# NUMERICAL SIMULATION OF STRESS AND STRAIN STATE INDUCED BY SHRINKAGE OF CONCRETE IN LARGE-SIZE PLATE

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***Abstract***

*The paper presents the numerical simulation of the shrinkage effect of concrete aprons. Three types of square aprons with the sides of 5, 10 and 20 mm and of the same thickness were considered. In simulations it was assumed the following properties of concrete: E = 32000 MPa, ν = 0.17, ρ = 2.4e-9t/mm³, $\alpha_t = 10^{-5}$. The shrinkage of concrete has been reported by the homogeneous temperature field in a plate. The plates were located on elastic foundation with stiffness k = 50 MN/m3. The foundation was simulated by using elements of GAP type, which did not transmit the drawings. Introducing the coefficients of rigidity in shear of GAP and the coefficient of friction between the foundation and the plate allowed to analysis the load plate friction forces caused by the shrinkage of concrete. The plate weight was taken into consideration, too. The numerical simulation was performed using the finite element method. Uniform changes in temperature –30° C and –60°C adopted in the analysis correspond to the values of contraction plates 0.3 mm/m and 0.6 mm/m. In practice, regular contraction joints are produced by notching of hardening concrete to a depth of about 1/3 the thickness of plate, for example, in a grid of 5 x 5 m. Studies show that in the case of protection of large-size plates from accidental shrinkage cracks, for example with the use of reinforcement, the stresses caused by the simultaneous change in temperature is not a major threat in slabs of large size.*

***Keywords**: airfield, concrete apron, temperature, thermal load, numerical analysis*

## 1. Introduction

According to the assumptions of the theory of elasticity, a homogeneous isotropic body, supported in a statically determined way, exposed to a temperature field which is a linear function of Cartesian coordinates, is deformed at zero stress state. When the temperature of plate along its thickness (z-axis) is constant, the plate deforms in the plane and remains flat, changing only its size. The linear change of temperature along the z-axis causes plate bending with a zero stress state. In the real case, where the plate can not be easily deformed by the action of its own weight, friction, foundation reactions and reactions of other plates, the stress caused by temperature appears in a plate. Summation of these stresses and the stress derived from additional load, such as aircraft wheels pressures, may lead to a situation in which the resulting stress exceeds the maximum allowable stress values and, consequently will lead to damage in the form of scratches or cracks in slabs.

History of research of concrete slabs under thermal loads dates back to the early twenties of the twentieth century. The first attempts to determine the stress arising from thermal loads were formulated by Westergaard and were based on the fundamental assumptions, that the dimensions of the plates are infinite along the x- and y-axis, and the field temperature is constant along the x- and y-axis and linear variable in the direction of the z-axis. Further works on the detailed description of the impact thermal loads carried out, inter alia, by J. Eisenman and I. Poliaĉek. Although they already introduced the concept of finite-dimensional plate along the x- and y-axis (critical length), did not take into account the nature of the changes in temperature along the z-axis, but they were only based on the difference of temperature between the bottom and the top of the plate (ΔT). Based on this assumption, the temperature gradient is the main parameter determining the state of stress caused by thermal loads.

Because of the complex nature and importance of the impact of thermal loads on the concrete labs, various studies are carried out around the world to calculate both the gradient occurring between the bottom and the top of the plate and to set detailed distributions of temperature along the thickness of the plate. The current state of knowledge about the impact and importance of the thermal loads on the concrete labs clearly confirms the validity of conducting detailed studies of temperature distributions.

The excellent tool for the analysis of the slabs strength under thermal load is the use of programs based on the finite element method, which allows further detailed examination not only of the influence of the gradient, but also the character of temperature changes on the thickness of the plates.

## 2. Description of numerical model of concrete plate on elastic foundation

Using experience from the analysis of the impact of thermal loads on the stresses state and displacements in concrete aprons conducted using FEM, the authors presented in [1], the use of this method to describe the shrinkage effect of concrete aprons was proposed. The analysis uses a system of FEM, MSC.NASTRAN. In all cases, the concrete has been reported by using the material properties:

- elastic modulus of concrete E=32000 MPa,
- Poisson’s ratio v = 0.17,
- concrete specific gravity $\gamma$ = 2400 kg/m$^3$,
- coefficient of thermal expansion $\alpha = 10^{-5}$ 1/°C.

The concrete plates were modelled by using 8-nodes solid elements with four layers of elements on the thickness of the plates and the regular distribution in the plane of plate – 0.1 m x 0.1m. The elastic foundation was simulated by using elements of GAP type, which did not transmit the drawings. These elements allow taking away the plate from the foundation. The ground reaction coefficient k = 50 MPa/m was assumed. In all cases, gravity load and temperature field were taken into consideration, too.

## 3. Model of 5m x 5m size concrete plate loaded by a uniform temperature field

The plates of this size were analysed in the work [2-4], both in the case of utility loads - the wheels of the aircraft, as well as at temperature field load. Variability of temperature along the thickness of the plate causes the bending. In the case of a positive temperature gradient (temperature increases from the bottom to the top of the plate), a plate is bending as shown in Fig. 1. In the case of a negative temperature gradient (temperature decreases from the bottom to the top of the plate), deformation of the plate is reversed to the previous one.

The uniform temperature field produces a different state of deformation of the plate. In the case of weightless plate, deformation occurs in a plane of the plate and the plate remains flat but the linear dimensions of the plate are changing in accordance with the change of temperature and the coefficient of thermal expansion. When including the weight of plate, displacements in the plane of the supporting plates cause the activation of friction forces directed opposite to the direction of displacement, proportional to the forces of pressure at a given point. In the FEM model, it takes place in the GAP element. Friction forces applied to the lower surface of the plate cause the deformation. Bending of the plate changes the force pressure on the GAP elements at the same time changing the friction forces (nonlinear problem). Constraints imposed on the plane plate prevent displacement the centre of the plate.

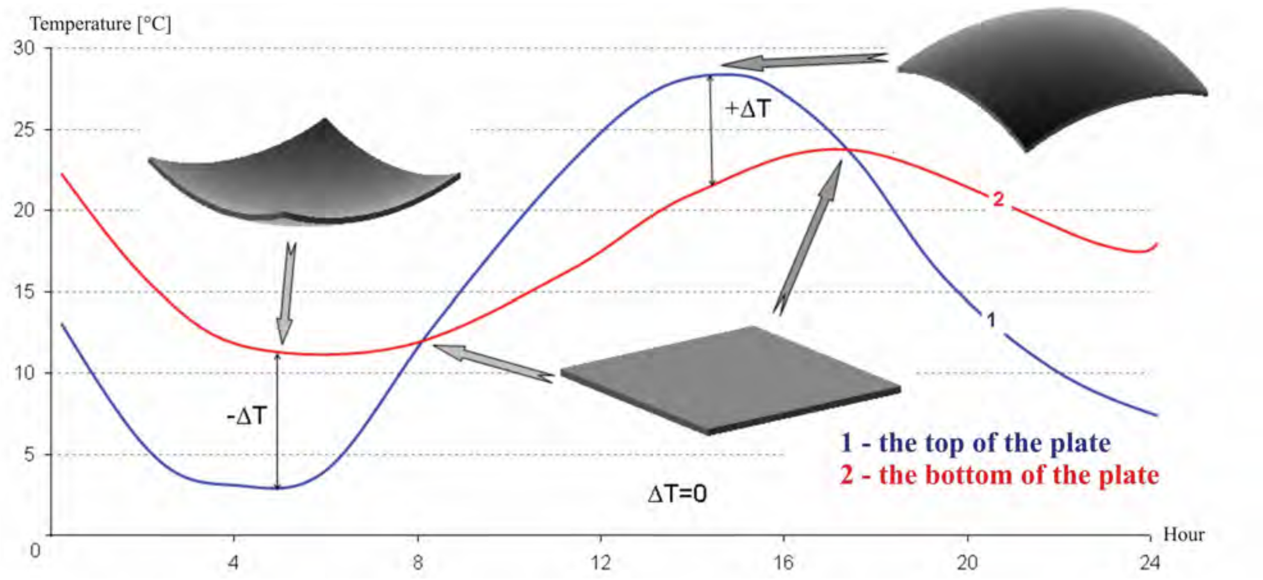


*Fig. 1. Deformation of plates under linear variation of temperature*

We will investigate this phenomenon on the example of a square plate of 5 x 5 m, in which the decrease of temperature by 30°C occurred. In the simulation it was assumed that the friction coefficient between the plate and the ground is equal 1. In Fig. 2 shows the deformation of the plate. Contours represent displacement in the normal direction to the surface. The maximum displacements are located in the middle of the plate and are equal 0.25 mm. Forces in GAP elements are shown in Fig. 3 and 4. The biggest pressure plate on the foundation (X component of force in the GAP elements) is in the middle of the plate top and the maximum value of pressure is 81.5 N (Fig. 3). The maximum value of the friction forces (Y component of forces in the GAP elements) is equal 80.3 N. It should be noted that the values of plate displacements and forces in the GAP elements, calculated in the considered task are greater than in the case where identical slabs are located freely on the elastic foundation. In that case, the average displacement of the plate is 0.141 mm.

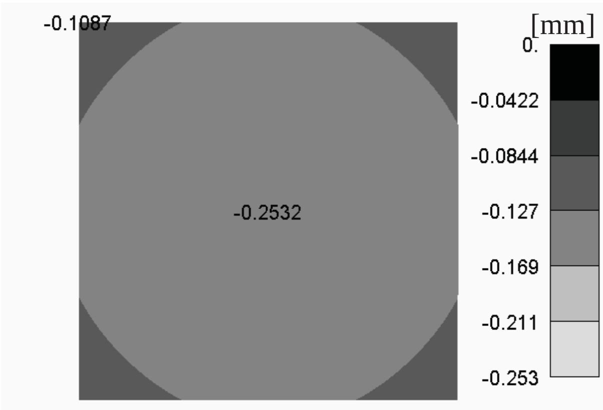


*Fig. 2. Displacements in the normal direction to the surface of the plate*

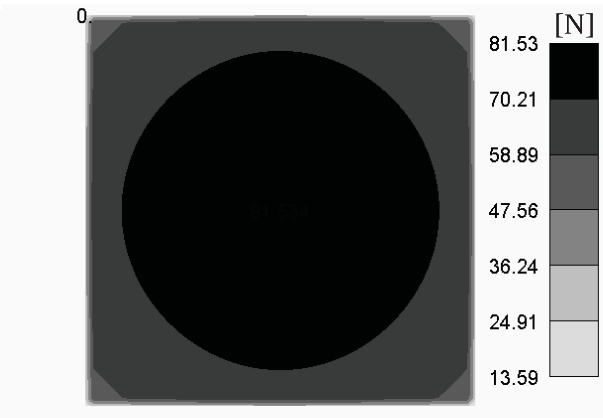


*Fig. 3. Force pressure plate on the foundation*

The reduced stresses calculated according to Huber's hypothesis for the lower surface of the plate are showed in Fig. 5. The maximum value of reduced stresses is equal 81.5 N and is located in the middle of the plate bottom. Fig. 6 and 7 show normal and tangential stresses on the lower surface of the plate. The maximum normal stress value is located in the middle of the plate bottom and is equal 138 kPa, while the normal stresses gain zero values in the corners. In this case, extreme values of shear stresses are equal 12 kPa. Fig. 8 and 9 show contours of maximum principal stress. Principal stress distribution on the plate bottom is similar to the reduced stress distribution.

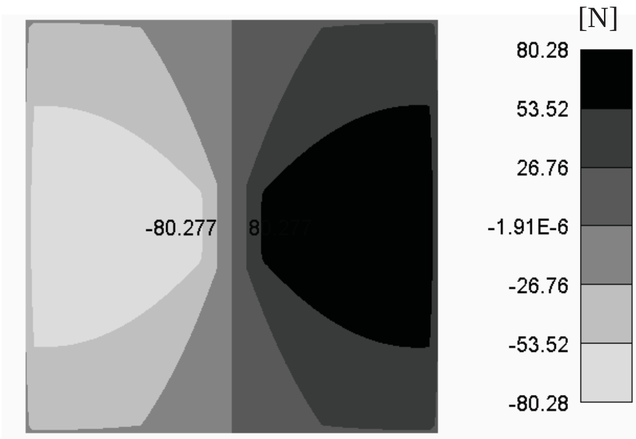


*Fig. 4. Friction forces between the plate and the ground*

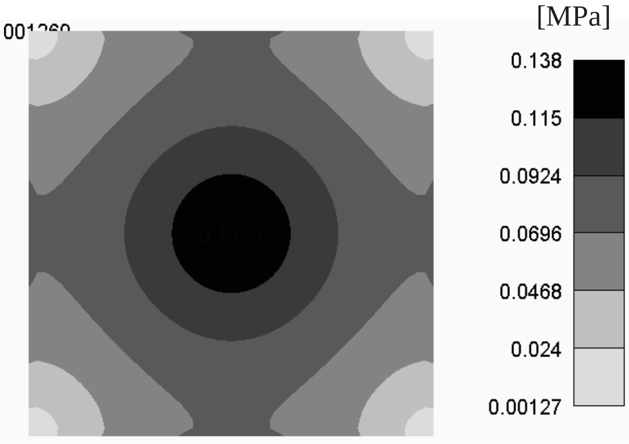


*Fig. 5. The Huber-Mises stresses*

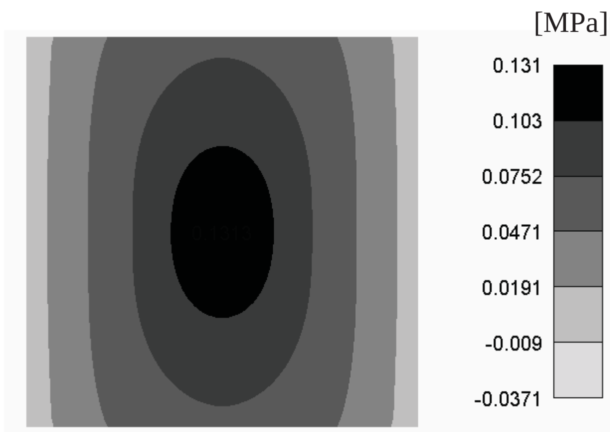


*Fig. 6. Normal stresses on the bottom plate*

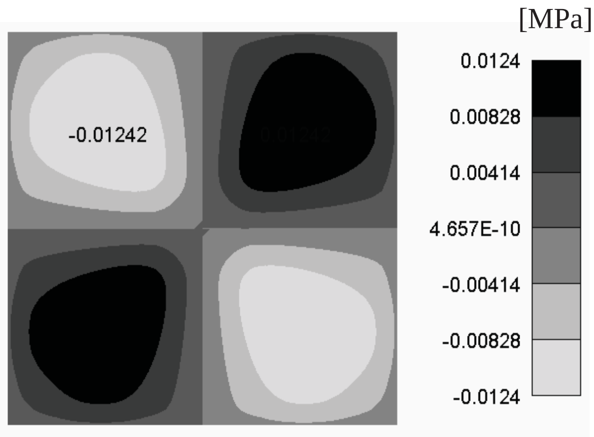


*Fig. 7. Tangential stresses on the bottom plate*

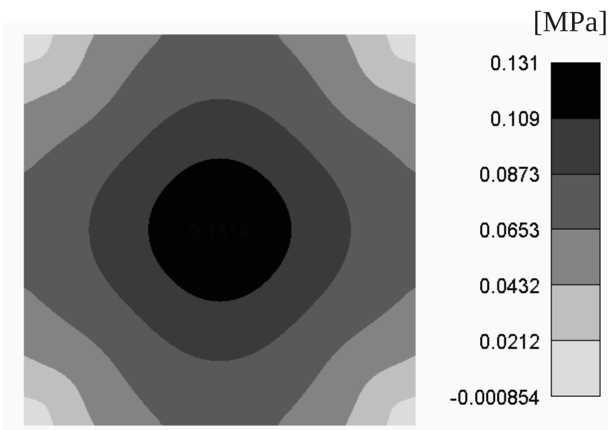


*Fig.8. The maximum principal stress on the bottom plate*

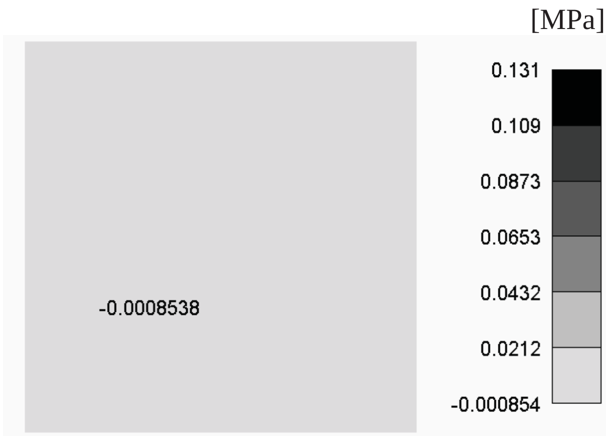


*Fig.9. The maximum principal stress on the top plate*

## 4. Model of 10m x 10m size concrete plate loaded by a uniform temperature field

The numerical analysis of concrete slabs 10 m x 10 m model, loaded by its own weight and uniform temperature field T =-30°C, was performed assuming that the friction coefficient between the plate and the ground is equal 1.3. Fig. 10 illustrates displacements contour in the normal direction to the surface of the plate. The maximum value of displacement in this direction is equal 0.269 mm. Fig. 11 shows the normal stresses distribution. The maximum value of stresses is equal 0.187 MPa and occurs in the middle of the plate bottom. The maximum value of shear stresses is located on the lower surface of the plate and is equal 21.2 kPa. Distribution of maximum principal stress - shown in Fig. 13 - is similar to the normal stress distribution.

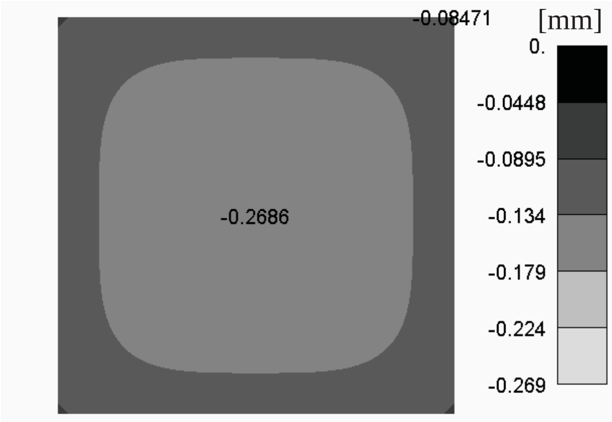


*Fig. 10. Displacements in the normal direction to the surface of the plate*

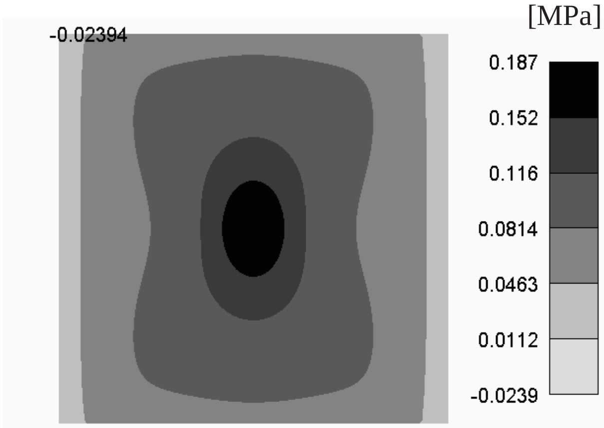


*Fig. 11. Normal stress on the bottom plate*

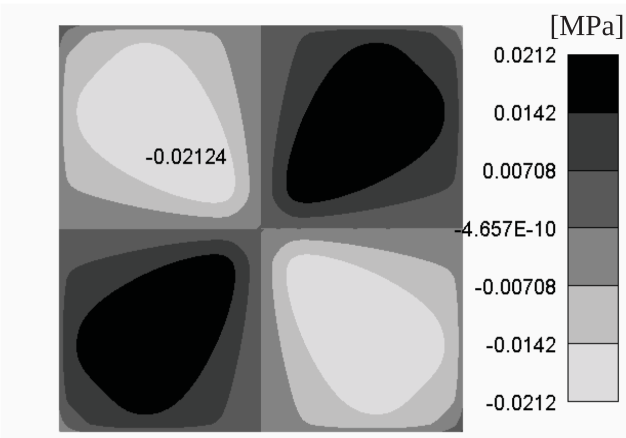


*Fig. 12. Tangential stresses on the bottom plate*

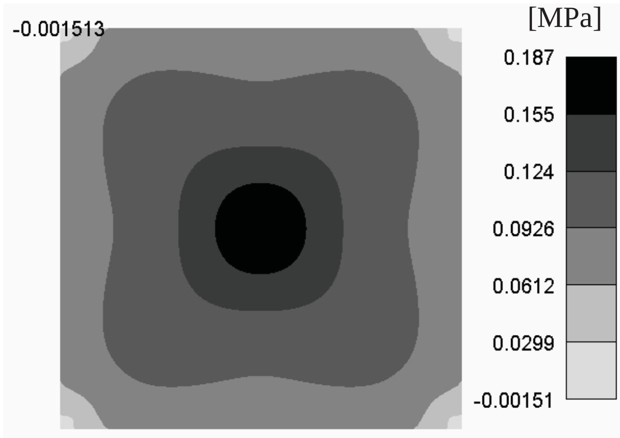


*Fig. 13. The maximum principal stress on the bottom plate*

## 5. Model of 20m x 20m size concrete plate loaded by a uniform temperature field

Using the two planes of symmetry, the quarter plate model was prepared. Constrains on the plate in the plane of symmetry provide a permanent location of the plate centre. Fig. 14-17 show the results of the analysis of concrete slabs 20 m x 20 m model, loaded by its own weight and uniform temperature field T =-30°C. In the analysis there was assumed the friction coefficient between the plate and the ground is equal 1. Fig. 14 illustrates displacements contour in the normal direction to the surface of the plate. The maximum value of displacement in this direction is equal 0.255 mm. Fig. 15 shows the normal stresses distribution. The maximum value of stress is equal 0.242 MPa and occurs in the middle of the plate. The maximum value of shear stresses is located on the lower surface of the plate and is equal 3.5 kPa (Fig. 16). The distribution of maximum principal stress - shown in Fig. 17 - is similar to the normal stress distribution. Maximum principal stress values are higher than in slabs of size 5 m x 5 m and 10 m x 10 m subjected to identical load conditions.

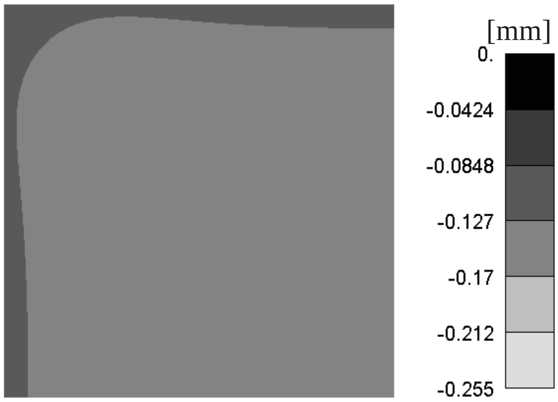


*Fig. 14. Displacements in the normal direction to the surface of the plate*

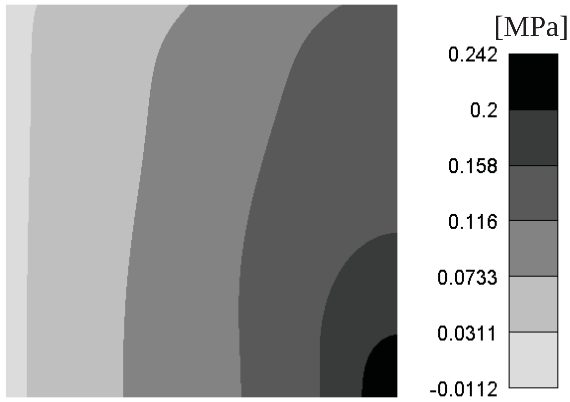


*Fig. 15. Normal stress on the plate bottom*

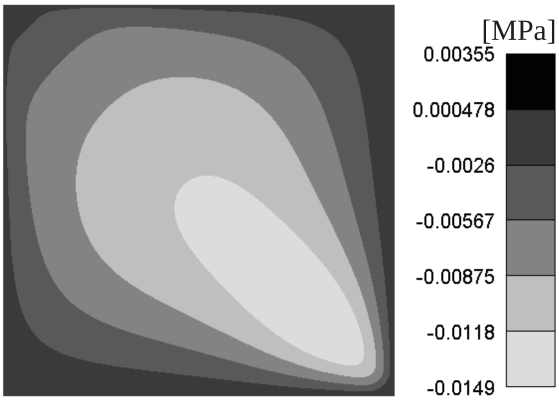


*Fig. 16. Tangential stresses on the plate bottom*

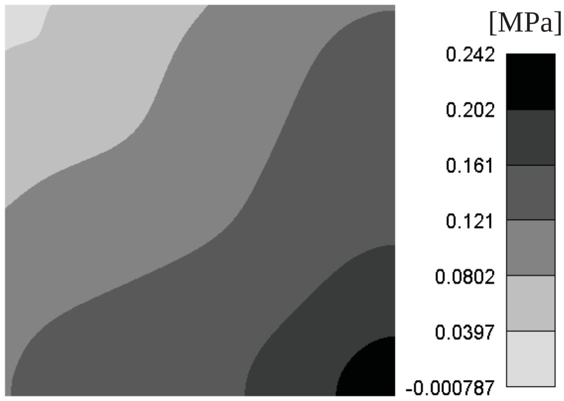


*Fig. 17. The maximum principal stress on the plate bottom*

## 6. Conclusion

*Tab. 1. Summary of maximum principal stress on the plate bottom*

| Maximum principal stress on the plate bottom [MPa] | | | | | | |
|---|---|---|---|---|---|---|
| load | | T=-30°C | | | T=-60°C | T=0→10°C |
| plate | | 5m x 5m | 10m x 10m | 20m x 20m | 20m x 20m | 20m x 20m |
| Friction coeff. | 0.5 | 0.07 | 0.075 | 0.124 | - | 2.00 |
| | 1 | 0.131 | 0.147 | 0.242 | 0.253 | 1.98 |
| | 1.3 | 0.165 | 0.187 | - | - | - |

Uniform changes in temperature –30°C and –60°C adopted in the analysis correspond to the values of contraction plates 0.3 mm/m and 0.6 mm/m. Reported in literature, the total shrinkage of concrete reaches 0.6 mm/m [5, 6]. As it can be seen from the values given in Tab. 1, the maximum principal stress values occurring in the middle of the bottom surface of the plate comes up to 0.25 MPa and, therefore, full-strength concrete plate in such loads will not be damaged. In reality, however, shrinkage occurs in the process of changes the strength and physical properties of concrete, what leads to a shrinkage cracks randomly distributed. In practice, regular contraction joints are produced by notching of hardening concrete to a depth of about 1/3 the thickness of plate, for example, in a grid of 5 m x 5 m. Studies show that in the case of protection of large-size plates from accidental shrinkage cracks, for example with the use of reinforcement, the stresses caused by the simultaneous change in temperature is not a major threat in slabs of large size.

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