liner circuital direction. At the Fig. 2 pressure distribution for individual coupling numbers at constant micropolar length  $\Lambda_1 = 20$ . The pressure increase effect is caused by oil dynamic viscosity efficiency increase as a result of coupling viscosity. At  $N^2 = 0.5$ , coupling viscosity is equal to shearing viscosity. Pressure graph in the Fig. 2 for micropolar oil lubrication ( $N^2 > 0$ ) find themselves above the pressure graph at the

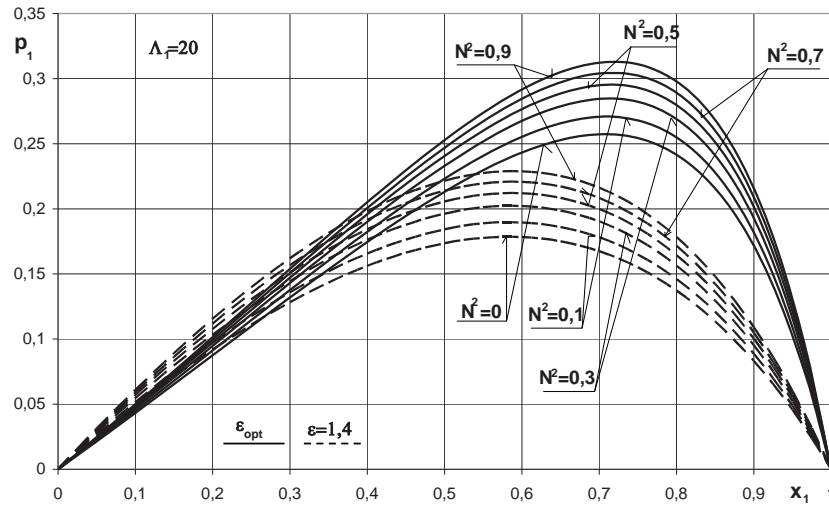


Fig. 2. The pressure distributions  $p_1$  in direction  $x_1$  in dependence on coupling number  $N^2$  by micropolar ( $N^2 > 0$ ) and Newtonian ( $N^2 = 0$ ) lubrication for convergence coefficient  $\varepsilon_{opt}$  and  $\varepsilon = 1.4$  from characteristic length of micropolar fluid  $\Lambda_1 = 20$

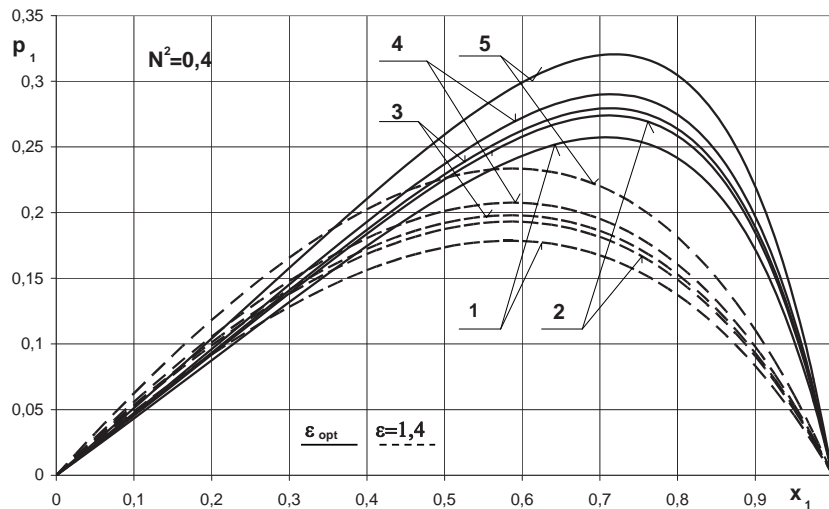


Fig. 3. The pressure distributions  $p_1$  in direction  $x_1$  in dependence on characteristic length of micropolar fluid  $\Lambda_1$ : 1) Newtonian oil, 2)  $\Lambda_1 = 40$ , 3)  $\Lambda_1 = 30$ , 4)  $\Lambda_1 = 20$ , 5)  $\Lambda_1 = 10$ , for convergence coefficient  $\varepsilon_{opt}$  and  $\varepsilon = 1.4$  from coupling number  $N^2 = 0.4$

Newtonian oil lubrication ( $N^2 = 0$ ). Pressure distribution is higher for higher coupling number. It is caused by oil viscosity dynamic efficiency. In the Fig.3 the course of dimensionless pressure  $p_1$  for few micropolar length quantity  $\Lambda_1$  is shown. Decrease of this parameter determines the increase of micropolar oil rotational dynamic viscosity. Pressure distribution are presented at the constant coupling number  $N^2 = 0.4$ . Newtonian oil pressure in the course 1. Rotational viscosity increase determines the pressure distribution increase and is caused, because both the oil flow and velocities of microrotation are coupled. Quantities of coupling number  $N^2$  and dimensionless micropolar length where taken from works [1, 2].

### 3. Velocity and velocity of microrotation distribution

The field equation of micropolar fluid with general lubrication theory assumptions is simplified into two systems of coupled ordinary differential equation. He interests us in the case of the bearing of the infinite breadth the arrangement the coupling the velocity  $V_x$  and the microrotation velocity  $\Omega_z$  according [9] and introduced in dimensionless form:

$$\begin{aligned} (1-N^2) \frac{\partial p_1}{\partial x_1} &= \frac{\partial^2 V_1}{\partial y_1^2} + N^2 \frac{\partial \Omega_3}{\partial y_1}, \\ \frac{1-N^2}{N^2 A_1^2} \frac{\partial^2 \Omega_3}{\partial y_1^2} - \frac{\partial V_1}{\partial y_1} - 2\Omega_3 &= 0. \end{aligned} \quad (11)$$

Dimensionless velocity  $V_1$  and velocity of microrotation  $\Omega_3$  are in formula:

$$V_x = V_1 U, \quad \Omega_z = \Omega_3 \frac{U}{h_e}. \quad (12)$$

The profiles of the schedule of velocity and microrotation velocity after the height of the gap (co-ordinate  $y$ ) comply following boundary conditions  $V_1(s_1)$  and  $\Omega_3(s_1)$  [9] on the surface of the bearing  $s_1 = 0$  and on the slide  $s_1 = 1$ :

$$\begin{cases} V_1(0) = 1 \\ V_1(1) = 0 \end{cases}, \quad \begin{cases} \Omega_3(0) = 0 \\ \Omega_3(1) = 0 \end{cases}, \quad s_1 = \frac{y_1}{h_1}. \quad (13)$$

Thus, the expressions for velocity  $V_x$  as results of the solutions of the above equations with boundary conditions (13) are [9] in dimensionless form  $V_1$ :

$$\begin{aligned} V_1 &= \frac{1}{2} s_1^2 h_1^2 \frac{\partial p_1}{\partial x_1} \left( \frac{1}{2} s_1^2 h_1^2 - \frac{N h_1}{A_1} \frac{\cosh s_1 N A_1 h_1 - 1}{\sinh N A_1 h_1} \right) + 1 + \\ &+ \frac{A_1}{1-N^2} \left\{ s_1 h_1 - \frac{N}{A_1} \left[ \sinh s_1 N A_1 h_1 - \frac{(\cosh s_1 N A_1 h_1 - 1)(\cosh N A_1 h_1 - 1)}{\sinh N A_1 h_1} \right] \right\}, \end{aligned} \quad (14)$$

where:

$$A_1 = \frac{1}{2} (1-N^2) \left( h_1 \frac{\partial p_1}{\partial x_1} + \frac{1}{\frac{h_1}{2} - \frac{N}{A_1} \frac{\cosh N A_1 h_1 - 1}{\sinh N A_1 h_1}} \right). \quad (15)$$

The profile velocity  $V_1$  was introduced on Fig. 4 along the height of the gap in the point where maximum hydrodynamic pressure steps out for three parameters of the micropolar oil. Nonlinearity visible is insignificant in the relation to the linear graph in the case of lubrication from Newtonian oil. Differences among the schedule of the velocity of micropolar and Newtonian oil it was introduced on Fig. 5. in points of the maximum pressure. The difference of the speed was marked from dependence:

$$\Delta V_1 = V_{1p} - V_{1N}. \quad (16)$$

where:

$V_{1p}$  - dimensionless velocity for micropolar oil flow marked from (15),

$V_{1N}$  - dimensionless velocity for Newtonian oil flow marked from (18).

$$V_{1N} = 1 - s_1 + \frac{h_1}{2} (s_1^2 - s_1) \frac{\partial p_1}{\partial x_1}. \quad (17)$$

Change velocities  $\Delta V_1$  in point of the maximal pressure are asymmetric in relation to the centre of the height of the gap. For  $0 < s_1 < 0.5$ , the velocity of the micropolar flow is larger from the velocity of the flow of the Newtonian oil and is for  $0.5 < s_1 < 1$  smaller. Velocity  $V_1$  of the flow dependent changes are from the coefficient convergence  $\varepsilon$  of the gap of bearing and are smaller for the optimum coefficient convergence  $\varepsilon_{opt}$ . The growth of the coupling viscosity (coupling number  $N$ ) causes the growth of the nonlinearity of velocity  $V_1$ .



Fig. 4. Velocities  $V_1$  in points of the maximal pressure of micropolar fluid  $\Lambda_1=20$ : 1)  $N^2=0.1$ , 2)  $N^2=0.5$ , 3)  $N^2=0.9$  from convergence coefficient  $\epsilon_{opt}$

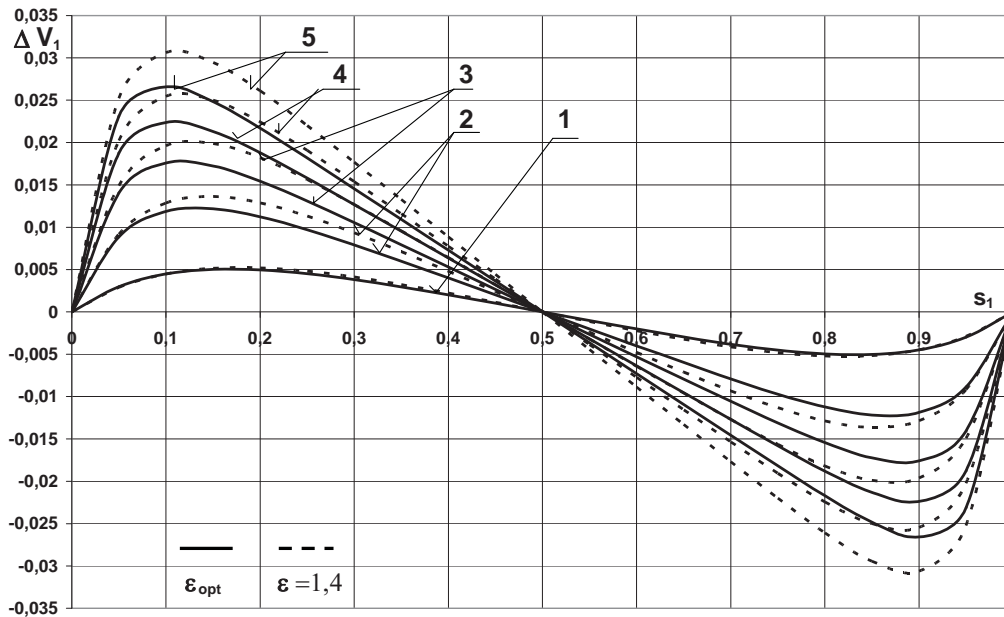


Fig. 5. Change velocities  $\Delta V_1$  in points of the maximal pressure of micropolar fluid  $\Lambda_1=20$ : 1)  $N^2=0.1$ , 2)  $N^2=0.3$ , 3)  $N^2=0.5$ , 4)  $N^2=0.7$ , 5)  $N^2=0.9$  from convergence coefficient  $\epsilon_{opt}$  and  $\epsilon=1.4$

The expressions for velocity of microrotation  $\Omega_z$  as results of the solutions of the above equations with boundary conditions (13) are [9] in dimensionless form  $\Omega_3$  :

$$\Omega_3 = \frac{\sinh s_1 N \Lambda_1 h_1}{\sinh N \Lambda_1 h_1} \left[ \frac{h_1}{2} \frac{\partial p_1}{\partial x_1} - \frac{A_1}{2(1-N^2)} (\cosh N \Lambda_1 h_1 - 1) \right] +$$

$$-s_1 \frac{h_1}{2} \frac{\partial p_1}{\partial x_1} + (\cosh s_1 N \Lambda_1 h_1 - 1) \frac{A_1}{2(1-N^2)}. \quad (18)$$

In the Fig. 6 are presented profile velocity of microrotation  $\Omega_3$  in points of the maximum pressure from the coefficient convergence  $\epsilon$  of the gap of bearing and in the coupling Number  $N^2$  function for chosen micropolar length  $\Lambda_1 = 20$ .



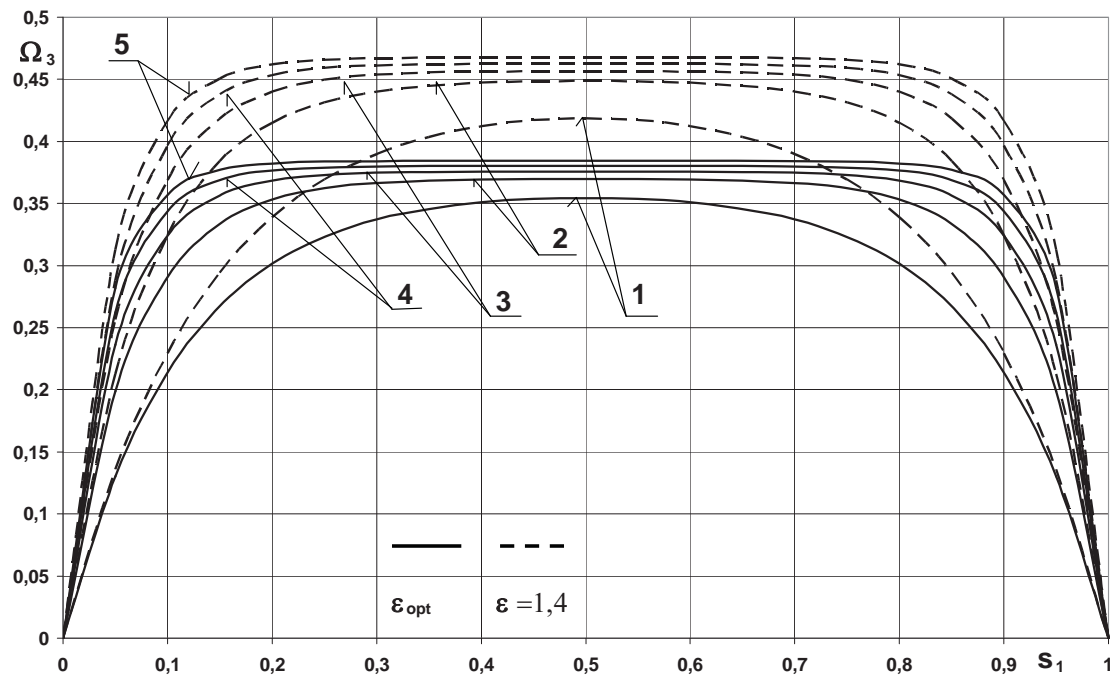


Fig. 6. Velocities of microrotation  $\Omega_3$  in points of the maximal pressure of micropolar fluid  $A_1=20$ : 1)  $N^2=0.1$ , 2)  $N^2=0.3$ , 3)  $N^2=0.5$ , 4)  $N^2=0.7$ , 5)  $N^2=0.9$  from convergence coefficient  $\varepsilon_{opt}$  and  $\varepsilon=1.4$

The profile of the velocity of microrotation  $\Omega_3$  of the flow dependent changes are from the coefficient convergence  $\varepsilon$  of the gap of bearing and are smaller for the optimum coefficient convergence  $\varepsilon_{opt}$ . The profile is symmetrical in relation to the centre of gap. He is more even (flat) after the height of the gap in the case of the growth of the coupling number  $N$ .

#### 4. Conclusions

Presented example of the Reynolds equation solutions for steady laminar non-Newtonian lubricating oil flow with micropolar structure, enable the hydrodynamic pressure distribution and velocity, velocity of microrotation introductory estimation as a basic exploitation parameter of slide plane bearing. Comparing Newtonian oil to oils with micropolar structure, can be used in order to increase hydrodynamic pressure and velocities of oil also to increase capacity load of bearing friction centre. Micropolar fluid usage has two sources of pressure increase in view of viscosity properties: increase of fluid efficient viscosity (coupling viscosity increase) and the rotational viscosity increase (characteristic length parameter  $\Lambda$ ). Author realize that he made few simplified assumptions in the above bearing centre model and in the constant parameter characterizing oil viscosity properties. Despite this calculation example apply to bearing with infinity length, received results can be usable in estimation of pressure distribution and of capacity force at laminar, steady lubrication of cylindrical slide bearing with infinity breadth. Presented results can be usable as a comparison quantities in case of numerical model laminar, unsteady flow Non-Newtonian fluids in the lubricating gaps of crosswise slide plane bearings.

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