# IDENTIFICATION OF PIERCED MATERIALS CHARACTERISTICS IN THE ASPECT OF SELECTED DEGENERATED MODELS

Krzysztof Jamroziak

Tadeusz Kosciuszko Military Academy of Land Forces, Institute of Command Czajkowskiego Street 10, 51-150 Wroclaw tel.: +48717658246, fax: +48717658099 e-mail: krzysztof.jamroziak@wso.wroc.pl

#### **Mirosław Bocian**

Wroclaw University of Technology Institute of Materials Science and Applied Mechanics Smoluchowskiego Street 25, 50-370 Wroclaw tel.: +48713202754, fax: +48713211235 e-mail : mirosław.bocian@pwr.wroc.pl

#### Abstract

A number of theoretical problems are discovered in the application of degenerated models in punching process of ballistic shields. Insufficient knowledge of dynamic behaviours of these models based on non-linear complex constitutive materials or combination of mixed rheological Maxwell models with linear elastic elements or dissipative is the main reason of the problems.

In the engineering application with t he influence of rheological forces the element of mass responsible for force of inertia in dynamic loads is always applied. Taking the forces i nto consideration in dynamic analysis the degenerated system is necessary to use. It is also utilitarian goal of material properties identification in punching process with the use of particle mass (projectile).

In this paper, the authors present further results of research in this area. Suppression and dispersion of energy is the crucial phenomena in the punching process of ballistic shields with the use of small arms. Therefore the research presented in the paper concerns systems based on viscotic damping with the dry friction parameter (h).

As a result of the research influence of h parameter on time and frequency characteristics under impact stress was estimated. Analytic calcul ations and simulations of the examination were validat ed by experimental test and measurements. With the basis on the results the final conclusions were formulated.

Keywords: analysis and modelling, light ballistic shields, materials penetration, degenerated models, identification

#### 1. Introduction

Phenomena of the punching process of materials dedicated to construction of ballistic shields are perceived as complicated problems caused by dynamic boundaries. Design of effective ballistic shields requires application of innovative models much more accurate than the models based on the Hooke's model and Young module. In the most cases it was sufficient for identifying the stiffness parameter and modal damping.

The idea of degenerated models implementation in the punching process of materials used in construction of ballistic shields encounters a number of theoretical difficulties [1-3]. It relates to the lack of comprehensive knowledge of dynamic behaviour of these models that are based on non-linear complex constitutive materials or combination of mixed rheological Maxwell models with linear elastic elements. However, in the engineering applications, aside from an effect of rheological forces, there is always a mass element, which causes inertia forces under the dynamic conditions. Taking the forces into consideration in dynamic analysis the degenerated system is

necessary to use [4-8]. Therefore development of reliable dynamic models in the space of degenerated systems is needed. This way of material properties determination seems to be suitable. Material engineering development provides many of modern composite materials. Some of them are dedicated to construction of ballistic shields. In this case only non-classical models are able to simulate precisely the punching process.

#### 2. Analysis of selected degenerated model with viscotic damping

Taking into account the change of the bullet velocity in the phase of material punching it is possible to establish, that function v(x) describes changes of the velocity depending on bullet position x in a shield. This function decreases (dv/dx<0 for each x {(0, h)} and the shape depends on the material properties. In this case the properties are decisive in the correct description of the punching process. Assuming that the accepted mathematical model and strength tests indicate selecting material properties of the ballistic shield, the model with Maxwell type element described by  $k_0$ ,  $c_d$  parameters in the parallel configuration with the resilient element (Fig. 1) was established.

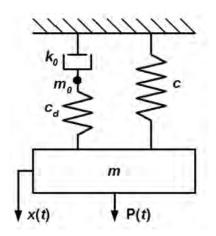


Fig. 1. Selected degenerated model with viscotic suppression

Differential equation of a movement of the punched material resistance force is presented in the following form:

$$m\ddot{x} + cx + c_d(x - \xi) = P,$$
  

$$c_d(x - \xi) = k_0 \dot{\xi},$$
(1)

where:

- m reduced casing mass,
- c tautness of casing,
- $c_d$  dynamic tautness of casing in the resilience range,
- $k_0$  suppression in the resilience range,
- $\xi$  variable representing a movement of fictional mass  $m_0 = 0$ . An analysis of proposed model was conducted on two different cases:
- a) analysis in quasi-static state of stretching (with constant velocity  $\dot{x} = v_0 = const$ ),
- b) analysis with forced type of impact (rectangular and sinusoidal impulse).

In the case (a), the analytical phrase representing relation of attached force P and distortion deformation x is presented in the following mathematical figure:

$$P(x) = cx + k_0 v_0 \left( 1 - e^{\frac{-c_0}{k_0 v_0} x} \right).$$
<sup>(2)</sup>

Therefore, the graphical figure of the relation (2) is presented in the following way (Fig. 2):

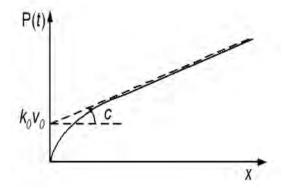


Fig. 2. The characteristic of quasi-static stretching of analyzed model reached from theoretical analysis

Executing further analysis and taking under consideration the computer simulation methods (Mathematica software), simulation of phrase (2), with assumed different velocities for selected constant numerical values of the model, was conducted, i.e.:

 $k_0 = 110\ 000\ [\text{kg/s}],$   $c = 11\ 500\ [\text{kg/s}^2],$  $c_d = 350\ 000\ [\text{kg/s}^2],$ 

m = 18 [kg],

where the achieved results are represented by the following charts (Fig. 3).

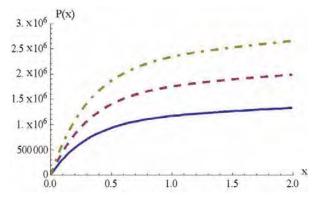


Fig. 3. Quasi-static characteristics for various i mpact velocities: (solid line - v0=10m/s, dashed line - v0=15m/s, dash-dot line - v0=20m/s)

In the second case (b) the analyzed system was subjected to the dynamic loads in the form of rectangular and sinusoidal impulses (positive half of the period of the function sin) with identical values of the model like above. As a result of this simulation the characteristics x(t) were obtained and presented on Fig. 4-5.

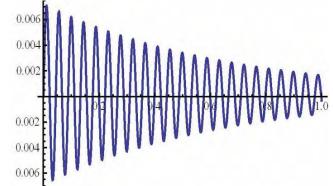


Fig. 4. Time responses on rectangular impulse extortion

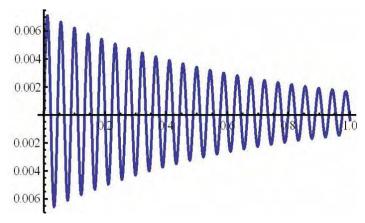


Fig. 5. Time responses on sinusoidal impulse extortion

### 3. Selected degenerated model analysis with dry friction consideration

Suppression and dissipation of energy plays main role in the process of destroying material by impact load of projectile. Therefore aside for viscotic suppression, the h parameter representing dry friction was utilized. In this stage of analysis, the influence of the h parameter on time and frequency characteristics under a percussive stress was conducted (Fig. 6). The dry friction is one of the reasons of decreasing kinetic energy and it should be taken into consideration in effective simulation of punching process.

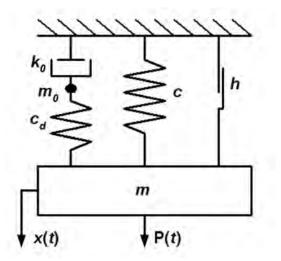


Fig. 6. Zener type degenerated model with dry friction

There was assumed that movement of overall mass in the shield reacts as impact load of single concentrated mass m. Resistant force of material shield S reduces the movement of punching mass in dependence of x position, and velocity v. In the simulation the resistant force is selected by model type. This relation is presented in the following equations:

$$S(x,v) = cx + c_d(x - \xi) + hSgn(v),$$
(3)

where variable  $\xi$  represents fictional mass movement  $m_0 = 0$  and fulfils the equation:

$$c_d = (x - \xi) = k_0 \dot{\xi}. \tag{4}$$

Hence, the equation of analyzed model movement is shown below:

$$m\ddot{x} + cx + c_d(x - \xi) + hSgn\dot{x} = P.$$
(5)

Based on relation (4) and (5), after removing variable  $\xi$ , the equation may be written as follows:

$$m\ddot{x} + hSgn(\dot{x}) + cx + \frac{k_0}{c_d} \left[ (c_d + c)\dot{x} + h\delta(\dot{x})\ddot{x} + m\ddot{x} - \dot{P} \right] = P.$$
(6)

As already mentioned,  $\delta(\dot{x})$  is a Dirac function from velocity and is removed for velocity values other than zero. Resistance force in degenerated structure may be described in the following way:

$$S = hSgn(\dot{x}) + cx + \frac{k_0}{c_d} \left[ (c_d + c)\dot{x} + h\delta(\dot{x})\ddot{x} + m\ddot{x} - \dot{P} \right].$$
(7)

In the case of quasi-static deformations (e.g. stretching attempts with constant set velocity  $v=const=v_z>0$ ), the force S will look as follows:

$$S_{q} = h + cx + \frac{k_{0}}{c_{d}} \Big[ (c_{d} + c) v_{z} - \dot{P} \Big].$$
(8)

Referring to movement equation (6), analyzed phrase will be written as:

$$P = h + cx + \frac{k_0}{c_d} [(c_d + c)v_z - \dot{P}].$$
(9)

Therefore, a final resolution of differential equation is a function P(x) expressed by the relation:

$$P(x) = h + cx + k_0 v_z \left( 1 - e^{\frac{-c_d x}{k_0 v_z}} \right).$$
(10)

Which graphical representation is presented in Fig. 7.

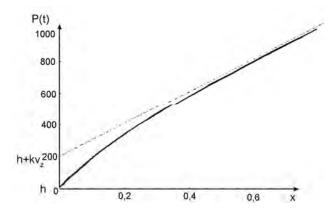


Fig. 7. Graphical representation of the relation (10) for an analysis of quasi-static penetration

As similarly as in case of analysis of the model taking viscotic suppression into account, the computer simulation of temporal and frequency characteristics changing the parameter of dry friction with the h was set. The research was conducted for four different values of h, assuming the value h as: 0, 1, 5 and 10 with considering following data in the model:

 $k_0 = 20 \text{ [kg/s]},$   $c = 1000 \text{ [kg/s^2]},$  $c_d = 1000 \text{ [kg/s^2]},$ 

$$m = 1 \, [kg].$$

In each case the impulse extortion was simulated, in the form: for t < 0.01, where force is equal to  $P(t)=100Sgn(Pi \ t/0.01)$ , for the rest of range the value is zero. Charts from simulation are presented in Fig. 8-9.

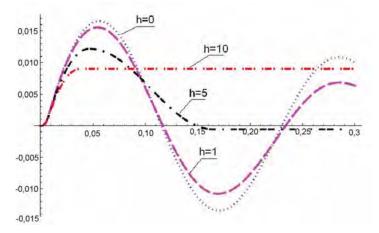


Fig. 8. Time characteristics of analyzed model for different values of h

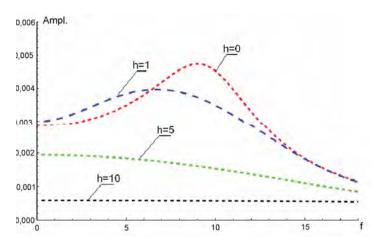


Fig. 9. Frequency characteristics of analyzed model for different values of h

### 4. Verification attempt of theoretical analysis in ballistic experiment

In the ballistic experiment of shield punching by 9 mm Parabellum bullet the casing-projectile acceleration was measured. The testing station (Fig. 10) was prepared with tautly fixed clamping of the sample of ballistic shield and two acceleration sensors PCB M350B21 (Fig. 11).



Fig. 10. Experiment station of punching the ballistic shield



Fig. 11. Acceleration sensor M350B21

The sensors ware designated for broad-range amplitude acceleration measurements. They are characterized by the following parameters:

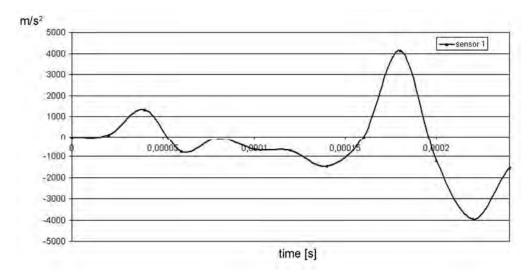
- amplitude  $\pm 100\ 000g\ (980k\ m/s^2)$ ,
- sensitivity  $0.05 \text{mV/g} [0.005 \text{mV/(m/s}^2)]$ ,
- frequency range 1-10 000 Hz,
- weight 4.4 grams,
- external connectors,
- titanium casing,
- non-filter resonance input  $\geq 200$  kHz.

The test was conducted according to the methodology of research described in regulation PN EN 1522 and PN EN 15 23 [8, 9].

Finally the following results were registered and analyzed in order to present the characteristics on Fig. 12-13.

# 4. Conclusions

Based on the chart presented in Fig. 12, it can be observed, that during an impact, there is no strong vibrations of composite material. The vibration was effectively suppressed. This observation suggests the high values of viscotic suppression and dry friction factors. Unlike dry friction and the viscotic suppression do not cause permanent deformation. After the visual observation of the tested shields there was confirmed that the dry friction does not influence on the punching process essentially. Future research in the area of material identification of efficient ballistic protection, the experiments with larger number of samples, what permits a statistical approach and determining precise values of models parameters is planed.



*Fig. 12. The acceleration registered by the sensor No.1 during impact of 9 mm Parabellum bullet with impact velocity* 358 m/s

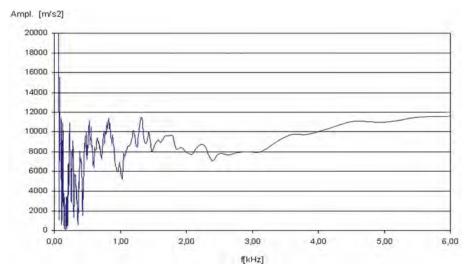


Fig. 13. Frequency characteristic during impact of 9 mm Parabellum bullet with impact velocity 358 m/s

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